Relationships between Fares, Trip Length and Market Competition

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ABSTRACT

This paper analyses equilibrium fares that arise from Collusion, Cournot, Stackelberg, Bertrand and Sequential Price Competition when two profit maximising transport firms produce symmetrically differentiable services and have identical costs. Special focus is placed on how different equilibrium fares are linked to trip length. Higher operator costs and higher demand from the authorities regarding the quality of transport supply result in steeper relationships (larger rate of change) between all fares and travel distance. Also, a higher degree of substitutability between the services will in most cases make these relationships steeper. The competitive situation has less influence on fares, both absolutely and relatively, the longer routes the operators compete on.

Keywords: Optimal fares, trip length, competitive situation.

1. INTRODUCTION

Throughout Europe and North America there has been a move in markets for public passenger transport towards a reduction in economic regulation (Banister et al., 1992). This has paved the way for increased competition between private companies to supply services both on subsidised contracts for the local government and commercially operated in the free-market. The deregulation has been the subject of many studies both with respect to privatization (Nash, 2005) and types of competitive contracts (e.g. Hensher and Stanley, 2003; Preston, 2005). The liberalization of the British bus industry has given examples of fierce competition (Nash, 1993). Also within the railway industry a scope for on-track competition has been identified with fare reductions as a feasible outcome (Preston et al., 1999). Preston et al. (1999) states that the competition within the rail industry is, much like coach and air transport, characterised by a higher potential for differentiation of products compared to that of the bus industry which to a larger extent provides a homogenous service.

On many transport routes, one, two or three suppliers are commonplace. Despite being competitors, such a limited number of suppliers of transport services can act in a collusive regime of some form when it comes to setting the price and/or the capacity. The strategic interactions between firms in such markets have been discussed theoretically by Pedersen (1999) and van Reeven (2003) and empirically found to be present e.g. in the Norwegian aviation industry (Salvanes et al., 2003) and in the British bus industry following the deregulation (e.g. Beesley, 1990). However, models of imperfect competition are scarcely used in order to analyse how the level of fare is related to travel distance or the length between the destinations.

Theoretical works the last five years have, admittedly, dealt with how a monopolist will design the relationship between fare and trip length when it: (1) maximises profits or a weighted combination of profits and consumer surplus and (2) under different assumptions regarding the relationship between transport users demand and their generalised travel cost, see Jørgensen and Pedersen (2004) and Jørgensen and Preston (2007). Later on these models are seen in the light of empirical studies from passenger transport in Norway (Mathisen, 2008b). Tsai et al. (2008) also develop a procedure in order to estimate simultaneously how fare, headway and transport quality on one hand should be related to trip length on the other hand, given one operator who wants to maximise profits. The latter analysis is carried out using an intercity transportation system as an example. Nevertheless, none of these works

analyse the relationship between optimal fare and travel distance when two or more rivals compete.

The aim of this paper is to analyse how fares are related to travel distance between locations when two profit maximising operators, producing symmetrically differentiated services, either collude or compete. In order to do so, we calculate equilibrium prices under Collusion, under quantity competition (Cournot and Stackelberg) and under price competition (Bertrand and Leader-Follower). The analysis is carried out under different degrees of substitutability in demand between the two transport services. Comparisons of the equilibrium solutions inform the transport authorities of the consequences of different regulatory policies for passengers travelling on routes of different lengths.

The structure of the article is as follows. In section 2, we present the model and emphasise its central assumptions. In section 3, we solve for the market equilibrium at both Collusion and simultaneous and sequential competition on both quantity and price. Section 4 provides the analysis where special focus is placed on deriving the link between equilibrium fares and trip length and implications of the equilibrium fare solutions on passengers' generalised travel costs. In section 5 we briefly relate the model results to empirical evidence. Lastly, in section 6, we make some concluding remarks and present the implications for policy makers.

2. THE MODEL

Let us assume that two transport firms provide their own version of a transport service at the given distance (D) between the two locations Y and Z, as illustrated in Figure 1. We want to analyse the changes in equilibrium prices under different competitive situations when the transport distance (D) between the locations changes. The degree of substitutability between the transport services depends on how equal the travellers perceive them to be; the more equal perceptions they have about them, the more substitutable are the services.

Insert Figure 1 about here.

Let us denote a representative transport user's generalised travel costs of using service 1 and service 2 by G_1 and G_2, respectively. G_i, where $i = \{1, 2\}$, is given by the sum of fare, P_i, and the time costs, $(b_0 + b_1 D)$, see for example Button (2010).

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(1) $\begin{aligned} G_1 &= P_1 + b_0 + b_1 D \\ G_2 &= P_2 + b_0 + b_1 D \end{aligned}$ where $b_0, b_1 > 0$

in which D denotes distance in km, b_0 distance-independent time costs and ($b_1 D$) time costs when travelling by the mode. The b_0 parameter depends on walking time, waiting time and time spent on boarding and alighting the mode (buses and trains) and/or transport time to airports (air transport), whilst the b_1 parameter denotes each passenger's time costs of travelling an extra km by the mode. The values of b_0 and b_1 are dependent on the quality of transport supply, the travellers' income and the purpose of the journey. A review of the influence of different quality factors in transport is, for example, given in Paulley et al. (2006).[1]

Note that we suppose that time costs of travelling with the two suppliers are the same and exogenous for them; i.e. we assume that the quality demands are set by the authorities. Hence, generalised travel costs using the two services can only differ through the fares P_1 and P_2. This means that our model applies to cases where two suppliers using the same type of mode compete, for example both offering bus transport, air transport etc. Since other factors than G_i such as the modes' departure- and arrival times, their safety records, travellers' habits and the firms' brand image also influence modal choice, the two services are not necessarily perfect substitutes, even though they offer the travellers the same G-values.

In the following we follow the model originally presented by Singh and Vives (1984) by assuming that a representative traveller has the following utility function based on the levels of the use of services supplied by firm 1, X_1, and firm 2, X_2

(2) $\qquad U(X_1, X_2) = X_1 + X_2 - \dfrac{(X_1)^2 + 2sX_1 X_2 + (X_2)^2}{2}$

where the parameter $s \in [0,1]$ measures the degree of substitutability between the services offered by the two operators. If $s = 1$ then the services are perfect substitutes and $s = 0$ is the

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[1] Assuming k is time costs per hour and h is the mode's speed measured in km per hour. Then $b_1 = k / h$. This suggests that b_1 is low (high) for fast (slow) modes. The b_0 parameter is in general high for air transport compared to other modes. The k-value and thereby the b_1-value will be lower the better the service and quality onboard.

case of independent markets; hence this parameter can be thought of as indicating the degree of competition between the firms in any specific market situation, see also comments below equation (5).

Assuming that the representative traveller maximises his consumer surplus, CS $(CS = U(X_1, X_2) - \sum_{i=1}^{2} G_i X_i)$, the utility function in (2) gives rise to the following linear and symmetric inverse demand functions

(3)
$$G_1 = 1 - X_1 - sX_2$$
$$G_2 = 1 - X_2 - sX_1$$

Since Singh and Vives (1984) focus on price (Bertrand) and quantity (Cournot) competition within a duopoly framework their model is relevant for the problem we aim to model. The clear-cut conclusions between Bertrand and Cournot which they made have, admittedly, been subject to critique for example in an *n*-firm specification where it is not evident which type of competition that is more efficient (Häckner, 2000). It has also been demonstrated that Bertrand could give higher prices than Cournot when the firms have goals that extend beyond profit maximisation (Clark et al., 2009) and even higher prices than monopoly when the spatial dimension is included (Sanner, 2007).

