

MASTER THESIS

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A liquidity-adjusted capital asset pricing model (CAPM) for Norwegian stocks market

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Abstract

This paper is about a liquidity-adjusted capital asset pricing model. This model was applied to Norwegian stock market. Liquidity measure is the bid-ask spread, and to measure it, it was chosen two estimators Corwin and Shultz (2002) and Roll (1984) estimators, because of its simple use. Also, it was used two-factor model, and built regression against market and liquidity.

Applying regressions it was found out that added liquidity measure did not give any significant results. Same regressions were used to the whole sample of 582 Norwegian companies and to the most 72 liquid companies. R-squared for liquid was higher than for the whole sample, but not as high as it could be expected.

Also it was decided to check some random element inside of each regression, and in some of them was quite high relation between liquidity factor and the market return. It could be explained by poor downloaded information from Titlon or wrongly chosen liquidity estimators.

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Introduction

There are a lot of researches about stocks and its returns. And it is not surprising, because stock is one of the most common securities. Investors interested in the price of security and in its liquidity. Liquidity is the measure, which shows how fast stock could transfer into cash without any losses. There are several measures of liquidity, which could be used in research.

Main asset pricing model is capital asset pricing model (CAPM) - a traditional model, which could show return of each stock or the whole stock portfolio. It consists of the risk-free rate and the excess return (from market return deducted risk-free rate). To summarize, it is interesting to take a look, how liquidity could change the return of the stock. It is possible to adjust, or add, liquidity measure directly to CAPM as one more index.

So, it was decided to take Norwegian stock market and apply this adjusted CAPM. To do this, it is needed to go through relevant theory and literature. This paper has 4 chapters. First one consists of main definitions, which related to stocks, liquidity and CAPM. Second – literature review. This part is very important in any type of research, because it is needed to find out relevant articles with model, which could be applied in this paper. Through literature review is important to confirm the relevance of the topic and choose models, in my case measures of liquidity, to work with. Third part is about data – where and how could it be calculated and to be useful in analyzing. Fourth part is results, which were received after applying models, programs.

In this paper the object is Norwegian stocks, and the subject – its characteristics, especially returns and liquidity. Liquidity measures by two estimators – Corwin and Shultz (2002) estimator and Roll (1984) estimator. These estimators apply daily data of high and low prices.

The main source of data collection is Titlon, which could provide with any type of stock characteristics. Also, there are no limits on number of companies, so it was taken about 582 companies. Year limit is 15 years – from 2000 until 2015 year. Stocks were analyzed by a two-factor model – market and liquidity, using regression models and applying RStudio.

After getting results of low relations in regressions with liquidity, it was decided to cut quite high amount of companies and work with 72, which are the most liquid according to “Trading economics” recourse.

Chapter 1. Theory

I would like to start from theory and introduce some relevant explanations and definitions, which are related to stock market, liquidity and CAPM. Here is not the full information, but just some introduction to each topic very shortly.

1.1. Asset classes and financial instruments

In this part I would like to write shortly about asset classes and financial instruments.

There are two main segments of financial markets (Mishkin, 2013):

- money markets;
- capital markets.

A financial market is a market that gathers buyers and sellers together to trade in financial assets such as stocks, bonds, commodities, derivatives and currencies. The aim of a financial market is to set prices for worldwide trade, increase capital and relocate liquidity and risk. A financial market can be divided into money markets and capital markets.

Money markets are used for short-term assets up to one year. So, *capital markets* are used for long-term assets obviously, and its maturity is more than one year. Capital markets consist of the equity market and debt market. Both of markets – money and market and capital market – are used to measure level of liquidity and risk for individual, companies and governments.

Capital markets are widely bigger than the money market. Both the stock and bond markets are commonly followed and their daily changes are analyzed as proxies for the general economic condition of the global markets. As a result, the institutions, which are in capital markets - stock exchanges, commercial banks and all types of companies, including such as insurance companies and mortgage banks - are examined in details.

The institutions dealing in the capital markets access them to increase capital for long-term goals, such as for a merger or acquisition, to widespread a line of business or enter into a new business, or for other capital projects. Organizations, which are increasing money for these long-term aims come to one or more capital markets. In the debt market, corporations may issue debt in the form of corporate bonds, and local and federal governments - government bonds. In the same way corporations can make a decision to increase money by issuing equity

on the stock market. Government organizations are typically privately held and, therefore, do not usually use equity. Companies and government organizations that issue equity or debt are considered the sellers in these markets.

The buyers buy security from the seller and trade it. If the stock or the bond is placed on the market for the first time, the market called – the primary market. If the stock or the bond was already placed to the market and is traded among buyers, so this market is called the secondary market. The sellers make money off the sale not in the secondary market, but in the primary, but they have a share in the pricing of the stock or the bond in the secondary market.

In the capital market the investors tends to use fund for long-term investments. Capital markets are not riskless markets and are not often used to invest short-term funds. Many buyers come to the capital markets to save for retirement or education, as long as the buyers have long time horizons - they are risk-takers and not so old.

The money market is an opposite market to the capital market. For investors this market is some kind of a ‘storage’ of funds, which have maturity shorter than one year, while in the capital market investors take more risk and need to have patience to invest in this market. Main instruments in money market are acceptance, bills of exchange, deposits and collateral loans. Organizations, which operate in money markets, are central banks, acceptance houses and commercial banks.

Money markets provide a wide range of functions for corporate, individual and government organizations. The main aim for entrance money market is liquidity. The money market occupies a main role in ensuring companies and governments support the needed level of liquidity on a daily basis, without decreasing short and needing a more expensive loan or without holding excess funds and missing the opportunity of setting up interest on funds.

Investors, on the other hand, entrance to the money markets to invest in a safe mode. Conversely to capital markets, money markets are thought about low risk; risk-averse investors are willing to approach them with the expectation that liquidity is readily available.

Some main differences and similarities between capital and money markets:

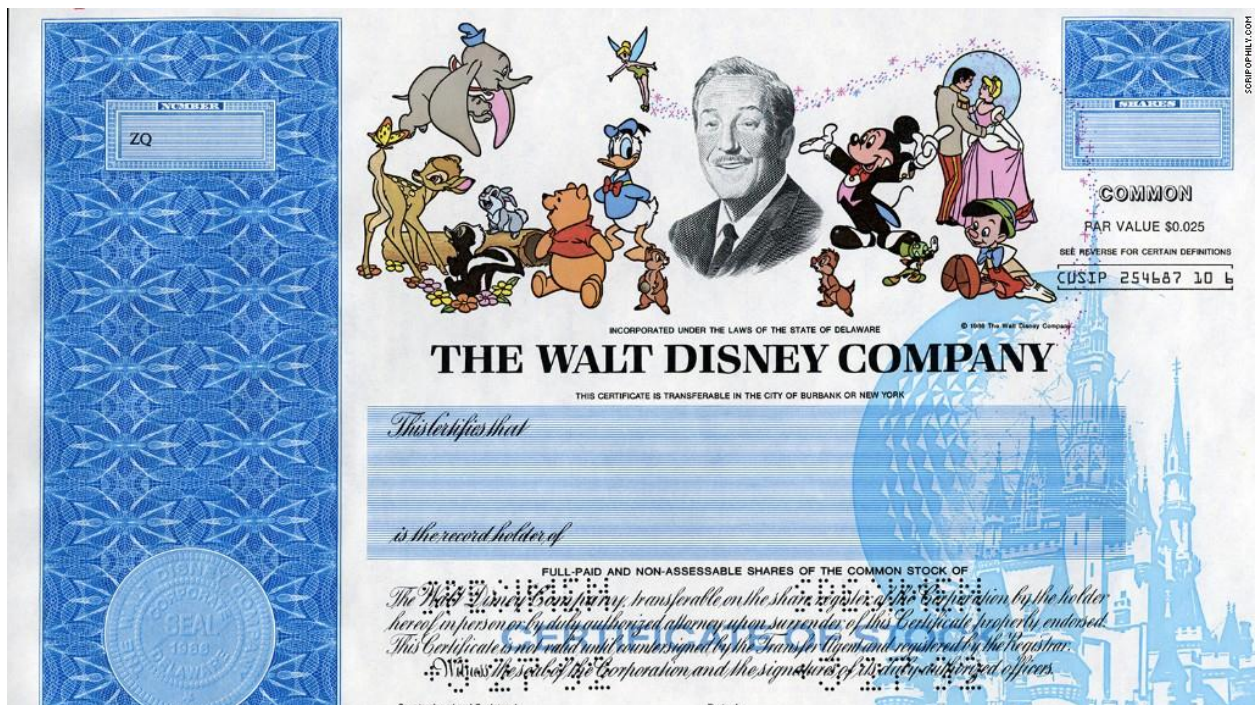
- Main function for both markets – support appropriate levels of funding;

- Main purpose for seller – depends on liquidity and time horizon.
- Risk – capital markets – high, money markets – low,
- Returns – capital market – high, money markets – low but steady.

1.2. Stock market

Stock is a security, which divide ownership of the company in shares. Stock shows companies assets and earnings, and as more stocks investor has, as more his ownership becomes greater.

Several years ago stocks were presented by certificate. But nowadays all information is based in electronic version, which usually keeps by brokerage.