The expressions of generalised costs in (1) are inserted in the inverse demand function in (3) giving the following linear symmetric demand fare functions

(4)
$$P_1 + b_0 + b_1 D = 1 - X_1 - sX_2 => P_1 = (1 - b_0 - b_1 D) - X_1 - sX_2$$
$$P_2 + b_0 + b_1 D = 1 - X_2 - sX_1 => P_2 = (1 - b_0 - b_1 D) - X_2 - sX_1$$

From (4) follows that higher values of b_0 and b_1 due to poorer transport quality lead to negative shifts in these inverse demand functions. The direct demand functions derived from (4) are presented in (5)

(5)
$$X_1 = \frac{1}{1 - s^2}(1 + sb_0 - s - b_0 + b_1(s-1)D - P_1 + sP_2)$$
$$X_2 = \frac{1}{1 - s^2}(1 + sb_0 - s - b_0 + b_1(s-1)D - P_2 + sP_1)$$
for $1 > s \geq 0$

From (5) follows that $\frac{\partial X_i}{\partial P_i} = -\frac{1}{1-s^2}$ and $\frac{\partial X_i}{\partial P_k} = \frac{s}{1-s^2}$ implying that a marginal increase in own price has always higher influence, in absolute terms, on own demand than on the rival's demand; that is $\left|\frac{\partial X_i}{\partial P_i}\right| > \left|\frac{\partial X_i}{\partial P_k}\right|$ when $s < 1$. The difference between $\left|\frac{\partial X_i}{\partial P_i}\right|$ and $\left|\frac{\partial X_i}{\partial P_k}\right|$ decreases, however, when the firms compete more fiercely (s increases).[2] One can also calculate the price elasticities of demand as $\varepsilon_i = \frac{\partial X_i}{\partial p_i}\frac{p_i}{X_i} = \frac{-p_i}{(1-b_0-b_1D)(1-s)-p_i+sp_k}$. The effect that increasing s has on the elasticity is $\frac{\partial \varepsilon_i}{\partial s} = -p_i\frac{(1-b_0-b_1D)-p_2}{((1-b_0-b_1D)(1-s)-p_i+sp_k)^2} < 0$. Hence as products become more similar, the elasticity becomes more negative, and demand is more elastic. Lower values of s gives the providers the possibility of increasing fare without so much loss of demand.

Since we assume operators with the same type of mode compete and additionally simplify the analysis by assuming no variation in efficiency levels between them, we specify the cost for providing the transport services with the following identical linear functions

(6)
$$C_1(X_1, D) = a_0 + a_1 X_1 + a_2(X_1 D)$$
$$C_2(X_2, D) = a_0 + a_1 X_2 + a_2(X_2 D)$$
where $a_0, a_1, a_2 > 0$

These functions imply linear positive relationships between the number of passengers (X_i) and the number of passenger km (X_iD), $i = \{1, 2\}$. Linear cost functions in transport are often good proxies of more advanced functions (Pels and Rietveld, 2000), and the above functions in particular are supported from several empirical cost studies carried out for bus (Jørgensen and Preston, 2003) and ferry transport (Jørgensen et al., 2004; Mathisen, 2008a) in Norway. From (6) it follows that

(7) $\quad \frac{\partial C_i}{\partial X_i} = a_1 + a_2 D$, $i = \{1, 2\}$

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[2] An s-value of for example 0.2 (0.6) implies that $\frac{\partial X_i}{\partial P_i} = -1.04\ (-1.56)$ and $\frac{\partial X_i}{\partial P_k} = 0.21\ (0.93)$ meaning that an increase in own price by one unit will decrease own demand by 1.04 (1.56) units and increase the rival's demand by 0.21 (0.93) units.

Marginal costs increase linearly with trip length. The a_1 parameter can be interpreted as the distance-independent marginal costs, while a_2 is the costs for the transport firm of carrying a passenger an extra km.

The profit for each firm, π_i, are expressed in (8) using the demand functions in (3) and the cost functions in (5):

(8)
$$\pi_1 = ((1 - b_0 - b_1 D) - X_1 - sX_2)X_1 - (a_0 + a_1 X_1 + a_2 X_1 D)$$
$$\pi_2 = ((1 - b_0 - b_1 D) - X_2 - sX_1)X_2 - (a_0 + a_1 X_2 + a_2 X_2 D)$$

The profit functions can also be written in terms of fares using expression (5) in combination with (8). We then get:

(9)

$$\pi_1 = \frac{1}{s^2 - 1}(a_0 + a_1 - P_1 - sa_1 + sP_1 + P_1^2 - a_1 b_0 - a_1 P_1 + b_0 P_1 - s^2 a_0 + sa_1 b_0 + sa_1 P_2 - sb_0 P_1 - sP_1 P_2 +$$
$$(a_2 - Da_2 b_1 - sa_2 - a_1 b_1 - a_2 b_0 - a_2 P_1 + b_1 P_1 + sDa_2 b_1 + sa_2 b_1 + sa_1 b_1 + sa_2 b_0 + sa_2 P_2 - sb_1 P_1)D)$$

$$\pi_2 = \frac{1}{s^2 - 1}(a_0 + a_1 - P_2 - sa_1 + sP_2 + P_2^2 - a_1 b_0 - a_1 P_2 + b_0 P_2 - s^2 a_0 + sa_1 b_0 + sa_1 P_1 - sb_0 P_2 - sP_1 P_2 +$$
$$(a_2 - Da_2 b_1 - sa_2 - a_1 b_1 - a_2 b_0 - a_2 P_2 + b_1 P_2 + sDa_2 b_1 + sa_2 b_1 + sa_1 b_1 + sa_2 b_0 + sa_2 P_1 - sb_1 P_2)D)$$

for $1 > s \geq 0$.

3. MARKET SOLUTIONS FOR DIFFERENT COMPETITIVE SITUATIONS

The conditions for maximising the profit functions in (8) and (9) give us equilibrium quantities and fares. The equilibrium fare expressions will be deduced with the purpose of deriving how optimal fares are linked to operators' costs (a_1 and a_2), the quality of transport supply (b_0 and b_1), the degree of substitutability between the services (s) and finally the distance travelled by the mode (D). Focus will be directed towards the collusive case and the four traditional forms of market competition where the firms act either simultaneously or sequentially and compete in either quantity or price. The five situations Collusion, Cournot, Stackelberg, Bertrand and Price Leader-Follower are denoted COLL, C, ST, B and SP,

respectively. [3] For the sequential games ST and SP the leader and follower are given the subscripts 1 and 2, respectively. Equilibrium fare and quantity are marked by an asterisk. Hence, a total of seven different equilibrium fares and quantities will be deduced.

The profit expressions for each firm in equilibrium, π_1^{*j} and π_2^{*j} ($j = \{COLL, C, ST, B, SP\}$), are given in the Appendix. By inspecting the profit functions it can be verified that $\pi_i^{*j} > 0$ when $D < D^{Max,j}$, $\dfrac{\partial \pi_i^{*j}}{\partial D} < 0$ and $\dfrac{\partial^2 \pi_i^{*j}}{\partial D^2} > 0$, $i = \{1, 2\}$ where $D^{Max,j}$ represents the maximum distance that can be operated profitably by the transport companies. Hence, for any of the competitive situations the relationship between equilibrium profits and trip length for each firm is convexly decreasing so that long distances ($D > D^{Max,j}$) cannot be profitably covered by two operators.[4] Our further analysis applies for D-values resulting in positive profits for both operators.

3.1 Equilibrium fares and quantities

Simultaneous Quantity Competition (Cournot)

When the transport operators maximise their profits by choice of the quantity variable we get the following common equilibrium quantity, X^{*C}, and fare, P^{*C} using equation (8)

(10)
$$X^{*C} = \frac{1 - a_1 - b_0 - (b_1 + a_2)D}{2 + s}$$
$$P^{*C} = \frac{1 + a_1(1+s) - b_0 + (a_2(1+s) - b_1)D}{2 + s}$$

Sequential Quantity Competition (Stackelberg)

Let firm 1 be the leader choosing its quantity, first. Using equation (8) the following equilibrium quantities (X_1^{*ST}, X_2^{*ST}) and fares (P_1^{*ST}, P_2^{*ST}) are derived

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[3] These standard market models from the industrial organization literature (see e.g. Carlton and Perloff, 2005; Frank, 2010) are the shared monopoly (collusion or cartel), the simultaneous quantity competition (Cournot), the sequential quantity competition (Stackelberg), the simultaneous price competition (Bertrand) and the sequential price competition (Price Leader-Follower).

[4] It can be verified from the firms' profit expressions in equilibrium that maximum travel distance, $D^{Max,j}$, varies for different *j*; that is for different competitive situations. $D^{Max,j}$ is derived by solving D when $\pi_i^{*j} = 0$.

$$X_1^{*ST} = \frac{(2-s)(1-a_1-b_0-(a_2+b_1)D)}{2(2-s^2)}$$

$$X_2^{*ST} = \frac{(4-2s-s^2)(1-a_1-b_0-(a_2+b_1)D)}{4(2-s^2)}$$

(11) $\quad P_1^{*ST} = \frac{1}{4}(2a_1 - s - 2b_0 + sa_1 + sb_0 + 2 + (2a_2 - 2b_1 + sa_2 + sb_1)D)$

$$P_2^{*ST} = \frac{1}{4(2-s^2)}(4a_1 - 2s - 4b_0 + 2sa_1 + 2sb_0 - 3s^2a_1 + s^2b_0 - s^2 + 4$$

$$+ (4a_2 - 4b_1 + 2sa_2 + 2sb_1 - 3s^2a_2 + s^2b_1)D)$$

Simultaneous Fare Competition (Bertrand)

Maximising π_i ($i = \{1, 2\}$) in (9) gives the common equilibrium quantity, X^{*B}, and fare, P^{*B}, in simultaneous fare competition shown in (12).

(12)
$$X^{*B} = \frac{1}{2+s-s^2}(1-a_1-b_0-(a_2+b_1)D)$$

$$P^{*B} = \frac{1}{2-s}(1+a_1-s-b_0+sb_0+(a_2-b_1+sb_1)D)$$

Sequential Fare Competition

Let us assume that firm 1 sets price first. The derived equilibrium quantities (X_1^{*SP}, X_2^{*SP}) and fares (P_1^{*SP}, P_2^{*SP}) are then derived from (9) and shown in (13)

$$X_1^{*SP} = \frac{(s+2)}{4s+4}(1-a_1-b_0-(a_2+b_1)D)$$

$$X_2^{*SP} = \frac{s^2-4-2s}{4s^3+4s^2-8s-8}(1-a_1-b_0-(a_2+b_1)D)$$

(13) $\quad P_1^{*SP} = \frac{1}{2(s^2-2)}(s - 2a_1 + 2b_0 - sa_1 - sb_0 + s^2a_1 - g^2b_0 + s^2 - 2$

$$+ (2b_1 - 2a_2 - sa_2 - sb_1 + s^2a_2 - s^2b_1)D)$$

$$P_2^{*SP} = \frac{1}{4(s^2-2)}(2s - 4a_1 + 4b_0 - 2sa_1 - 2sb_0 + s^2a_1 - 3s^2b_0 + s^3a_1 + s^3b_0 + 3s^2 - s^3 - 4$$

$$- (4a_2 - 4b_1 + 2sa_2 + 2sb_1 - s^2a_2 + 3s^2b_1 - s^3a_2 + s^3b_1)D)$$

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Collusion

In this case the operators maximise $\pi = \pi_1 + \pi_2$ and we the get the following equilibrium quantity and fare for each operator

$$X^{*COLL} = \frac{1 - a_1 - b_0 - (b_1 + a_2)D}{2(1+s)}$$

(14)

$$P^{*COLL} = \frac{1 + a_1 - b_0 + (a_2 - b_1)D}{2}$$

Influence on fares of changes in a, b and s parameters

Using equations (10), (11), (12), (13) and (14) it can be determined that all equilibrium fares are increasing in the operators' costs (a_1 and a_2) and decreasing when the passengers' time costs (b_0 and b_1) increase. Consequently, higher demands from the authorities regarding the quality of the transport supply, causing lower values of b_0 and b_1, will increase fares. Furthermore, higher degree of substitutability between the firms' services (s increases) will reduce all fares except for the collusive case; fare is then independent of the value of s.

3.2 Ranking of Fares

All equilibrium fares are lower when the firms compete than when they collude. In summary, the following ranking can be verified when all parameter values are positive

$$P^{*B} < P_2^{*SP} < P_1^{*SP} < P_1^{*ST} < P_2^{*ST} < P^{*C} < P^{COLL}$$

Bertrand competition yields lowest fares and Collusion highest fares when $s > 0$. This ranking corresponds with the results found in ordinary textbooks dealing with duopoly models, profit maximising entities and linear cost- and demand functions, see for example Frank (2010). Using this ranking and making pair wise comparison of fares gives a total of 21 fare gaps[5]. Common for all of these is that they will all be reduced when the firms' cost (a_0 and a_1) and

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[5] Seven different fares imply $(7 \cdot 6) / 2 = 21$ possible fare differences.

trip length (D) increase. Also, lower demands regarding transport quality resulting in increased passengers' time cost (b_0 and b_1) will reduce the fare differences.

More substitutable services (s increases), will increase the differences between the Collusion fare and all other equilibrium fares. Generally speaking, the differences in equilibrium fares under quantity competition on one hand and price competition on the other hand increase in s. The difference between the Stackelberg follower's fare and leader's fare, ($P_2^{*ST} - P_1^{*ST}$), increases (decreases) when $s < (>) 0.73$. Less intense competition, i.e. a sufficiently low s, gives the leader the freedom to increase fare even more above the following rival firm; this is due to the fact that prices decisions as s falls, have less effect on the rival's quantity. Also the gaps between the Bertrand fare and the equilibrium fares under sequential fare competition ($P_i^{*SP} - P^{*B}$), $i = \{1, 2\}$ are ambiguous and depend on the magnitude of s; they are larger (smaller) for the leader when $s < (>) 0.73$ and for the follower for $s < (>) 0.83$. The difference between the fare leader's price and the follower's price, ($P_1^{*SP} - P_2^{*SP}$) also increases when $s < 0.73$. Summing up, an increase in s will make the gaps in equilibrium fares higher providing that $s < 0.73$, i.e. that competition between the rivals is not too intense. When s is low, the demand for trips is less sensitive to fare changes, giving the scope for this result.

4. TRAVEL DISTANCE AND FARES - FURTHER ANALYSIS

4.1 Conditions for Increasing Fares with Distance

The conditions for increasing fares with distance for Cournot, Stackelberg, Bertrand, Sequential fare setting and Collusion can be found using equations (10), (11), (12), (13) and (14), respectively. From (10) we can deduce the following conditions at Cournot competition:

$$(15) \quad \frac{\partial P^{*C}}{\partial D} = \frac{1}{2+s}(a_2(1+s) - b_1) \implies \frac{\partial P^{*C}}{\partial D} \geq (<) \text{ when } \frac{a_2}{b_1} \geq (<)\frac{1}{1+s}$$

The derivative of P^{*C} with respect to distance (D) shows that the function is monotonic and will be either positive or negative depending on the parameter values a_2, b_1 and s. The higher the costs of transporting a passenger an extra km (a_2), the lower the extra time costs for each passenger of being transported an extra km (b_1) and the more substitutable services the

operators produce the more likely it is that fare increases with travel distance. When $s = 0.5$, for example, the fare increases with distance provided that the $a_2 / b_1 > 2/3$.

Under Stackelberg competition we get the following conditions using equation (11)

$$
\begin{aligned}
(16) \quad & \frac{\partial P_1^{*ST}}{\partial D} = \frac{1}{4}(2(a_2 - b_1) + s(a_2 + b_1)) \Rightarrow \frac{\partial P_1^{*ST}}{\partial D} \geq (<) \, 0 \text{ when } \frac{a_2}{b_1} \geq (<) \frac{2-s}{2+s} \\
& \frac{\partial P_2^{*ST}}{\partial D} = \frac{1}{4} \frac{b_1(s^2 + 2s - 4) + a_2(2s - 3s^2 + 4)}{2 - s^2} \\
& \Rightarrow \frac{\partial P_2^{*ST}}{\partial D} \geq (<) \, 0 \text{ when } \frac{a_2}{b_1} \geq (<) \frac{4 - 2s - s^2}{4 + 2s - 3s^2}
\end{aligned}
$$

Also for the Stackelberg case, the parameter values a_2, b_1 and s determine whether both leader's and follower's fares increase or decrease in distance. It can easily be worked out from the conditions under (16) that the threshold values of the a_2 / b_1 ratios resulting in positive relationships between both leader's and follower's fares and travel distance decrease when the firms compete more intensely; that is when s increases. If for, example, $s = 0.5$, $\frac{\partial P_1^{*ST}}{\partial D} > 0$

and $\frac{\partial P_2^{*ST}}{\partial D} > 0$ when the a_2 / b_1 ratio is higher than 0.60 and 0.65, respectively.

Under Bertrand competition the variation of common fares with respect to distance is derived by the differentiation of equation (12)

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(17) \quad \frac{\partial P^{*B}}{\partial D} = \frac{1}{2 - s}(a_2 - b_1 + sb_1) \Rightarrow \frac{\partial P^{*B}}{\partial D} \geq (<) \, 0 \text{ when } \frac{a_2}{b_1} \geq (<) 1 - s
$$

From (17) follows that a sufficient, but not necessary, condition for increasing fare with respect to trip length under simultaneous price competition is that $a_2/b_1 > 1$. If, for example, $s = 0.5$ then a_2 / b_1 must be higher than 0.5.