Pic. 1 Example of stock certificate.

For shareholder there is no need to go to work in particular firm, which stocks he has. To increase value of companies stocks is a goal of managers, who work there. Also shareholder can easily sell his shares, because he keeps it in electronic way.

One more important point about stock is liabilities, which relies to a person, who keeps it. Shareholder is not personally liable, if company can not pay debts.

There are two main types of stocks: common stock and preferred stock.

Common Stock

- represent ownership in a company and a claim (dividends) on a portion of profits;
- investors get one vote per share to elect the board members, who oversee the major decisions made by management;
- shareholders - not receive money until the creditors, bondholders and preferred shareholders are paid;
- variable dividends that are never guaranteed.

Preferred stock

- represents some degree of ownership in a company ;
- investors are usually guaranteed a fixed dividend forever.
- in the event of liquidation - are paid off before the common shareholder .
- preferred stock may also be callable - the option to purchase the shares from shareholders at anytime for any reason (usually for a premium).

And moreover, it is important to tell about difference of the primary market and the secondary market. The primary market is the market, where securities are created, but in the secondary market investors trade already existed securities.

1.3. Stock exchange

A stock exchange - legal entity, which is responsible for the regular functioning of the organized commodity market, currencies, securities and derivatives. Trade is conducted in standard contracts or lots, which regulate the size of the regulatory stock exchange documents.

Previously, the stock exchange was called the place or building, where are collected at certain times people trade and intermediaries, stockbrokers for transactions in the securities or commodities.

Before the era of computerization of transactions the parties agreed orally. Now most of the bids are held in electronic form with the use of specialized software. Brokers in their own interests or the interests of clients are portrayed in commercial systems applications for the purchase or sale of securities (currency, commodity). These requests can be accommodated counter orders of other traders. Stock exchange keeps track of executed transactions,

implements, organizes and ensures calculations (clearing), provides a mechanism for the interaction of "delivery versus payment".

Usually stock exchange receives a commission on each prisoner deal with them, and it is the main source of exchange earnings. Other sources may be membership fees, fees for access to trading, sale of market data.

1.4. Liquidity

Liquidity describes how easy assets can be converted to cash. One of the most liquid assets is known to be cash because it can be used immediately in all the cases. And the least liquid asset is considered to be real estate because it may take from several weeks to several months to sell it.

When the investment is done it is always important to keep in mind the liquidity level of the asset because it can be very difficult to convert the asset back to cash again.

Instead of selling assets, we can get cash by borrowing against them. This transaction can be done between two people or, what is more common, through the bank. The bank has in reservations so much cash from several depositors at the same time so it can easily face the needs of any borrower.

What is more, if a depositor needs cash immediately, he can just withdraw it from the bank instead of going to the borrower and demanding the whole payment. Besides, banks are acting as financial intermediaries between borrowers and lenders, and it allows a smooth flow of money while meeting the needs of both sides.

When talking about market, liquidity has a slightly different meaning, although still tied to how easy assets can be converted to cash. The market for a stock can be considered liquid if shares can be sold fast and the process of selling does not have or have a little impact on the stock's price. Usually it translates to where these shares are sold and the level of interest that investors have in the company. Company stock can normally be considered liquid if it is traded on the major exchanges. It is common that precisely 1% of the float trades hands daily, and that indicates a high level of interest in the stock. At the same time company stock that is

traded on the pink sheets or over the counter often happens to be non-liquid with only few (maybe even zero) shares traded daily.

Also it is possible to look at bid-ask spread to judge company's stock liquidity. For liquid stocks, such as Microsoft or General Electric, the spread is often just a few pennies - much less than 1% of the price. For illiquid stocks, the spread can be much larger, amounting to a few percent of the trading price.

One thing to mention here for investors that are placing an order is the liquidity of the stock. A good price that investor need can be found during normal market hours on the major exchanges if a limit order will be placed. This statement is particularly true for non-liquid companies or during after-hours trading when only few traders are acting; so far it is better to place a limit order because the lower liquidity may result in the price the investor is not ready to pay.

1.5. CAPM

The expected return of the security is related to the beta (riskiness) of the asset. The capital asset pricing model (CAPM) is the traditional model, which measures risk and return.

The expected return of an asset could be written as (Damodaran, 2012):

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f],$$

where $E(R_i)$ – expected return on asset i , R_f – risk-free rate, $E(R_m)$ – expected return of the market, β_i – beta of an asset i .

Main inputs of CAPM:

- the riskless asset – the expected return of which is known by investor with certainty for the time horizon of the analysis;
- the risk premium – premium of the marker portfolio with all risky assets, exclude a rickless asset;
- the beta – covariance, which divided by a market portfolio, and measures a risk.

This model provides useful measure of risk, which could help investors estimate what return they would get.

Table 1.4.1 Drawbacks and advantages of CAPM

Drawbacks	Advantages
Risk-free rate (R_f) is the measure on short-term government securities. It changes daily, creating volatility.	Ease-of-use: CAPM is a simply calculated model which could derive a range of possible results to provide confidence around the required rates of return.
Return on the market (R_m) is the sum of the capital gains and dividends for the market. It could be negative, and returns are backward-looking.	Diversified portfolio – investors would like to hold such kind of portfolio with unsystematic risk.
Ability to borrow at a risk-free rate: CAPM is built on four major assumptions: <ul style="list-style-type: none"> • investors could borrow and lend at a risk-free rate, • individual investors could not borrow at the rate the US government. 	Systematic risk (beta) is an important variable because it is hardly predictable because it is often not fully expected.
Determination of project proxy beta is necessary to find proxy for it, because it is needed to be reflective to the investment.	Business and financial risk variability, because financing needs to calculate required return, while other models could not do.

It was necessary to take a quick look at some important definitions, which relate with the topic. More information would be found out in literature review part. There could be possible to find different view on the same topics of different authors, understand how to measure liquidity, understand stock behavior in market relations and how return and risk-free rate could adjust security.

Chapter 2. Literature review

My paper is about liquidity adjusted capital asset pricing model. This topic is quite interesting not only nowadays, but also for several years – to be more specific, researchers are interested in it for more than 30 years. And it is not surprised, because liquidity and stock always come together, because investor makes decision to buy stock, when it is liquid, in other case he is not interested in it.

I decide to go through different articles, which are relevant for my topic to take closer look on different views on such issue. I prefer not to mix this articles with different point of view, I would like to write about them article-by-article and make common conclusion about some relevant points.

2.1. Market and main definitions review

Market microstructure: a survey (Madhavan, 2000). As I can see from the topic, the main idea of this article is to study market microstructure. Authors of the paper divided it into main topics:

- price formation,
- market structure and design,
- transparency,
- applications to other areas of finance.

Authors made literature review and made some link to related article of Coughenour and Shastri (1999), who provide a detailed summary of empirical studies in four select areas:

- the estimation of the components of the bid-ask spread,
- order flow properties,
- the Nasdaq controversy,
- linkages between option and stock markets.

I will come back to this article later to give some comments about first part, which interested me the most – the estimation of the components of the bid-ask spread.

Ananth Madhavan writes that ‘A central idea in the theory of market microstructure is that asset prices need not equal full-information expectations of value because of a variety of frictions’. Investments and theory of marker microstructure are closely related to each other.

In this article I interested in topics, which directly related to mine, so I skipped some part and went straightly to the bid-ask spread. Madhavan looks to the determinants of the bid-ask spread and gives some explanations – ‘Market makers quote two prices: the bid price, at which they will buy securities and the ask price, at which they will sell. The difference between the bid and the ask price is the market maker's spread’. Also he gives links to Demsetz (1968), who said that the market maker provides a service of directness in an organized exchange market, for which the bid-ask spread – return under competition. The role of market maker is not active – just simply adjusting the bid-ask spread in response to changing conditions. To measure determinants of the bid-ask spread author suggests to use a cross-sectional regression equation:

$$S_i = \beta_0 + \beta_1 \ln(M_i) + \beta_2 \left(\frac{1}{p_i}\right) + \beta_3 \sigma_i + \beta_4 \ln(V_i) + \varepsilon_i, \quad (2.1)$$

where, for of security i , s_i is the average (percentage) bid-ask spread modeled as a function of independent variables: log market capitalization (firm size), M_i , price inverse, $1/p_i$, the riskiness of the security measured by the volatility of past returns σ_i , and a proxy for activity such as log trading volume, V_i . To explain the variability in the bid-ask spread, appear volume, risk, price and firm size. The coefficient of volume is commonly not positive, since dealers can achieve quicker turnaround in stock lowering their potential liquidation costs and reducing their risk. Spreads are wider for riskier securities.

Also Madhavan talks about asymmetric information, which relates to dealer’s behavior. Author links to paper of Jack Treynor (1971), who suggested the difference between liquidity influenced traders, who possess no special advantages of information and informed traders with private information.

Author suggests Glosten and Milgrom’s (1985) model – formulation, which says about the idea that the provider of liquidity quotes price conditional on the directional of the trade – an ask price for a buy order and a bid price – a sell order:

$$p_t^{ask} = E[v_t | x_t = 1] = v^H \Pr[\theta = i | x_t = 1] + \bar{v}_t \Pr[\theta = u | x_t = 1], \quad (2.2)$$

Thus, the combination of public information includes all information at time t including knowledge of the trade itself. Assuming symmetry, the bid-ask spread is

$$p_t^{ask} - p_t^{bid} = \omega\sigma, \quad (2.3)$$

which is increasing in information asymmetry ω and in the degree of asset value uncertainty σ .