Furthermore, the following conditions can be worked out for sequential fare setting using equation (13)

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$$\frac{\partial P_1^{*SP}}{\partial D} = \frac{1}{2} \frac{a_2(s+1)(s-2) + b_1(s-2)(1-s)}{s^2 - 2}$$

$$\Rightarrow \frac{\partial P_1^{*SP}}{\partial D} \geq (<) 0 \text{ when } \frac{a_2}{b_1} \geq (<) \frac{(s+2)(1-s)}{(s+1)(2-s)}$$

(18)

$$\frac{\partial P_2^{*SP}}{\partial D} = \frac{1}{4} \frac{-4a_2 + 4b_1 - 2sa_2 - 2sb_1 + s^2a_2 - 3s^2b_1 + s^3a_2 + s^3b_1}{s^2 - 2}$$

$$\Rightarrow \frac{\partial P_2^{*SP}}{\partial D} \geq (<) 0 \text{ when } \frac{a_2}{b_1} \geq (<) \frac{4 + s^3 - 3s^2 - 2s}{2s - s^2 - s^3 + 4}$$

Equation (18) implies that the conditions for increasing fares with respect to distance also for sequential price setting are determined by a_2, b_1 and s. It can be verified that the higher the degree of substitutability between the services (increasing s) the more likely it is that equilibrium prices increase with trip length. If for, example, $s = 0.5$, $\dfrac{\partial P_1^{*SP}}{\partial D} > 0$ and $\dfrac{\partial P_2^{*SP}}{\partial D} > 0$ when the a_2 / b_1 ratio is higher than 0.56 and 0.51, respectively.

Finally, the expression in (14) for the collusive fare leads to:

(19) $\qquad \dfrac{\partial P^{*COLL}}{\partial D} = \dfrac{1}{2}(a_2 - b_1) \quad \Rightarrow \quad \dfrac{\partial P^{*COLL}}{\partial D} \geq (<) 0 \text{ where } \dfrac{a_2}{b_1} \geq (<) 1$

In the collusive case equilibrium fare increases with trip length when the operators' costs of transporting a passenger an extra km (a_2) exceed each passengers time cost of being transported an extra km (b_1). For a more thorough discussion of the latter, we refer to Jørgensen and Preston (2007).

The above conditions are summarized in Figure 2 where the horizontal axis ranges from 0 (independent services) to 1 (perfect substitutes) and the vertical axis follows the a_2/b_1 ratio, scaled from 0 to 1. Parameter combinations above the seven solid curves indicate increasing fares with respect to distance.

When $a_2/b_1 > 1$ all forms of competition give increasing fares with distance. Opposite, combinations of a_2/b_1 and s below the Bertrand curve result in decreasing fares in distance. Except for the collusive case, Figure 2 shows that the necessary value of a_2/b_1 resulting in increasing fares with travel distance decreases in s. Under low degree of substitutability

between the services ($s < 0.1$), the threshold levels of a_2/b_1 are broadly speaking the same for all situations where the firms compete. When the firms' services are more substitutable as measured by s moving towards 1, there is, however, a clear division between the curves related to competition in prices in the lower part of Figure 2 and competition in quantities in the upper part. The curves follow the same ranking for the value of a_2/b_1 from top to bottom in Figure 2 for all values of $s > 0$. The threshold levels of a_2/b_1 implying increasing fares with distance for all s under Cournot competition and Stackelberg competition are 1/2 and 1/3, respectively.

Summing up, even though empirical studies show that fares usually increase with travel distance (see section 5), the model's results do not give rise to such an unambiguous conclusion, having the original restrictions imposed on the a_2, b_1 and s parameters in mind. For certain combinations of a_2/b_1 and s values, it follows also from Figure 2 that the Stackelberg leader's fare may increase in distance whilst the follower's fare may decrease in distance. The same can be the case for the fare follower and the fare leader.

Insert Figure 2 about here.

4.2 Ranking of the Equilibrium Fares Regarding their Dependence on Trip Length

The differentiations of the fare functions with respect to distance under different forms of competition given in equations (15), (16), (17) (18) and (19) enable us to derive the following unambiguous results when $s > 0$:

$$(20) \quad \frac{\partial P^{*COLL}}{\partial D} < \frac{\partial P^{*C}}{\partial D} < \frac{\partial P_2^{*ST}}{\partial D} < \frac{\partial P_1^{*ST}}{\partial D} < \frac{\partial P_1^{*SP}}{\partial D} < \frac{\partial P_2^{*SP}}{\partial D} < \frac{\partial P_i^{*B}}{\partial D}$$

The interpretation of the above derivatives must be seen in relation to Figure 2. Let us first focus on a_2 and b_1 combinations such that $a_2/b_1 > 1$ which is the area above the Collusion curve in Figure 2. All fares are then increasing in distance. Then we can conclude unambiguously that trip length influences fares least under Collusion and most under Bertrand competition.

When we have combinations of a_2/b_1 and s below the Bertrand curve in Figure 2, all competitive situations give the unusual results that fares decrease when trip length increases.

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We then get the opposite conclusions regarding the importance of the competitive situation on the relationships between fares and travel distance (D); changes in trip length have a considerably lower impact on fares under price competition than under quantity competition and Collusion.

4.3 The Influence of Parameter Values

It has been demonstrated in the previous sections that, for all competitive situations, the influence that the trip length exerts on equilibrium fares depends on the three parameters a_2, b_1 and s. From equations (15), (16), (17), (18) and (19) it follows that $\dfrac{\partial(\partial P^{*j}/\partial D)}{\partial a_2} > 0$, $j =$ {C, ST, B, SP, COLL} meaning that when the costs of transporting a passenger an extra km increase, the relationships between all equilibrium fares and trip length become steeper when fares increase in trip length and less steep when fares decrease in trip length. Furthermore, lower demands from the authorities regarding the quality of transport, which subsequently increase the time costs for the passengers of travelling an extra km (b_1), have the opposite impact; that is $\dfrac{\partial(\partial P^{*j}/\partial D)}{\partial b_1} < 0$. In words: when the quality of transport increases (b_1 decreases), travel distance will influence all fares by more (less) when fares increase (decrease) in distance. Finally, it can be shown from the $\dfrac{\partial P^{*j}}{\partial D}$ expressions that $\dfrac{\partial(\partial P^{*j}/\partial D)}{\partial s} > 0$, $j =$ {C, ST, B, SP}; that is except for the collusive case. More intense competition between the firms (increasing s) results in steeper (less steep) relationships between all fares and travel distance when fares increase (decrease) in distance.

Even though the ranking of the derivatives in (20) is independent of a_2, b_1 and s, the magnitudes of their differences are; it can easily be determined that the values of all the 21 derivative differences are increasing in a_2 and b_1. Provided that s is not very high ($s < 0.83$), it can be verified that increasing s will also make the differences between the derivatives larger.[6]

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[6] The differences between the derivatives for (1) Stackelberg leader's and follower's fares and (2) between the fares under price competition increase provided that $s < 0.83$. This s-value represents a direct price derivative and a cross derivative of about -3.21 and -2.67, respectively (see equation (5)). For all other cases the differences in the derivatives increase unambiguously in s.

4.4 Summarised Model Results for Fare and Distance

The most important conclusions above are summarised in Figure 3 and Figure 4. Figure 3 describes the most common situation, namely that all fares increase in distance. This applies to all combinations of a_2 and b_1 such that $a_2/b_1 > 1$. Figure 3 shows that the differences between all equilibrium fares, both relatively and absolutely decrease when trip length (D) increases. Increasing a_2 and decreasing b_1 make all relationships shifting upwards and being steeper. Except for the collusive case higher degree of substitutability between the services (higher s) makes all curves shifting downwards but being steeper. From section 3.2 follows too that all curves get closer to each other when a_2 and b_1 increase. An increase in s has an ambiguous effect, making some curves move closer together, while others do not.

Insert Figure 3 about here.

Insert Figure 4 about here.

Figure 4 illustrates the more rare case, namely that fares decrease in distance. Opposite to the situation in Figure 3, increasing a_2 and s and decreasing b_1 make all curves less steep when the firms compete.