Other relevant topic – estimation of price formation, which covering in article. An expression for price is depends on the expected value of asset and the dealer's inventory.

$$p_t^{ask} = E[v_t | x_t = 1] - \phi(I_t - I^*) + s = \bar{v}_t + \left(\frac{\omega\sigma}{2}\right) - \phi(I_t - I^*) + s, \quad (2.4)$$

$$p_t^{bid} = E[v_t | x_t = -1] - \phi(I_t - I^*) - s = \bar{v}_t - \left(\frac{\omega\sigma}{2}\right) - \phi(I_t - I^*) - s, \quad (2.5)$$

Last topic, which interesting for me in this article is liquidity and expected returns. Some researches give definition of modeled expected return as functions of variables, which include proxies for size and default risk. And author links to Amihud and Mendelson (1986), who show that expected returns are a decreasing function of liquidity. Simply they suppose that economy's agents are risk-neutral and the risk-free rate is r_f . Under risk-neutrality, a buyer with a T period had to be compensated for round-trip trading costs:

$$m = m^* - (1 + (1 + rf)^{-T})(\lambda + s), \quad (2.6)$$

where $m^* = d/rf$ – expected present value of security, purchase price $p = m + (\lambda + s)$, m – midquote.

Liquidity, asset prices and financial policy (Amihud and Mendelson, 1991). Liquidity is one of the main factors, which is important for asset pricing – the lower the liquidity, the higher the return. If investor is a long shareholder, he could prefer an asset with low liquidity. Higher price depends on the high liquidity measure. Liquidity, in other words, could be named as marketability, because risk-averse investor – who is riskiness, would expect higher return of the security. Illiquidity could be separated in several measures:

- bid-ask spread;
- market-impact costs;
- delay and search costs;
- direct transaction costs.

The effect of beta, bid-ask spread, residual risk and size on stock returns (Amihud, Mendelson, 1989). The bid-ask spread is related to investors, who have the asset, because of availability of information about it. Spread falls down, when more information is available for the publicity, as marker maker set it to compensate their losses.

Moreover, the bid-ask spread is associated with the residual risk, which reflects price answer to specific information of the firm and deals with opportunity to trade against dealers.

Authors apply the CAPM, which asset returns are determined by their systematic (beta) risk:

$$R_{jt} = \alpha_j + \beta_j R_{mt} + \varepsilon_{jt}, \quad (2.7)$$

where R_{jt} – the return of asset j in period t , α_j and β_j – constant terms, ε_{jt} – residual return, which assumed cross-sectionally independent, R_{mt} – the market return, which also independent.

Amihud and Mendelson (1989) provided a joint test of several numbers of hypotheses. According to CAPM, expected return is an increasing function of the systematic (beta) risk and the bid-ask spread, as a measure of liquidity.

Market microstructure and asset pricing: on the compensation for illiquidity in stock returns (Brennan, Subrahmanyam, 1996). In this article was investigated the relation between monthly stock returns and measures of illiquidity. The adverse selection is a primary cause of illiquidity. Generally there are two way to calculate liquidity – the bid-ask spread and effect of asymmetric information.

Authors in this paper compare empirical techniques from asset pricing and market microstructure to examine relation between return and liquidity. To estimate illiquidity factors, they use two models – the Glosten-Harris (GH) model thr Hasbrouck-Foster-Viswanathan (HFV) model for price formation.

$$m_t = m_{t-1} + \lambda q_t + y_t \quad (2.8)$$

where m_t – marker maker, q_t – the order flow, y_t – a public information signal, λ – the market depth parameter.

$$p_t = m_t + \psi D_t, \quad (2.9)$$

where D_t – the incoming order at time t (if it is buyer trade $t+1$, if seller – $t-1$), ψ – the fixed cost component, p_t – the transaction price.

By substituting formulas below, they get

$$p_t - m_{t-1} + \lambda q_t + \psi D_t + y_t, \quad (2.10)$$

and

$$\Delta p_t = \lambda q_t + \psi [D_t - D_{t-1}] + y_t, \quad (2.11)$$

where y_t – the unobserved error term. This formula represents GH model.

HFV model relates on the price response to unexpected volume, which is a measure of price change.

$$q_t = \alpha_t + \sum_{j=1}^5 \beta_j \Delta p_{t-j} + \sum_{j=1}^5 \gamma_j q_{t-j} + \tau_t, \quad (2.12)$$

$$\Delta p_t = \alpha_p + \psi [D_t - D_{t-1}] + \lambda \tau_t + v_t, \quad (2.13)$$

where q_t – trade quantity, which corresponds to the price change, D_t – direction of the trade.

Common risk factors in the returns on stocks and bonds (Fama, French, 1992). This article does through five main factors in the returns of stocks and bonds. They are:

- market beta;
- size;
- leverage;
- earnings/price;
- book-to-market equity.

Moreover, authors go through several asset-pricing tests:

- the set of asset returns – used common stocks;
- the set of variables – size and book-to-market variables related to stock;
- the approach to test asset-pricing model – cross-section regression.

There are two asset-pricing issues, which could be explained by the time series regressions:

- it could explain that the price of the stock is rational, because of variables, which are related to average returns – size and book-to-marker equity – proxy for common risk factor in returns.
- it uses excess returns as dependent variable and returns on zero-investment portfolio as explanatory variable.

Research process consists of three main steps of regressions:

- use excess market return to explain excess stock returns;
- use SMB (small minus big) and HML (high minus low) – for the size and book-to-market factors;
- use RM-RF, SMB and HML.

Generally regressions look like as follows:

$$R_t - RF_t = a + b[RM_t - RF_t] + sSMB_t + hHML_t + e_t . \quad (2.14)$$

Also authors applied other regression for bonds, but this one is general. Their results could be used in different researches, where need estimation of stock returns:

- portfolio selection;
- performance of portfolio evaluation;
- abnormal returns measures;
- the cost of capital estimation.

2.2. Liquidity review

Liquidity and asset pricing model (Amihud, Mendelson and Pedersen, 2006). In this article they discussed connection between liquidity and required return. Liquidity is the ease of trading a security. Sources of illiquidity: demand pressure, inventory risk, exogenous transaction costs.

Security trading can be costly because of private (asymmetric) information.

What can be explained by liquidity:

- the cross-section of assets with different liquidity,
- the time series relationship between liquidity and securities return,
- the pricing of securities,
- the return on hedge funds,
- equities – high return, liquid risk-free treasuries – low return, small sticks – high return.

Standard asset pricing – perfectly liquid market securities are traded at the price, which is given.

In other words, it can be called frictionless, and three main concepts for it – no arbitrage, agent optimality and equilibrium. But this assumption about market's frictionless is crucial because it means that securities with the same prices must have the same cash flows, and it means that there are no trading costs for buyer and seller – immediate arbitrage profit at no risk. Normally, in real world securities have different prices while having same cash flows (Amihud and Mendelson, 1991). So, this is one of the main differences between standard asset pricing and liquidity asset pricing. To summarize, it can be said that for liquidity asset pricing is an important behavior of the trading activity.

Illiquidity and stock returns: cross-section and time-series effects (Amihud, 2002). Main assumption in connection between liquidity and return is that return increases in illiquidity. Proposition: "...the ex ante stock excess return is increasing in the expected illiquidity of the stock market".

The illiquidity measure (ILLIQ) – daily ratio of stock return to its volume, which is averaged over some period. More measures of illiquidity: the bid-ask spread, transaction-by-transaction market impact, the probability of trading information.

Illiquidity shows the influence of order flow on price – discount or premium, which buyer pays when executing an order (Amihud and Mendelson, 1980).

Illiquidity measure (Amihud, 2002) is calculated from daily data on returns and volume, in other words, stock illiquidity is an average ratio of the daily return to the trading volume on that day.

$$ILLIQ_{iy} = 1/D_{iy} \sum_{d=1}^{D_{iy}} |R_{iyd}| / VOLD_{iyd} \quad (2.15)$$

where D_{iy} – the number of days for stock i in year y , R_{iyd} - the return on stock i on day d of year y , $VOLD_{iyd}$ – daily volume. It gives percentage price change.

To talk about measures of liquidity on volume, there are some findings about negative relations between stock return and its size (Brennan et al., 1998). Also it is a negative relation

to illiquidity costs against stock's turnover (Amihud and Mendelson, 1986). But it is positive relationship between the turnover ratio and the bid-ask spread (Atkins and Dyl, 1997).

The average market illiquidity:

$$AILLIQ_y = 1/N_y \sum_{t=1}^{N_y} ILLIQ_{iy} \quad (2.16)$$

where N_y – the number of stocks in year y .

The estimator of the cross-section model by its mean-adjusted value:

$$ILLIQMA_{iy} = ILLIQ_{iy}/AILLIQ_y. \quad (2.17)$$

The cross-section model includes $SIZE_{iy}$, which can be a proxy for liquidity.

To sum up, this paper shows new test that the asset's expected return is increasing in illiquidity. Also it implies that the risk premium referred excess return $R_m - R_f$. And it contributes that the equity premium is too high.

Asset pricing with liquidity risk (Achrya, Pedersen, 2002). Different measures of liquidity vary over time for the market and the individual stocks. Authors use CAPM, which shows that stock's required return, which depends on sensitivity of its return.

Model shows:

- investor require a return premium for a security, when the market is illiquid – security is also illiquid;
- if security has a high return, while market is illiquid, investor is ready to pay premium for it;
- if security is liquid and market is down, investor also ready to pay for a premium for it;
- future returns could be predicted by liquidity;
- liquidity co-moves with contemporaneous returns.

Firstly, authors talk about dividends of security and illiquidity cost. To measure them, they apply formulas:

$$D_t = D + \gamma(D_{t-1} - D) + \varepsilon_t, \quad (2.18)$$

$$C_t = C + \gamma(C_{t-1} - C) + \eta_t, \quad (2.19)$$

where D_t - dividends at time t , C_t - liquidity costs at time t , ε_t and η_t - an independent identically distributed normal process with its mean and variance and covariance.

Liquidity adjusted CAPM could be characterized by four betas and covariances.

$$r_t^i = \frac{D_t^i + P_t^i}{P_{t-1}^i}, \quad (2.20)$$

$$c_t^i = \frac{C_t^i}{P_{t-1}^i}, \quad (2.21)$$

$$r_t^M = \frac{\sum_t S^t (D_t^i + P_t^i)}{\sum_t S^t P_{t-1}^i}, \quad (2.22)$$

$$c_t^M = \frac{\sum_t S^t C_t^i}{\sum_t S^t P_{t-1}^i}, \quad (2.23)$$

where r_t^i - the expected return, c_t^i - the relative illiquidity cost, r_t^M - the market return, c_t^M - the relative market illiquidity.

To get equilibrium returns - which asset I has a dividend of $D_t^i - C_t^i$ and no illiquidity cost. Author's CAPM transforms into CAPM in net returns with illiquidity costs.

$$E_{t-1}(r_t^i - c_t^i - r^f) = \lambda_{t-1} cov_{t-1}(r_t^i - c_t^i, r_t^M - c_t^M), \quad (2.24)$$

where λ_{t-1} - the market price of risk.

And liquidity measure, which they apply is ILLIQ (Amihud, 2002):

$$ILLIQ_t^i = 1/D_t^i \sum_{d=1}^{D_t^i} |R_{td}^i| / V_{td}^i, \quad (2.25)$$

where R_{td}^i - the return on day d in month t , V_{td}^i - dollar volume on day d in month t , D_t^i - number of days in month t .

To sum up, model shows why contemporaneous liquidity and returns co-move and why high illiquidity forecasts high future returns.

Liquidity and stock returns: an alternative test (Datar, Nail, Radcliffe, 1998). General acceptance of this article is that the most important attributes of assets are liquidity, marketability and transaction costs, which influence investor's decisions.

Authors paper is about relation between liquidity and asset return, and proxy for liquidity is not usual the bid-ask spread. They propose the turnover rate as a proxy for liquidity.

The generalized least-squares (GLS) methodology was used to observe cross-sectional variation in stock returns, which could be explained by difference in the turnover rates.

Empirical model has form as follows:

$$R_{it} = \gamma_{0t} + \sum_{k=1}^k \gamma_{kt} x_{it} + \varepsilon_{it}, i = 1, 2, \dots, N_t, t = 1, 2, \dots, T, \quad (2.26)$$

where R_{it} – the return on security i in month t , x_{it} – the turnover rate, firm size, book-to-market ratio etc. of stock i in month t , ε_{it} – the deviation of the realized return from its expected value, N_t – number of securities.

To sum up, it was found out that the security return is negatively related to its turnover rate, accepting that illiquid stock provide higher average returns.

Liquidity commonality and return co-movement (Domowitz, Hansch, Wang, 2005). In this article measure of liquidity is intended from economic supply and demand. liquidity in the market is dependent from area between two schedules – the farther away supply from demand – the lower liquidity.

$$l_t(q) = \int_0^q [S_t(Q) - D_t(Q)] dQ, \quad (2.27)$$

measures the liquidity at time t for round-trip trade of size q .

There are two sides of liquidity – sell and buy.

$$l_t^{sell}(q) = \int_0^q [S_t(Q) - P_{0,t}(Q)] dQ, \quad (2.28)$$

$$l_t^{buy}(q) = \int_0^q [P_{0,t}(Q) - D_t(Q)] dQ, \quad (2.29)$$

with

$$l_t(q) = l_t^{sell}(q) + l_t^{buy}(q). \quad (2.30)$$

This aggregates all the orders on supply and demand sides through the quoted spread.

Main results of paper is that there are different causes of return and liquidity commonality.

2.3. The bid-ask spread review

Bid, ask and transaction prices in a specialist market with heterogeneously informed traders (Glosten, Milgrom, 1985). Market is a place, where buyer and seller come together to trade a common stock. Issue about measuring the bid-ask spread arises, when appears adverse selection.

Several propositions in article:

- the bid and ask prices fluctuate the price;
- the price occurs form a martingale;
- bound on the size of the spread;
- the value expectations tend to converge;
- how the spread responds to variation of the model.

The optimal decision of an investor at time t is given by:

$$\begin{aligned} & \text{buy if } Z_t > A, \\ & \text{sell if } Z_t < A, \end{aligned}$$

where Z_t is given by

$$Z_t = p_t E[V | F_t] = p_t(1 - U_t)E[V | H_t, J_t, A, B] + p_t U_t E[V | H_t, A, B], \quad (2.31)$$

where B – bid, A - ask, U_t – one or zero (uninformed/informed), J_t – private information, H_t – public information, F_t – a refinement of H_t (information about quoted bid and ask).

The spread is important because it offers potential explanation of the excess returns.

A simple implicit measure of the effective bid-ask spread in an efficient market (Roll, 1984). The effective bid-ask spread can be measured:

$$\text{Spread} = 2\sqrt{-cov} \quad (2.32)$$

where cov – first-order serial covariance of price changes.

In paper says that bid-ask spread can be measured by using a time series of market prices.

Two main assumptions:

- The asset is traded in an informationally efficient market,
- The probability distribution of observed price changes stationary.

If trading costs are zero, it means that the market is informationally efficient. If change in price appears, so, there is unanticipated information on the market. Usual compensation for broker is the bid-ask spread, which defined by Roll as a small interval of price, which

brackets the underlying value of the asset. When a market maker is involved in transactions, negative serial dependence in price changes (Niederhoffer and Osborne, 1966).

A simple way to estimate bid-ask spread from daily high and low prices (Corwin, Schultz, 2002). High-low ratio reflects the variance and the bid-ask spread of the stock. Some advantages of the high-low spread estimator:

- outperforms the LOT estimator of Lesmond, Ogden and Trzcinka (1999) and the popular Roll(1984) estimator;
- easy to use and calculate;
- not computer-time intensive – ideal for large samples;
- derived under general conditions.

$$\left[\ln\left(\frac{H_t^0}{L_t^0}\right) \right]^2 = \left[\ln\left(\frac{H_t^\wedge(1+\frac{S}{2})}{L_t^\wedge(1-\frac{S}{2})}\right) \right]^2, \quad (2.32)$$

where H_t^\wedge (L_t^\wedge) – actual high (low) stock price, H_t^0 (L_t^0) – observed high (low) stock price, S – spread. Parkinson (1980) and Garman and Klass (1980) show that this model could be rewritten as:

$$E \left\{ \frac{1}{T} \sum_{i=1}^T \left[\ln\left(\frac{H_t}{L_t}\right) \right]^2 \right\} = k_1 \sigma_{HL}^2, \quad (2.33)$$

where H_t (L_t) – high (low) and $k = 4\ln(2)$.

I would like to pass some formulas with rewrite equations, and go directly to the unknowns β , γ , α .

$$\beta_t = E \left[\sum_{j=0}^1 \left[\ln\left(\frac{H_{t+j}^0}{L_{t+j}^0}\right) \right]^2 \right], \quad (2.34)$$

where β_t – the expectation of the sum of the price ranges over two days, H_t^0 – the high price at day t , L_t^0 – low price at day t .

$$\gamma_t = \left[\ln\left(\frac{H_{t-1,t}^0}{L_{t-1,t}^0}\right) \right]^2, \quad (2.35)$$

where γ_t – the max range of the high-low price ratio over two days, $H_{t-1,t}^0$ – the maximum price for two days of $t-1$ and t , $L_{t-1,t}^0$ – the minimum price for two days of $t-1$ and t .

Corwin and Shultz (2002) use $(t, t-1)$, because the measure of high and low prices - backward looking.

After computing β_t and γ_t , everything prepared to estimate α_t using following difference:

$$\alpha_t = \frac{\sqrt{2\beta_t} - \sqrt{\beta_t}}{3-2\sqrt{2}} - \sqrt{\frac{\gamma_t}{3-2\sqrt{2}}} = (1 + \sqrt{2})(\sqrt{\beta_t} - \sqrt{\gamma_t}). \quad (2.36)$$

So, the bid-ask spread (S) is calculates as follows:

$$S = \frac{2(e^{\alpha_t} - 1)}{1 + e^{\alpha_t}}. \quad (2.37)$$

It was quite difficult to choose relevant literature, because there are a lot of researches about CAPM and liquidity. CAPM is a traditional model in asset pricing, and liquidity is unseparated factor with securities. It is important to go through, at least part, of them for better understanding different views on this issue. Different authors applied different models, methods, use different factors to measure liquidity and stock returns.