4.5 Influence on Traveller's Generalised Cost

Inserting the equilibrium fares in equation (1) shows that for all competitive situations equilibrium generalised travel costs (G^{*j}, $j = \{$C, ST, B, SP, COLL$\}$) will increase when operators costs (a_1 and a_2), travellers' time costs (b_0 and b_1) and trip length (D) increase. If we disregard the collusive case, increasing competition between the operators will decrease generalised travel costs. In summary, we get the following:

$$\frac{\partial G^{*j}}{\partial s} < 0, j = \{\text{C, ST, B, SP}\}, \quad \frac{\partial G^{*j}}{\partial a_1}, \frac{\partial G^{*j}}{\partial a_2}, \frac{\partial G^{*j}}{\partial b_0}, \frac{\partial G^{*j}}{\partial b_1}, \frac{\partial G^{*j}}{\partial D} > 0, j = \{\text{C, ST, B, SP, COLL}\}$$

The signs of $\dfrac{\partial G^{*j}}{\partial b_0}$ and $\dfrac{\partial G^{*j}}{\partial b_1}$ imply that higher demands from the authorities regarding the quality of the transport supply give lower generalised travel costs; the negative effect for the travellers of higher fares when b_0 and b_1 decrease do not outweigh the positive effect of

reduced time costs. For all competitive situations and for all combinations of a_2, b_1 and s values we can conclude unambiguously that generalised travel costs will increase with trip length. A closer look at these derivatives shows that when the firms compete, the magnitude of $\dfrac{\partial G^{*j}}{\partial b_0}$ depend on s only whilst $\dfrac{\partial G^{*j}}{\partial b_1}$ depend on s and D in the following ways

$$\partial(\frac{\partial G^{*j}}{\partial b_0})/\partial s\,,\ \partial(\frac{\partial G^{*j}}{\partial b_i})/\partial s > 0\,,\ \partial(\frac{\partial G^{*j}}{\partial b_0})/\partial D = 0\,,\ \partial(\frac{\partial G^{*j}}{\partial b_1})/\partial D > 0\,,\ j = \{C,\ ST,\ B,\ SP\}$$

Higher demands regarding transport quality (b_0 and b_1 decrease) have, thus, a larger positive effect on travellers' well-being the more alike the perception of the services (s increases) and the longer they travel (D increases). In the collusive case it is easily inferred that $\dfrac{\partial G^{*COLL}}{\partial b_0} = \dfrac{1}{2}$ and $\dfrac{\partial G^{*COLL}}{\partial b_1} = \dfrac{1}{2}D$.

Note that the above results rest upon the assumption that that higher quality demand from the authorities will boost the operators' productivity such that their costs are held constant. This assumption is not unreasonable; in particular for subsidised and/or publically own transport firms X-inefficiency may be present leading to scope for productivity improvements (Button, 2010).[7] If operators' cost increase when quality is raised, the effect on G^{*j} is ambiguous. [8]

5. MODEL RESULTS SEEN IN THE LIGHT OF PREVIOUS NORWEGIAN STUDIES

In the work carried out by Jørgensen and Preston (2007) parameter values for a_2 and b_1 for the year 2002 are worked out using earlier empirical studies concerning bus transport and car ferry transport in Norway. For bus and ferry, respectively, they estimated that the parameter

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[7] Jørgensen et al. (1997) estimated, for example, the average inefficiency in the regulated Norwegian bus industry in 1991 to be between 7 % and 14 %.

[8] When the firms collude ($j = \{COLL\}$), Jørgensen and Pedersen (2004) conclude that generalised travel costs (G^*) will decrease (increase) as the quality (Q) improves when the marginal costs of serving passengers increase less (more) than travellers reductions in generalised costs of the service improvement; that is when $\dfrac{\partial^2 C}{\partial X \partial Q} < (>) - \dfrac{\partial G}{\partial Q}$.

values for a_2 are 1.70 NOK[9] and 4.79 NOK and for b_1 are 1.00 NOK and 6.35 NOK. Consequently, the fraction a_2/b_1 is about 1.70 for bus transport and 0.75 for ferry transport.

When relating these findings to Figure 2 we can conclude that bus operators will design a fare system implying increasing fares in distance for all forms of competition and for all degrees of substitutability. For ferry operators, however, fare will decrease with the length of the service when they collude. If they compete and the degree of substitutability between the services is sufficiently high ($s > 0.4$), Figure 2 shows that also ferry fares will be positively related to distance. If the competition between the operators is moderate such that $s < 0.3$, fares will decrease with distance for all forms of competition. When the ferry operators offer their services between the same destinations and therefore probably use the same quays, the variation in these services is low both respect to route choice and ferry size. It is, therefore, likely that the services provided by two operators are highly substitutable meaning that the s-value is high so that fares increase with distance.

Norwegian studies that review the relationships between ordinary fares and travel distance for subsidised domestic bus, ferry, train and air transport in Norway, show close positive relationships for all these modes (Mathisen, 2008b). Bearing in mind that every Norwegian ferry operator meets low competition from other operators such that s is low, the above observations seem to be in conflict with the model's conclusions as far as ferry transport is concerned. In that respect it is, however, worth noting that our model assumes profit maximising operators whilst the ferry fares in Norway are set by the central authorities and they have goals that extend beyond profit maximization; for example the well-being of the travellers. It can be deduced that the more weight a single operator (monopolist) places on consumer surplus compared to profits, the positive relationship between fares and trip length will be steeper and the increase in actual fare with distance will approach the marginal costs for the operator of transporting a passenger (car) an extra km (a_2). Using the above mentioned a_2 and b_1 figures for buses and ferries, it can be derived that fares for bus transport always increase with distance irrespective of the weight the bus operator places on profit versus consumer surplus. For ferry transport, fares will increase with distance provided that the

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[9] 1€ ≈ 8 Norwegian kroner (NOK). Note that for ferry transport one passenger car is used as the numeraire.

single company puts less than four times higher weight on profit than on consumer surplus.[10] A thorough discussion of this matter is given in Jørgensen and Preston (2007).

Historical fares on unregulated air transport between central destinations in Norway support the model's results in the sense that the collusive case gives higher prices than any of the other equilibrium prices above. A survey carried out by Amundsveen (2004) shows, for example, that full fare prices between Oslo and Bodø (802 km) and between Oslo and Tromsø (1115 km) set by the only operator Scandinavian Airlines Systems (SAS)[11] prior to 2002 were about 20 % lower one year after it met competition from the entrant low cost air company Norwegian Air Shuttle (NAS). A random check (26th August 2009) on the two companies full fare price offers on these two routes shows that fares increase in distance. For SAS the full fare prices were 2593 NOK and 2793 NOK between Oslo and Bodø and between Oslo and Tromsø, respectively. Similar figures for NAS were 1800 NOK and 1999 NOK. The market share for SAS, measured in seat kms offered, is about 67 % between Oslo and Bodø and about 62 % between Oslo and Tromsø (Norwegian Air Shuttle, 2009).

SAS has, thus, both higher transport production and higher full fare prices than NAS on these two services. These results seen in combination with the models' results above indicate that neither Stackelberg competition nor sequential fare competition are present on these routes. Stackelberg competition implies namely that the leader has highest fare but lowest quantity whilst sequential fare competition implies that the leader has lowest price and highest quantity; that is $P_1^{*ST} > P_2^{*ST}, X_1^{*ST} < X_2^{*ST}, P_1^{*SP} < P_2^{*SP}, X_1^{*SP} > X_2^{*SP}$. The absence of both Stackelberg and sequential fare competition in the Norwegian airline industry is supported from a study of the competitive situation in the industry after its deregulation in 1994, see Salvanes et al. (2003). Their study indicates that the air companies had a semicollusive behaviour; that is they collude in fares and compete in capacities. It is also worth noting that our model assumes that both firms have identical cost structure. This is not true for as far as

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[10] According to Jørgensen and Preston (2007) fares increase with distance if $(a_2 - \tau b_1) > 0 \Rightarrow \dfrac{a_2}{b_1} > \tau$ in which

τ is a function of the weight put on profit α compared to consumer surplus $(1-\alpha)$, $0.5 \le \alpha \le 1$, defined by $\tau = (2\alpha - 1)/\alpha$. $\alpha = 1$ and $\alpha = 0.5$ represent a pure profit maximizing firm and a firm aiming to maximise social surplus, respectively. For ferries $(a_2 - \tau b_1) > 0$ implies $\tau < 0.75$, $\alpha < 0.80$ and $(1-\alpha) > 0.2$.

[11] The Norwegian subdivision of Scandinavian Airlines System (SAS) bought the other dominant Norwegian air transport company, Braathen, in 2001 and the two firms cooperated until 2005 when they merged and took the name SAS Braathen. The name was changed back to SAS in 2007.