The bid-ask spread is an important liquidity measure. I need to estimate it in my paper, and it was decided to use two estimators – Corwin and Shultz (2002) estimator and Roll (1984) estimator. It was decided because of specific characteristic of data – daily. And estimation of high and low prices in CS model is perfectly suits my work. Moreover, these two measures are quite simple to estimate using summaries of articles.

As I plan to work with big amount of data, to make my work easier, it was decided to use simple CAPM with adding there liquidity spreads. So, regressions look like general CAPM equation. In next chapters, there is more detailed information about working process.

Chapter 3. Data

This paper is in the financial field, so research generally could be called - quantitative research, as I work with huge amount of data.

3.1. Collection of all needed data

Firstly, I need to download all necessary information about stocks. Main source of data collection is Titlon, where you can find all useful information about stocks, with which I work. Before downloading data, it was filtered with only Norwegian stocks, because the main goal is to go only through them.

General data set consists from 582 different companies, chosen period 2000 – 20015 and data is daily. Also I need quite much information about stock – date, company’s id, name, prices information, etc.

To summarize all necessary data, it was decided to put information in Table 3.1.

Table 3.1.1. Data summary

Data collection	Titlon
Security type	Norwegian stocks
Period	2000 – 2015 years
Type of data	Daily
Stock’s information	Date
	Company’s id
	Symbol
	Name
	Open
	High
	Low
	Close
	Volume
	Adjusted price
	Indeltaadjusted price
	Indeltaosebxb

Downloaded data looks quite unbalanced, because trading days of different companies were not the same in different months and years; some days, months or years were missed; some

companies were in the market only until several years in period, which was defined, because they are bankrupt, other companies appear during chosen period. So, I work with unbalanced panel data.

Also needed information has gaps, where no data about stocks at all. But it was decided not to delete such information – during calculations just use not zero, but empty space.

3.2. Calculations of liquidity estimators

Secondly, I need to calculate liquidity estimators – Corwin and Schultz (2002) estimator; Roll (1984) estimator. About both of them I wrote in previous part. Measure of liquidity is the bid-ask spread, so for different estimators, it is needed different information.

Corwin and Shultz (2002) estimator requires daily high and low prices, beta, gamma and alpha. High and low prices should be adjusted as:

$$Adj\ H = \left(1 + \frac{(High-Close)}{Close}\right) * Adj\ Price ;$$

$$Adj\ L = \left(1 + \frac{(Low-Close)}{Close}\right) * Adj\ Price .$$

Also this estimator applies maximum and minimum of adjusted high and low prices. To find max (min), it is needed to compare high (low) price from t period and $t + 1$ period. And for max choose bigger price and for min – smaller price.

Moreover, some words about β , γ , α .

$$\beta_t = E \left[\sum_{j=0}^1 \left[\ln\left(\frac{H_{t+j}^0}{L_{t+j}^0}\right) \right]^2 \right],$$

where β_t – the expectation of the sum of the price ranges over two days, H_t^0 – the high price at day t , L_t^0 – low price at day t .

$$\gamma_t = \left[\ln\left(\frac{H_{t-1,t}^0}{L_{t-1,t}^0}\right) \right]^2 ,$$

where γ_t – the max range of the high-low price ratio over two days, $H_{t-1,t}^0$ – the maximum price for two days of $t-1$ and t , $L_{t-1,t}^0$ – the minimum price for two days of $t-1$ and t .

Corwin and Shultz (2002) use $(t, t-1)$, because the measure of high and low prices - backward looking.

After computing β_t and γ_t , everything prepared to estimate α_t using following difference:

$$\alpha_t = \frac{\sqrt{2\beta_t} - \sqrt{\beta_t}}{3 - 2\sqrt{2}} - \sqrt{\frac{\gamma_t}{3 - 2\sqrt{2}}} = (1 + \sqrt{2})(\sqrt{\beta_t} - \sqrt{\gamma_t}).$$

So, the bid-ask spread (S) is calculates as follows:

$$S = \frac{2(e^{\alpha_t} - 1)}{1 + e^{\alpha_t}}.$$

Main difference between Corwin and Shultz (2002) estimator and Roll (1984) estimator, that Roll (1984) uses prices to measure the bid-ask spread.

Adjusted price is available in Titlon, so I calculated ΔP_t and ΔP_{t-1} . Next step was the computation of the serial covariance of the price changes $Cov(\Delta P_t, \Delta P_{t-1})$.

And effective bid-ask spread (S) is given by:

$$S = 2\sqrt{-Cov(\Delta P_t, \Delta P_{t-1})}.$$

Generally, on this level I have already got information, which I need to apply regression.

3.3. Building models

Thirdly, I need to estimate CAPM model with liquidity measures. General model looks like:

$$R_i = R_f + \beta_i(R_m - R_f),$$

where R_i – the return of stock i , R_f – the risk free rate, $\beta_i - \beta$ of stock i , R_m – the market return.

Using liquidity estimators, CAPM is given by:

$$R_i = R_f + \beta_i(R_m - R_f) + LIQ,$$

where LIQ is liquidity measure – CS and Roll spreads, in this case.

There are two ways how to download information of market returns – use Titlon “Indeltaadjustedosebx” or download it directly from OSE. As I use daily data, so market returns also should be daily. Same with risk free rate, it can be found at OSE, and data should be daily. Also stocks returns are already computed at Titlon, what is very convenient.

For my paper, I also decided to use one more extra liquidity measure, which I do not need to calculate, it is already calculated by Fama and French for Norwegian market, so I apply it to see a difference between all liquidity measures. This liquidity measure is available also at OSE, and I use daily one, as I work with only daily data.

3.4. Working with codes

When I gathered all data from different files and resources, everything was ready to do regressions. It was decided to use RStudio to apply statistical analysis. In Table 3.2 I summarize data, which was downloaded in RStudio to work with.

Table 3.4.1. Factors for CAPM

Stocks data	582 stocks, 15 years, daily
Risk free rate	15 years, daily
Market return	15 years, daily
Liquidity measure	15 years, daily

As I wrote previously, I applied general model of CAPM and, then, add in this model liquidity estimators. So, I have 5 different regressions, which look as follows:

1. $R_i = R_f + \beta_i(R_m - R_f) ;$

2. $R_i = R_f + \beta_i(R_m - R_f) + liq$;
3. $R_i = R_f + \beta_i(R_m - R_f) + CS$;
4. $R_i = R_f + \beta_i(R_m - R_f) + Roll$;
5. $R_i = R_f + \beta_i(R_m - R_f) + CS + Roll$,

where liq – liquidity, which was taken from OSE and calculated by Fama and French, CS – spread estimator of Corwin and Shultz (2002), Roll – Roll (1984) estimator.

Also I decided to rewrite main regression functions:

1. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f)$;
2. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 liq$;
3. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 CS$;
4. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 Roll$;
5. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 CS + \beta_3 Roll$.

All these regressions were applied for all 582 companies. As I wrote previously, there are some problems with data, because there are quite a lot of gaps with empty information, so results seem a little bit bias.

3.5. Regressions

After main regressions, I decided to take the most liquid companies and make the same procedure. These companies were taken from “Trading Economics¹”. List of companies is in Appendix (). It was decided to save all information about these companies, especially 2000-2015 years and daily data.

Also all regressions were decided to save and apply:

1. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f)$;
2. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 liq$;
3. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 CS$;
4. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 Roll$;
5. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 CS + \beta_3 Roll$.

¹ <http://www.tradingeconomics.com/norway/stock-market>

In this chapter it is important to write about process in details to understand the procedure. Main data of stocks characteristics is available on Titlon, from where it was downloaded and rewritten in a specific way easier to work with. Data is daily, so there are quite a lot of information, because I take mostly all Norwegian stocks during 15 years from 2000 until 2015. Risk-rate and market return were downloaded from external resource.

As I estimate liquidity by measuring the bid-ask spread, I go through different formulas, which I need to apply in my work. It was decided to take two estimators in previous chapter: Corwin and Shultz (2002) estimator and Roll (1984) estimator. I work with daily data, so there two estimators are relevant to apply. From articles it follows that it is quite easy to use.

Chapter 4. Data analysis and discussion

Data analysis starts from working with RStudio. To start process it is needed to upload all necessary data to program. In my case it was general information about stocks, liquidity measures, risk free rate and market return.

I work with quite big amount of companies and data, it was needed to do some actions in program to get normal results.

Firstly, it was decided to work with all data, so to download all information about each daily stock. Main problem was that information about stocks was daily for each stock and market return and risk free rate should be connected with each company in each day of the month and year. So, it was used code (Appendix 2), which helped to combine this information. Secondly, after receiving some results it was decided to take only 72 companies from 582, to combine results and try to mention difference between first five regressions and next five regressions.

I would like to start from appearance of estimators. As I wrote previously, it was used Corwin and Shultz (2002) – CS – estimator and Roll (1984) estimator. Because of gaps in downloaded information, all zero values were deleted to calculate average bid-ask spread (CS and Roll). An average CS estimator is 0.003469, and an average Roll estimator – 0.047286. These values are quite significant, because generally spread should be as lower as it could be. If the bid-ask spread is low, the investor is more interested in stock value, so it means that security is demanded. Normal spread meanings should be less than 5%, if not – stock is illiquid.

I rewrite it in follow dependence: low spread => low trade costs => high potential return. Also on Figure 4.1.1. and Figure 4.1.2. it is possible to find values, which close to average values of both estimators. Some data was cleared, because of big amount of information. So, here it is data on the last trading day in December 2014.

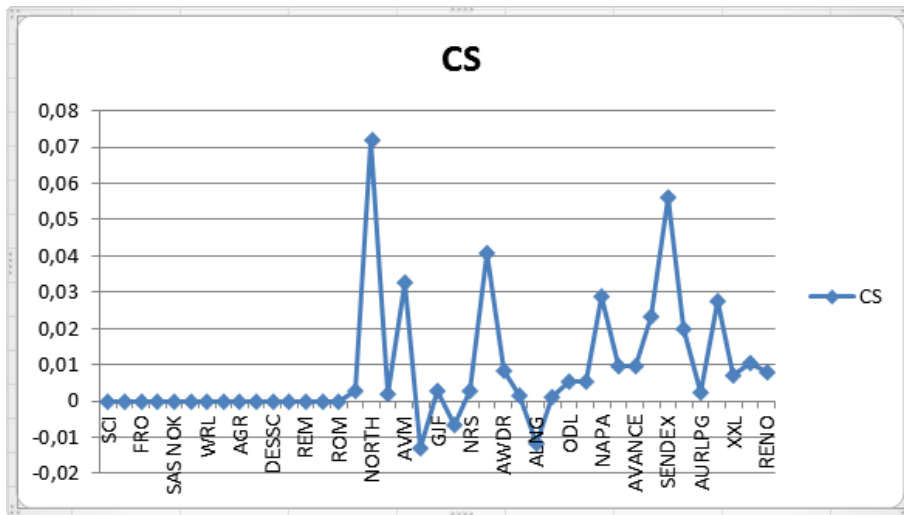


Figure 4.1.1. Corwin and Shultz estimator, December 2014

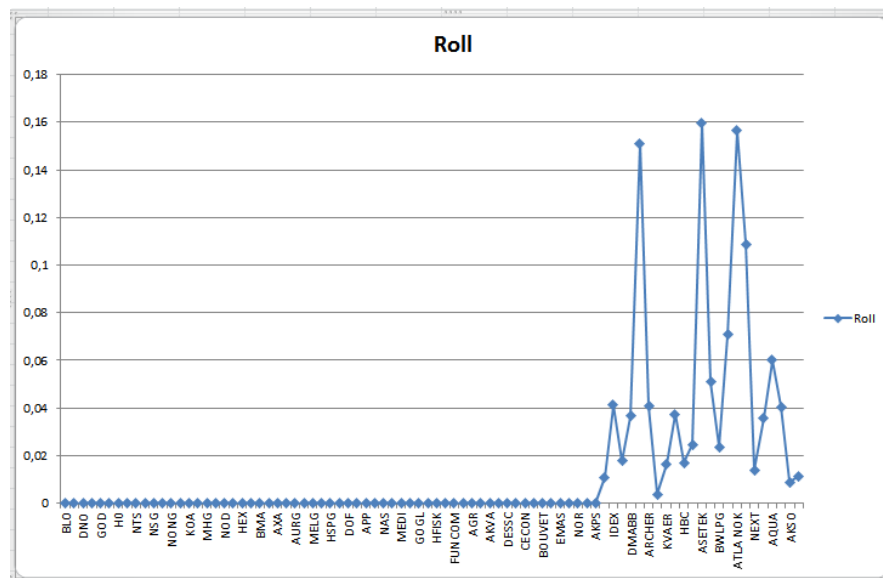


Figure 4.1.2. Roll estimator, December 2014

In these graph is possibly to see that spread not constant through different companies. And it is not surprising. Later in this part I would like to look at 72 companies, which are the most liquid in Norway. May be picture will change.

Risk free rate was taken from OSE Figure 4.3. R_f represents the investor's expectations about his absolutely risk-free investment. In other words, risk-free rate is the minimum return which expected by investor. Risk-free rate decreases in several years, it means that

values of stocks are increase, but it could not be applied because stocks are more dependent on expected earnings.

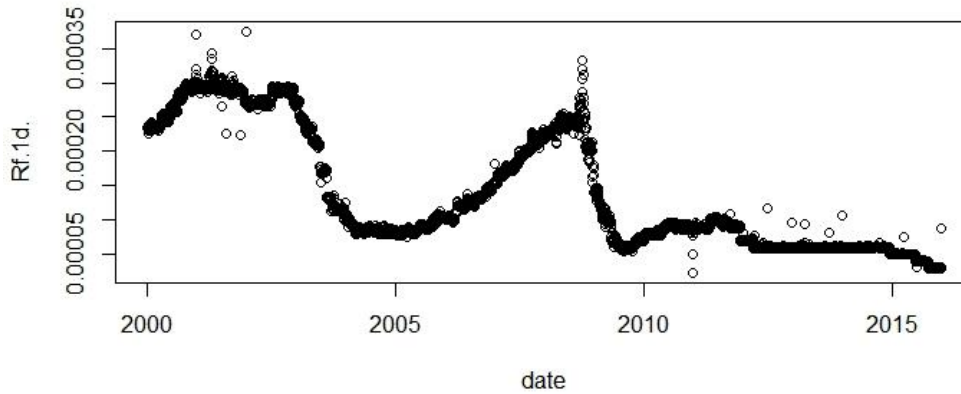


Figure 4.1.3. Risk-free rate

4.1. Regressions for all sample elements

1. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) ;$
2. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 liq ;$
3. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 CS ;$
4. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 Roll ;$
5. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 CS + \beta_3 Roll .$

There are five different regressions, in which I am interested. After applying regression codes, RStudio divided all companies in five regression groups. See Table 4.1.1.

Table 4.1.1 Regressions for the whole sample

Regression type	Number of companies
$R_i - R_f = \alpha_0 + \beta_1(R_m - R_f)$	456 elements
$R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 liq$	456 elements
$R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 CS$	305 elements
$R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 Roll$	201 elements
$R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 CS + \beta_3 Roll$	305 elements

Table 4.1.2 Results of all regressions

Regression	α	β_1	β_2	β_3	R-squared
CAPM	-0.008593	0.059			0.08038
CAPMliq	-0.00091	0.53839	0.05567		0.0797
CAPMcs	0.0385	-0.063	0,061		0.0363
CAPMroll	0.00622	0.44620	0.0591		0.07682
CAPMcr	0.0023	-0.0251	0.04837	0.0372	0.0483

First regression is general capital asset price model without any liquidity estimators. It was made to compare results and find out differences.

$R_i - R_f = \alpha_0 + \beta_1(R_m - R_f)$ is first regression, which could be rewritten as $R_1 = \alpha_0 + \beta_1 R_{ex}$, where R_1 – regression one, R_{ex} – excess return, which equals $R_m - R_f$. With coefficients model returns into $R_1 = -0.008593 + 0.059R_{ex}$.

Beta coefficient is a slope coefficient, which is positive. R squared is only 8%, what means that only 8% of variation could be explained by this model.

Also it was interesting to take a look to random stocks in this regression to compare result, because R squared is quite, what usually in general CAPM model is strange.

For example, I took 400 and 456 elements, it was randomly.

```
> print(CAPM[[400]])
```

```
Call:
```

```
lm(formula = r1 ~ rm1)
```

```
Residuals:
```

```
      Min       1Q   Median       3Q      Max
-0.096550 -0.011306 -0.000647  0.011233  0.192828
```

```
Coefficients:
```

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.0013183  0.0005667   2.326   0.0201 *
rm1          0.5758695  0.0450717  12.777  <2e-16 ***
```

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual standard error: 0.02155 on 1445 degrees of freedom
 Multiple R-squared: 0.1015, Adjusted R-squared: 0.1009
 F-statistic: 163.2 on 1 and 1445 DF, p-value: < 2.2e-16

Regression for this exact element is follows: $R_{1.400} = 0.0013183 + 0.5758695R_{ex}$, where slope coefficient is quite high, what means that the stock is riskiness. Also R-squared is about 10%, what says that factors in general CAPM model work normally, and this 10% of variation could be explained by this model.

And the same for 456 element, which is last in this model.

```
> print(CAPM[[456]])
```

Call:

```
lm(formula = r1 ~ rm1)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.148298	-0.017616	0.000517	0.018501	0.085135

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-0.002957	0.001656	-1.786	0.075 .
rm1	0.628359	0.138205	4.547	7.55e-06 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.03089 on 346 degrees of freedom
 Multiple R-squared: 0.05638, Adjusted R-squared: 0.05365
 F-statistic: 20.67 on 1 and 346 DF, p-value: 7.55e-06

Regression is $R_{1.456} = -0.002957 + 0.628359R_{ex}$, where also the slope coefficient is high and positive. R-squared is low – 5%.