SAS and NAS are concerned; NAS is regarded as a low cost company and is run more cost efficiently than SAS.

6. CONCLUDING REMARKS

The main purpose of this paper has been to analyse how the equilibrium fares that arise from Collusion, Cournot, Stackelberg, Bertrand and Price leader-follower competition are linked to: (1) transport operators' costs; (2) the quality of transport supply; (3) the degree of substitutability between the transport services and (4) the trip length. The analysis is carried out by assuming profit maximising transport operators that have identical costs and produce symmetrically substitutable services. Moreover, the quality of transport supply is set by public authorities and, thus, exogenous for the transport operators.

The analysis shows, as expected, that all equilibrium fares increase when the operators' costs increase and when the authorities demand a higher quality of transport supply. If we disregard the Collusion case, more intense competition between the transport firms will decrease fares. The Collusion fare is globally the highest, whilst Cournot competition yields highest fares and Bertrand competition lowest fares when the firms compete. The internal ranking of the equilibrium fares is independent of the firms' costs, the quality of their transport services, the degree of substitutability between their services and trip length. These factors influence, however, the magnitudes of the differences in price values. Less cost efficient firms, lower demands regarding transport quality and longer trip lengths will reduce the differences between all equilibrium fares. On the other hand, more substitutable services (higher value of s) will increase fare gaps across different market situations; the gap between fares under quantity competition and fare competition is especially affected.

A more in-depth look at the relationships between equilibrium fares and trip length shows that all competitive situations imply increasing relationships between fares and travel distance when the costs for the operators of transporting a passenger an extra km exceed each passenger's time costs of travelling another km; that is when $(a_2 - b_1) > 0$ or $(a_2 / b_1) > 1$. When $(a_2 - b_1) < 0$, fares are decreasing in distance when the firms collude. Also when the firms compete, fares may decrease with trip length when the competition between them is fairly low. Moreover, fares that increase with trip length are more likely under fare (price)

competition than under quantity competition. It is, thus, worth noting that the model's results do not rule out that fares may decrease in distance when both the a_2 / b_1 ratio and s value are sufficiently low. Hence, the probability for decreasing fares with distance raises if a transport mode is slow and holds passengers with high time costs (high b_1 value), differs significantly from the alternatives (s low) and has low costs of transporting passengers an extra km (a_2 low). Since empirical studies from Norway show positive relationships between fares and travel distance both for regulated and unregulated transport services, such combinations of the a_2, b_1 and s parameters are probably rare.

Given a_2, b_1 and s values such that all equilibrium fares increase in distance, increasing costs of transporting a passenger an extra km and higher demands regarding transport quality, result in steeper relationships between all equilibrium fares and trip length. Except for the collusive case, all fares increase faster according to trip length the more intense the firms compete. Fares under quantity competition are less dependent on trip length than under price competition implying that all equilibrium fares get closer as travel distance increases. Increasing a_2, b_1 and s values also result in how much travel distance influences equilibrium fares.

The above relationships between all equilibrium fares and travel distance result in positive relationships between generalised travel cost and trip length for all positive values of a_2, b_1 and s. Since all aspects with transport quality captured in generalised travel costs are equal for both operators, ranking of these costs are in accordance with the fare rankings; that is Collusion gives highest generalised costs and Bertrand lowest generalised costs etc. Given that higher demands from the authorities regarding transport quality not will increase the operators' costs but improve their productivity[12], we can conclude unambiguously that generalised travel costs will be reduced. Hence, such transport policy initiatives will benefit the transport users.

To sum up, the competitive situation between transport firms has higher importance on fares and thereby on passengers' generalised travel costs, the more productive the firms are, the higher demands the authorities set regarding the quality of transport supply and, broadly

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[12] It is not unreasonable to believe that increased quality demands from the authorities will give the operators incentives to boost their efficiency.

speaking, the more substitutable services they produce. This suggests, for example, that it is more important for the transport users that collusion is hindered when operators' productivity is high, passengers' time costs are low (fast speed modes) and competition is high. Since passengers' time costs in general have increased over time due to rises in income, the way the transport firms compete has become more important for the transport users. Another important finding is that the competitive situation exerts more influence on fares, both absolutely and relatively, on shorter trips than on longer trips. Transport regulators should, thus, focus most on the organisation of transport supply on short routes. Higher productivity among the transport firms, higher degree of substitutability between their services and lower demands regarding transport quality makes the latter recommendation even more relevant.

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APPENDIX

The profit functions for the different types of competition with corresponding first- and second order derivatives with respect to distance are presented in this appendix. The positive restrictions on the equilibrium quantities imply that $(a_1 + b_0 + Da_2 + Db_1 - 1) < 0$.

Cournot competition (using eq. (10) in eq. (9))

$$\pi^{*C} = \frac{1}{(s+2)^2}(-a_0 s - 4a_0 s + D^2 a_2^2 + 2D^2 a_2 b_1 + D^2 b_1^2 + 2Da_1 a_2 + 2Da_1 b_1 + 2Da_2 b_0 - 2Da_2$$
$$+ 2Db_0 b_1 - 2Db_1 + a_1^2 + 2a_1 b_0 - 2a_1 + b_0^2 - 2b_0 - 4a_0 + 1)$$

$$\frac{\partial \pi^{*C}}{\partial D} = 2\frac{a_2 + b_1}{(s+2)^2}(a_1 + b_0 + Da_2 + Db_1 - 1) < 0$$

$$\frac{\partial^2 \pi^{*C}}{\partial D^2} = 2\frac{(a_2 + b_1)^2}{(s+2)^2} > 0$$

Stackelberg competition (using eq. (11) in eq. (9))

Leader's profit:

$$\pi_1^{*ST} = -\frac{1}{8s^2 - 16}(s^2 a_1^2 - 16a_0 - 8a_1 - 8b_0 - 4s + s^2 b_0^2 + 8sa_1 + 8sb_0 + 4a_1^2 + 4b_0^2 + 4D^2 a_2^2 + 4D^2 b_1^2$$
$$- 8Da_2 - 8Db_1 + 8a_1 b_0 + 8s^2 a_0 - 4sa_1^2 - 2s^2 a_1 - 4sb_0^2 - 2s^2 b_0 + s^2 + 2s^2 a_1 b_0 + 8D^2 a_2 b_1 - 4sD^2 a_2^2$$
$$- 4sD^2 b_1^2 + 8sDa_2 + 8sDb_1 - 8sa_1 b_0 + 8Da_1 a_2 + 8Da_1 b_1 + 8Da_2 b_0 + 8Db_0 b_1 + s^2 D^2 a_2^2 + s^2 D^2 b_1^2$$
$$- 2s^2 Da_2 - 2s^2 Db_1 + 2s^2 Da_1 a_2 + 2s^2 Da_1 b_1 + 2s^2 Da_2 b_0 - 8sD^2 a_2 b_1 + 2s^2 Db_0 b_1 + 2s^2 Da_2 b_1 - 8sDa_1 a_2$$
$$- 8sDa_1 a_2 - 8sDa_1 b_0 - 8sDb_0 b_1 + 4)$$

$$\frac{\partial \pi_1^{*ST}}{\partial D} = -\frac{a_2 + b_1}{4s^2 - 8}(s-2)^2(a_1 + b_0 + Da_2 + Db_1 - 1) < 0$$

$$\frac{\partial^2 \pi_1^{*ST}}{\partial D^2} = -\frac{(a_2 + b_1)^2}{4s^2 - 8}(s-2)^2 > 0$$