One explanation about these differences is the data gaps in some period of time, which come to misrepresentation of results.

Second one is adjusted with Fama and French liquidity factor and could by rewritten as:

$$R_2 = -0.0091 + 0.53839R_{ex} + 0.05567liq.$$

Beta coefficients are positive, so the slope is also positive. And R-squared is also not significant only 7,9% of variation could be explained with this liquidity factor adjusted model.

Also it is possible to check exact company, which is in range of this model and look at the result.

```
> print(CAPMliq[[300]])
```

```

Call:
lm(formula = r1 ~ rm1 + liq)

Residuals:
    Min       1Q   Median       3Q      Max
-0.41910 -0.01152 -0.00071  0.01155  0.31973

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0003757  0.0009119  -0.412   0.6804
rm1          0.4834706  0.0792284   6.102  1.4e-09 ***
liq         -0.1807977  0.0998205  -1.811   0.0704 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.03174 on 1213 degrees of freedom
Multiple R-squared:  0.1489, Adjusted R-squared:  0.1475
F-statistic: 106.1 on 2 and 1213 DF, p-value: < 2.2e-16

```

Regression for stock number 300 looks as follows:

$R_{2,300} = -0.0003757 + 0.4834706R_{ex} - 0.1807977liq$, and its R-squared is 14.89%, which is almost twice higher than in general CAPMliq regression.

It should be done the same procedure for next three regressions. Third one is $R_3 = 0.0385 - 0.063R_{ex} + 0.061CS$, which add CS estimator, and when it was used Roll estimator is $R_4 = 0.00622 + 0.44620R_{ex} + 0.0591Roll$. For these two models R-squared is also not so high as could be expected, 3.6% and 7.6% respectively.

Last model, where added both CS and Roll spreads looks like: $R_5 = 0.0023 - 0.0251R_{ex} + 0.04837CS + 0.0372Roll$ with R-squared 4.8%.

To take a look at all results, it seems that factors are not correlated with each other. One explanation for all this is data. Data has some missing characteristics, so one way to get other results is to change daily data to monthly data, or not to take in consideration missing prices. But before writing paper I do not have any reason for such assumption.

It was found list of the most liquid companies in Norway (Appendix 2). It could be possible to apply the same procedure and compare information.

4.2. The most liquid Norwegian companies

It was decided to cut main companies and leave 72 of them, which are the most liquid.² The list of the companies is in Appendix 1.

Before applying regressions to these companies, it was taken a closer look to its spread. I chose 2015 year and tried to calculate average spread. For Roll estimator, it is about 0.0316. It is quite high spread, especially for liquid companies. The CS estimator was decided to look through some companies separately. And average spread is 0.00617. Figure 4.2.1. shows only part of companies – it is for easier observations, not to make it heavy.

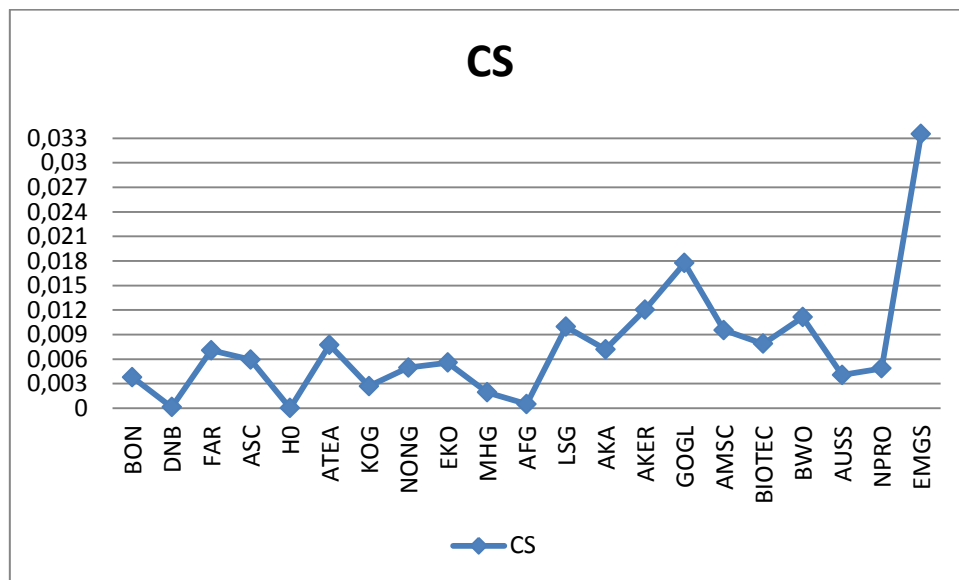


Figure 4.2.1. Corwin and Shultz for liquid companies

Regressions

1. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) ;$
2. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 liq ;$
3. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 CS ;$
4. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 Roll ;$
5. $R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 CS + \beta_3 Roll .$

² According to: <http://www.tradingeconomics.com/norway/stock-market>

There are still five different regressions, in which number of companies reduced from 582 to 72. After applying regression codes, RStudio divided all companies in five regression groups. See Table 4.2.1.

Table 4.2.1 Regressions for liquid companies

Regression type	Number of companies
$R_i - R_f = \alpha_0 + \beta_1(R_m - R_f)$	64 elements
$R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 liq$	64 elements
$R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 CS$	55 elements
$R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 Roll$	10 elements
$R_i - R_f = \alpha_0 + \beta_1(R_m - R_f) + \beta_2 CS + \beta_3 Roll$	10 elements

Also in this time, it was decided to remove all empty data, because it could give wrong results.

Table 4.2.2 Results of regressions of liquid companies

Regression	α	β_1	β_2	β_3	R-squared
lqCAPM	-0.00168	0.68829			0.17567
lqCAPMliq	-0.00011	0.63673	0.07415		0.17757
lqCAPMcs	0.001212	-0.03585	0.72601		0.17948
lqCAPMroll	0.00293	0.14912	0.52593		0.11088
lqCAPMcr	0.00109	0.59543	0.3186	0.02733	0.20398

In these regressions there are new coefficients. And first regression as general model for liquid companies looks as follow: $R_1 = -0.00168 + 0.68829R_{ex}$, where slope coefficient beta equals to 0.68829, which is positive. Also R-squared in this case is not significantly high, but it is bigger than the previous, equals 17.567%, what means that 17.567% of variation could be explained applying this CAPM. Moreover, it is possible to take a look to a general stock in this model. And, again randomly, was chosen 64 element.

```
> print(lcCAPM[[64]])
```

```
Call:
lm(formula = r1 ~ rm1)
```

```
Residuals:
      Min       1Q   Median       3Q      Max
-0.119749 -0.008667 -0.000211  0.010442  0.057014
```

```
Coefficients:
      Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0002495  0.0009339  -0.267   0.789
rm1          0.8744058  0.0820633  10.655 <2e-16 ***
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 0.01932 on 426 degrees of freedom
Multiple R-squared:  0.2104, Adjusted R-squared:  0.2086
F-statistic: 113.5 on 1 and 426 DF, p-value: < 2.2e-16
```

Its regression looks like: $R_{1.64} = -0.0002495 + 0.8744058R_{ex}$, with R-squared 21.04%.

Second model with general coefficients looks as follow: $R_2 = -0.0011 + 0.63673R_{ex} + 0.07415liq$, slope coefficients are positive, and R-squared 17.757%, what is a little bit higher than previous one. Also I took 10 element to look at its statistics.

```
> print(lcCAPMliq[[10]])
```

```
Call:
lm(formula = r1 ~ rm1 + liq)
```

```
Residuals:
      Min       1Q   Median       3Q      Max
-0.109450 -0.010596 -0.000405  0.009907  0.100410
```

```
Coefficients:
      Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.0004349  0.0002948   1.475   0.14
rm1          0.3210669  0.0289200  11.102 < 2e-16 ***
liq         -0.1287926  0.0314435  -4.096 4.29e-05 ***
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Residual standard error: 0.01859 on 4012 degrees of freedom
Multiple R-squared:  0.1112, Adjusted R-squared:  0.1108
F-statistic: 251.1 on 2 and 4012 DF, p-value: < 2.2e-16
```

Its regression looks like: $R_{2.10} = 0.0004349 + 0.320669R_{ex} - 0.1287926liq$, with R-squared 11.12%.

Third model with general coefficients looks as follow: $R_3 = 0.001212 - 0.03585R_{ex} + 0.72601CS$, and R-squared is 17.948%. And 45 element gives following statistics.

```
> print(lcCAPMcs[[45]])
```

```

Call:
lm(formula = r2 ~ rm2 + cs2)

Residuals:
    Min       1Q   Median       3Q      Max
-0.216769 -0.017754 -0.000059  0.017319  0.166887

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0007439  0.0050397  -0.148  0.88269
rm2          0.4157483  0.0520038  27.224 < 2e-16 ***
cs2          0.0748403  0.0248545   3.011  0.00269 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0345 on 736 degrees of freedom
Multiple R-squared:  0.5112, Adjusted R-squared:  0.5092
F-statistic: 256.6 on 3 and 736 DF, p-value: < 2.2e-16

```

Its regression looks like: $R_{3.45} = -0.0007439 + 0.4157483R_{ex} + 0.0748403CS$, with R-squared 51.12%, what is significantly high, and it means that for this exact element model works normally.

Fourth model with general coefficients looks as follow: $R_4 = 0.00293 + 0.14912R_{ex} + 0.52593Roll$, with positive slope coefficients and R-squared – 11.088%. Its 5 element gets following statistics.

```
> print(lcCAPMroll[[5]])
```

```

Call:
lm(formula = r3 ~ rm3 + roll2)

Residuals:
    Min       1Q   Median       3Q      Max
-0.079851 -0.008420 -0.000813  0.008051  0.059026

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.0002738  0.0025581   0.107   0.915
rm3          0.7164899  0.0454992  15.747 <2e-16 ***
roll2       -0.0005736  0.0339055  -0.017   0.987
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01509 on 659 degrees of freedom
Multiple R-squared:  0.2767, Adjusted R-squared:  0.2734
F-statistic: 84.03 on 3 and 659 DF, p-value: < 2.2e-16

```

Regression looks like: $R_{4.5} = 0.002738 + 0.7164899R_{ex} - 0.0005736Roll$, with R-squared 27.67%.

And last fifth model: $R_5 = 0.00109 + 0.59543R_{ex} + 0.3186CS + 0.02733Roll$, it has all positive slope coefficients. Its R-squared is 20.398%. This model has the highest measure of R-squared, comparing to all previous one, but it is still not as significant as it could be. And its first element statistics.

```
> print(lccAPMcr[[1]])
```

```
Call:
```

```
lm(formula = r4 ~ rm4 + cs4 + roll4)
```

```
Residuals:
```

```
      Min       1Q   Median       3Q      Max
-0.080595 -0.016527  0.004036  0.014910  0.074709
```

```
Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.002218	0.020290	0.109	0.914
rm4	-0.833885	0.719606	-1.159	0.258
cs4	-0.019730	0.110773	-0.178	0.860
roll4	0.167785	0.136452	1.230	0.231

```
Residual standard error: 0.0356 on 24 degrees of freedom
```

```
Multiple R-squared: 0.1014, Adjusted R-squared: -0.04834
```

```
F-statistic: 0.6772 on 4 and 24 DF, p-value: 0.6144
```

Regression looks like: $R_{5.1} = 0.002218 - 0.833885R_{ex} - 0.0197330CS + 0.167785$, with R-squared 10.14 %.

In this chapter it is important to look through relations between asset pricing and liquidity measures using regression. Traditional model is CAPM, which does not need much information to complete it. I was using R_f – risk free-rate, R_m – the market return, excess return, which is difference between risk-free rate and the market return ($R_m - R_f$). In codes it was used excess return. These two measures are possible to download from OSE, in my case I use daily data.

First part was related to the whole sample 582 companies, but during calculations in RStudio some of them are missing, it is because of special condition not to use zero. As I wrote previously, all data was downloaded from Titlon, which has problems, as there are a lot of missing important elements about prices. So, it was better not to take in consideration this numbers.

When I got first results it seems that model do not work properly, because summary statistics shows very low R-squared, which is the most important measure, which shows the exact % of variation, which could be explain by applied model. Regressions for general models for the whole sample have R-squared between 3% and 8% - it is very low. But when I took random element from each model, they show higher relations, but still not high as I would expect. More or less, the same situation appears with the most liquid companies. Model element's relations are higher in general, and quite good for exact elements.

All of these issues could be explained in two ways. First, results are distorted because of some emptiness of data. Or it could change situation, if I take not daily, but monthly data, and limit year interval for 7-10 last years. Secondly, as these stocks are still on the Norwegian market, now I speak only about the most liquid securities, so they are liquid and CAPM was chosen as wrong model to estimate them.

Conclusion

My work is about liquidity estimation of CAPM. It consists from four chapters: first one is theory, where it is possible to find just short part of main definitions, which I use – stocks, liquidity and capital asset pricing model. CAPM is the traditional model, which usually used for finding return of the stock, because it includes market factor, risk-free rate and beta, which is responsible for riskiness of an asset. Liquidity is closely correlated with stock, because any type of security should be liquid.

Second part consists of the literature review. It quire hard to limit article for work, because liquidity and asset pricing are two important factors, in which interested a lot of researchers. Literature review helped me to choose liquidity measure – the bid-ask spread. Spread could be calculated in different way, but I prefer two simple – Corwin and Shultz (2002) estimator and Roll (1984) estimator. And also it was decided to use two-factor regression against market and liquidity.

Third part is about data collection and calculations. I work with Norwegian stock market, so, all needed information is possible to download from Titlon, but it has some problems with data – some measures are missing, what in future could be a cause of bias results. Generally, I need to work with 582 companies, daily data, period of time is 15 years – from 2000 until 2015. So, it is quite huge amount of data. CS and Roll estimators were calculated using all necessary data and applying specific formulas. The risk-free rate and the market return were downloaded from external resource - Oslo Stock Exchange (OSE). Moreover, it was decided to use liquidity factor, which is given by Fama and French at OSE.

Fourth part is practical one. There were used five different regressions to compare them. It is quite hard to make any objective results, because CAPM usually works, as its traditional model and it applies all necessary factors to calculate the return of the stock. But in my case, results were not as good as it could be expected. Inside each regression there were some sample elements, when I checked them, they gave quite good R-squared, especially, that, which were in cut group of 72 the most liquid companies. But there could be two explanations. First one, because of missing data, it could give wrong results,

because during calculations it was taken in consideration. Secondly, change measure of liquidity.

What I was trying to do, to show close relation between stock and its liquidity measure. As CAPM uses all general information in building model, liquidity should add there more value and make results significant.

Some suggestions for future research – try to use not daily data, but monthly or quarterly. It could help to average numbers and not to take in consideration zero values. Also it could be possible to cut time interval to 10 or 5 years. For last five years, for example, is easier to find relevant information without missing values. Moreover, I think it could be good to cut amount of companies or take only those, which are more or less has the same indexes. And one more way, how to work with stocks, try to apply other measures of liquidity.

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List of companies

1. ABG Sundal Collier
2. AF Gruppen
3. Agasti Holding
4. Akastor
5. Aker
6. Aker Solutions
7. AKVA Group
8. American Shipping Company
9. Apptix
10. Arendals Fossekompagni
11. Atea
12. Austevoll Seafood
13. Avance Gas
14. Avocet
15. Bakkafrost
16. Belships
17. Biotec Pharmacon
18. Bonheur
19. Borgestad
20. Borregaard
21. BW LPG
22. BW Offshore
23. Det Norske
24. DNB
25. DNO
26. DOF Group
27. Eidesvik Offshore
28. Ekornes
29. Electromagnetic Geoservices
30. Emas Offshore Reg S
31. Entra

32. Europris
33. Farstad Shipping
34. Fred. Olsen Energy
35. Gjensidige
36. Golden Ocean
37. Hafslund
38. Höegh LNG
39. IDEX
40. Kongsberg Automotive
41. Kongsberg Gruppen
42. Lerøy Seafood
43. Marine Harvest
44. Multiconsult
45. Nordic Nanovector
46. Nordic Semiconductor
47. Norwegian Property
48. Ocean Yield
49. Opera Software
50. Orkla
51. PGS
52. Photocure
53. Prosafe
54. Protector Forsikring
55. Q-Free
56. REC Silicon
57. Salmar
58. Scatec Solar
59. Schibsted
60. Seadrill
61. SpareBank 1
62. Statoil
63. Stolt Nielsen
64. Storebrand
65. Subsea 7

- 66. Telenor
- 67. TGS
- 68. Thinfilm
- 69. Tomra
- 70. Wilh. Wilhelmsen Holding
- 71. XXL
- 72. Yara

Code for RStudio

(to combine each stock with daily Rf and Rm)

```
(CDlist <- list());
(for (j in 1:(length(ids) )){ ( CDlist[[j]] <- subset(CDD, V4 == ids[j]) ) })
remove(a,q, CData)
(CDlist <- list());
(for (j in 1:(length(ids) )){ ( CDlist[[j]] <- subset(CDD, V4 == ids[j]) ) })
(A <- list()); (B <- list()); (C <- list()); (D <- list());
(for (j in 1:length(ids)){
(input <- data.frame()); (input <- CDlist[[j]] ); (output <- data.frame())
(k <- 1); (for (i in 1:nrow(rf)){
if ( isTRUE( (input[k, 2] == month(rf[i,1]) ) &
(input[k, 3] == year(rf[i,1]) ) &
( input[k, 1] == day(rf[i,1]) ) ) == TRUE ){ (input[k, 8] <- rf[i, 2]);
(k <- k + 1) } } );
(k <- 1) ;(for (i in 1:nrow(rm)){
if ( isTRUE( (input[k, 2] == month(rm[i,1]) ) &
(input[k, 3] == year(rm[i,1]) ) &
(input[k, 1] == day(rm[i,1]) ) ) == TRUE ){
(input[k, 9] <- rm[i, 5] ); (k <- k + 1) } } );
(k <- 1) ;(for (i in 1:nrow(FFD)){
if ( isTRUE( (input[k, 2] == month(FFD[i,1]) ) &
(input[k, 3] == year(FFD[i,1]) ) &
(input[k, 1] == day(FFD[i,1]) ) ) == TRUE ){
(input[k, 10] <- FFD[i, 6] ); (k <- k + 1) } } )
```