Follower's profit:

$$\pi_2^{*ST} = \frac{1}{16s^4 - 64s^2 + 64}(4s^3 a_1^2 - 64a_0 - 32a_1 - 32b_0 - 4s^2 a_1^2 - 4s^2 b_0^2 - 16s + 4s^3 b_0^2 + s^4 a_1^2 + s^4 b_0^2$$

$$+ 32sa_1 + 32sb_0 + 16a_1^2 + 16b_0^2 + 16D^2 a_2^2 + 16D^2 b_1^2 - 32Da_2 - 32b_1 + 32a_1 b_0 + 64s^2 a_0 - 16sa_1^2$$

$$+ 8s^2 a_1 - 16sb_0^2 + 8s^2 b_0 - 8s^3 a_1 - 16s^4 a_0 - 8s^3 b_0 - 2s^4 a_1 - 2s^4 b_0 - 4s^2 + 4s^3 + s^4 - 8s^2 a_1 b_0$$

$$+ 8s^3 a_1 b_0 + 2s^4 a_1 b_0 + 32D^2 a_2 b_1 - 16sD^2 a_2^2 - 16sD^2 b_1^2 + 32sDa_2 + 32sDb_1 - 32sa_1 b_0 + 32Da_1 a_2$$

$$+ 32Da_1 b_1 + 32Da_2 b_0 + 32Db_0 b_1 - 4s^2 D^2 a_2^2 - 4s^2 D^2 b_1^2 + 4s^3 D^2 a_2^2 + 4s^3 D^2 b_1^2 + s^4 D^2 a_2^2 + s^4 D^2 b_1^2$$

$$+ 8s^2 Da_2 + 8s^2 Db_1 - 8s^3 Da_2 - 8s^3 Db_1 - 2s^4 Da_2 - 2s^4 Db_1 - 8s^2 Da_1 a_2 - 8s^2 Da_1 b_1 - 8s^2 Da_2 b_0$$

$$+ 8s^3 Da_1 a_2 - 32sD^2 a_2 b_1 - 8s^2 Db_0 b_1 + 8s^3 Da_1 b_1 + 8s^3 Da_2 b_0 + 2s^4 Da_1 a_2 + 8s^3 Db_0 b_1 + 2s^4 Da_1 b_1$$

$$+ 2s^4 Da_2 b_0 + 2s^4 Db_0 b_1 - 8s^2 D^2 a_2 b_1 + 8s^3 D^2 a_2 b_1 + 2s^4 D^2 a_2 b_1 - 32sDa_1 a_2 - 32sDa_1 b_1 - 32sDa_2 b_0$$

$$- 32sDb_0 b_1 + 16)$$

$$\frac{\partial \pi_2^{*ST}}{\partial D} = \frac{a_2 + b_1}{8s^4 - 32s^2 + 32}(s^2 + 2s - 4)^2 (a_1 + b_0 + Da_2 + Db_1 - 1) < 0$$

$$\frac{\partial^2 \pi_2^{*ST}}{\partial D^2} = \frac{(a_2 + b_1)^2}{8s^4 - 32s^2 + 32}(s^2 + 2s - 4)^2 > 0$$

Bertrand competition (using eq. (12) in eq. (9))

$$\pi^{*B} = -\frac{1}{(s+1)(s-2)^2}(a_0 s^3 - 3a_0 s + sD^2 a_2^2 + 2sD^2 a_2 b_1 + sD^2 b_1^2 + 2sDa_1 a_2 + 2sDa_1 b_1$$

$$+ 2sDa_2 b_0 - 2sDa_2 + 2sDb_0 b_1 - 2sDb_1 + sa_1^2 + 2sa_1 b_0 - 2sa_1 + sb_0^2 - 2sb_0 + s$$

$$- D^2 a_2^2 - 2D^2 a_2 b_1 - D^2 b_1^2 - 2Da_1 a_2 - 2Da_1 b_1 - 2Da_2 b_0 + 2Da_2 - 2Db_0 b_1 + 2Db_1 - a_1^2$$

$$- 2a_1 b_0 + 2a_1 - b_0^2 + 2b_0 + 4a_0 - 1)$$

$$\frac{\partial \pi^{*B}}{\partial D} = -2(a_2 + b_1)\frac{s-1}{(s+1)(s-2)^2}(a_1 + b_0 + Da_2 + Db_1 - 1) < 0$$

$$\frac{\partial^2 \pi^{*B}}{\partial D^2} = -2(a_2 + b_1)^2 \frac{s-1}{(s+1)(s-2)^2} > 0$$

Sequential fare competition (using eq. (13) in eq. (9))

Leader's profit

$$\pi_1^{*SP} = \frac{1}{8s^3 + 8s^2 - 16s - 16}(16a_0 + 8a_1 + 8b_0 + 3s^2 a_1^2 + 3s^2 b_0^2 + s^3 a_1^2 + s^3 b_0^2 + 16sa_0 - 4a_1^2 - 4b_0^2$$
$$- 4D^2 a_2^2 - 4D^2 b_1^2 + 8Da_2 + 8Db_1 - 8a_1 b_0 - 8s^2 a_0 - 6s^2 a_1 - 8s^3 a_0 - 6s^2 b_0 - 2s^3 a_1 - 2s^3 b_0 + 3s^2 + s^3$$
$$+ 6s^2 a_1 b_0 + 2s^3 a_1 b_0 - 8D^2 a_2 b_1 - 8Da_1 a_2 - 8Da_1 b_1 - 8Da_2 b_0 - 8Db_0 b_1 + 3s^2 D^2 a_2^2 + 3s^2 D^2 b_1^2$$
$$+ s^3 D^2 a_2^2 + s^3 D^2 a_1^2 - 6s^2 Da_2 - 6s^2 Db_1 - 2s^3 Da_2 - 2s^3 Db_1 + 6s^2 Da_1 a_2 + 6s^2 Da_1 b_1 + 6s^2 Da_2 b_0$$
$$+ 2s^3 Da_1 a_2 + 6s^3 Db_0 b_1 + 2s^3 Da_1 b_1 + 2s^3 Da_2 b_0 + 2s^3 Db_0 b_1 + 6s^2 D^2 a_2 b_1 + 2s^3 D^2 a_2 b_1 - 4)$$

$$\frac{\partial \pi_1^{*SP}}{\partial D} = (a_2 + b_1)(s - 1)\frac{(s+2)^2}{4s^3 + 4s^2 - 8s - 8}(a_1 + b_0 + Da_2 + Db_1 - 1) < 0$$

$$\frac{\partial^2 \pi_1^{*SP}}{\partial D^2} = (a_2 + b_1)^2(s - 1)\frac{(s+2)^2}{4s^3 + 4s^2 - 8s - 8} > 0$$

Follower's profit

$$\pi_2^{*SP} = -\frac{1}{16s^5 + 16s^4 - 64s^3 - 64s^2 + 64s + 64}(64a_0 + 32a_1 + 32b_0 + 20s^2 a_1^2 + 20s^2 b_0^2 - 5s^4 a_1^2$$
$$- 5s^4 b_0^2 + s^5 a_1^2 + s^5 b_0^2 + 64sa_0 - 16a_1^2 - 16b_0^2 - 16D^2 a_2^2 - 16D^2 b_1^2 + 32Da_2 + 32Db_1 - 32a_1 b_0 - 64s^2 a_0$$
$$- 40s^2 a_1 - 64s^3 a_0 - 40s^2 b_0 + 16s^4 a_0 + 10s^4 a_1 + 16s^5 a_0 + 10s^4 b_0 - 2s^5 a_1 - 2s^5 b_0 + 20s^2 - 5s^4 + s^5$$
$$+ 40s^2 a_1 b_0 - 10s^4 a_1 b_0 + 2s^5 a_1 b_0 - 32D^2 a_2 b_1 - 32Da_1 a_2 - 32Da_1 b_1 - 32Da_2 b_0 - 32b_0 b_1 + 20s^2 D^2 a_2^2$$
$$+ 20s^2 D^2 b_1^2 - 5s^4 D^2 a_2^2 - 5s^4 D^2 b_1^2 + s^5 D^2 a_2^2 + s^5 D^2 b_1^2 - 40s^2 Da_2 - 40s^2 Db_1 + 10s^4 Da_2 + 10s^4 Db_1$$
$$- 2s^5 Da_2 - 2s^5 Db_1 + 40s^2 Da_1 a_2 + 40s^2 Da_1 b_1 + 40s^2 Da_2 b_0 + 40s^2 Db_0 b_1 - 10s^4 Da_1 a_2 - 10s^4 Da_1 b_1$$
$$- 10s^4 Da_2 b_0 + 2s^5 Da_1 a_2 - 10s^4 Db_0 b_1 + 2s^5 Da_1 b_1 + 2s^5 Da_2 b_0 + 2s^5 Db_0 b_1 + 40s^2 D^2 a_2 b_1 - 10s^4 D^2 a_2 b_1$$
$$+ 2s^5 D^2 a_2 b_1 - 16)$$

$$\frac{\partial \pi_2^{*SP}}{\partial D} = -(a_2 + b_1)(s - 1)\frac{(-s^2 + 2s + 4)^2}{8s^5 + 8s^4 - 32s^3 - 32s^2 + 32s + 32}(a_1 + b_0 + Da_2 + Db_1 - 1) < 0$$

$$\frac{\partial^2 \pi_2^{*SP}}{\partial D^2} = -(a_2 + b_1)^2(s - 1)\frac{(-s^2 + 2s + 4)^2}{8s^5 + 8s^4 - 32s^3 - 32s^2 + 32s + 32} > 0$$

Collusion (using eq. (14) in eq. (9))

$$\pi^{*COLL} = \frac{1}{4s+4}(D^2 a_2^2 + 2D^2 a_2 b_1 + D^2 b_1^2 + 2Da_1 a_2 + 2Da_1 b_1 + 2Da_2 b_0 - 2Da_2 + 2Db_0 b_1$$

$$- 2Db_1 + a_1^2 + 2a_1 b_0 - 2a_1 + b_0^2 - 2b_0 - 4a_0 - 4sa_0 + 1)$$

$$\frac{\partial \pi^{*COLL}}{\partial D} = \frac{1}{2s+2}(a_2 + b_1)(a_1 + b_0 + Da_2 + Db_1 - 1) < 0$$

$$\frac{\partial^2 \pi^{*COLL}}{\partial D^2} = \frac{1}{2s+2}(a_2 + b_1)^2 > 0$$

Figure Captions

Figure 1: Substitutable transport services over the distance D between the locations Y and Z.

Figure 2: Conditions for increasing fares with respect to distance, dP^*/d$D > 0$, for substitutable transport services under different forms of competition.

Figure 3: The relationships between equilibrium fares and travel distance when fares increase in distance.

Figure 4: The relationships between equilibrium fares and travel distance when fares decrease in distance.
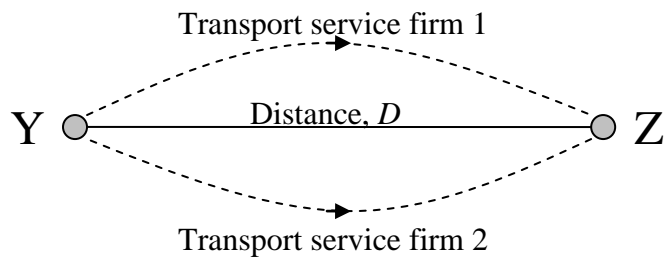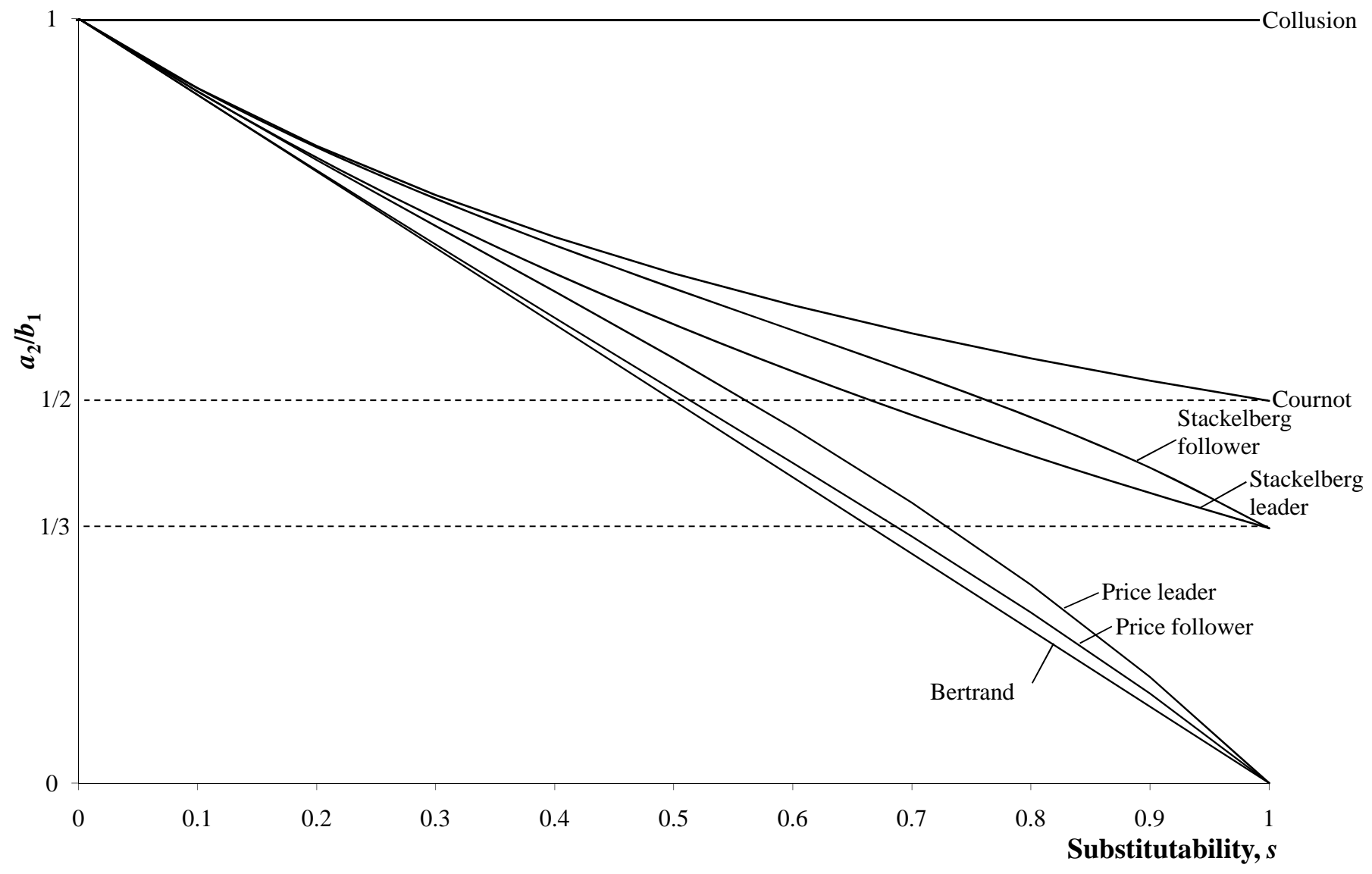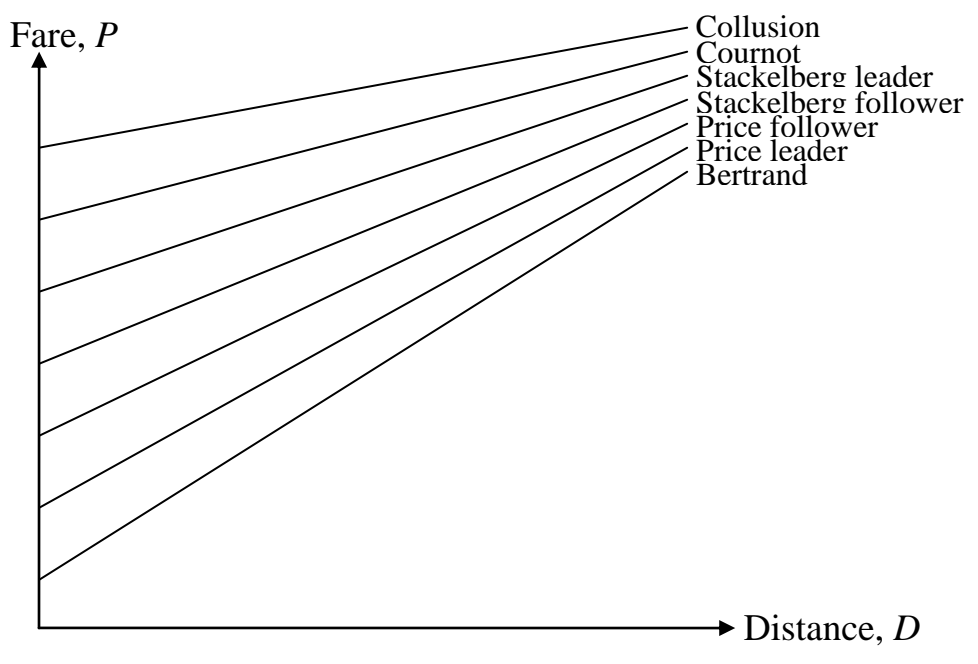
Figure 1

Transport service firm 1

Distance, *D*

Y   Z

Transport service firm 2

Figure 2

Figure 3

Figure 4

Fare, *P*

Collusion
Cournot
Stackelberg leader
Stackelberg follower
Price follower
Price leader
Bertrand

Distance, *D*