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Compare the out-of-sample performance of mean-variance optimization relative to equally weighted or naïve 1/N portfolio

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PREFACE

This paper is the master thesis and has been written in the final part of the Master program at Nord University. The view of this thesis is finance and investment in the field of Exchange Traded Fund (ETF). The task was a bit challenging but informative. We have faced the challenge to choose the appropriate benchmark and at the end it goes well. We have used the data from yahoo finance. We have created diversified portfolios and measure the performances of those portfolios. We have seen different diversification perform differently in the arena of exchange traded funds.

This thesis paper has two major parts. The first part, we called it POPP. This part contains the thesis question and theoretical discussion. The second part is the MOPP which has been written as an article format and this part is developed according to the guideline MDPI. MDPI will be publishing special issues on exchange traded funds and we wish to publish this article in the MDPI.

ACKNOWLEDGEMENT

For this thesis paper many people have inspired us in different ways. First and foremost, we have constructed the idea from our supervisor Thomas Leirvik and special thanks goes to him for providing such an interesting and informative topic. We are grateful to him for all sorts of guidance and constructive discussion throughout the project period. We have written this thesis during the corona pandemic situation and he has guided us through several online meetings and written feedback even in holidays. Without his supervision, it would not be possible to write such an analytical paper.

The next thanks deserve to Yevheniia Antoniuk for the technical support throughout the process. This thesis paper has been written in R Studio and she guided us whenever necessary. We are thankful to our classmates, friends and families for their inspiring words. We have collected data from the yahoo finance and the authority deserves special thanks. Finally we are thankful to each other of this thesis for providing our constructive discussion during the whole process. The task would not be so constructive and analytical without the help and guidance of you guys.

Mesbah Uddin Suruj

Maruf Yakubu Ahmed

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PART 1: POPP

1.INTRODUCTION

Exchange traded funds are a relatively new investment tool that has become popular in recent times. The simplest investment method is through an indirect capital market like ETFs, mutual funds, equity, bond, treasury etc. The direct investment method required adequate knowledge and understanding on how the complex financial market operates. Many researchers have conducted research on actively managed mutual funds and the majority concluded that only a relatively small number of actively managed mutual funds outperform their benchmark index (Elton, Gruber & Blake, 1996). This can probably be associated with the rise in passive investment vehicles like exchnage funds in recent years since its inception. The available studies on exchange traded funds in recent times are limited and the scopes are not adequate. The study on the use of international traded funds as the vehicle to mitigate against risk exposure particularly during bear market is inadequate due to strong correlation between international markets (Vermeulen, 2013). Many studies also show that the majority of investors favor investing in their domestic market rather than foreign market due to home bias phenomenon (Blitz & Huij 2012, Berril & Kearney 2010). In this research paper we will use the mean variance portfolio approach (Kono, Yatrakis, Simon and Segal, 2015) to construct a diversified funds portfolio that will increase return per unit of risk which is superior to the benchmark index.

We have aimed to construct a portfolio using an equally weighted approach and mean-variance approach with different asset classes. The construct portfolio performance is measured and compared with their respective constructed benchmark. We WILL choose the ETF with the highest information ratio in each asset class to construct the portfolios. The monthly dataset will be collected from yahoo finance from the period January 2010 to January 2020. The monthly closing price will be used for calculation and measuring performance in this research paper.

Problem statement: In this study we will compare the out-of-sample performance of mean-variance optimization relative to equally weighted or naïve $\frac{1}{N}$ portfolio

1.1 PURPOSE OF THE STUDY

The objective of this study is to construct a diversified portfolio of equally weighted and mean variance portfolio approaches, measure their in-sample and out-of-sample performance. This is to ascertain how a portfolio is developed based on an equally weighted approach and mean variance approach, will outperform or underperform compared to the respective benchmarks index. This performance measurement will be done using various concepts under portfolio management analysis which will include active return, tracking error, information ratio, Sharpe ratio, and Treynor ratio. The portfolio consists of bonds, equity, leveraged and commodity ETFs traded in the capital markets. The ETFs are passive investment and one of the growing financial innovations in recent decades, but are not traded on the Oslo Stock Exchange.

We hope our research study will throw more light in the potential of passive investment such as ETFs introduction in the Norwegian capital market. The use of different ETFs asset classes in our study will create a diversified portfolio will reduce the risk exposure to the financial market. At the same time this study will also enrich the financial literature in Norway and the world as well.

1.2 SIGNIFICANCE OF THE STUDY

This study will evaluate the performance of two portfolios that will be constructed using the equally weighted and mean-variance portfolio approaches. The mean variance portfolio will be constructed on modern portfolio theory (Markowitz, 1952). The study will empirically test the out-of-sample performance of diversified mean variance portfolios relative to diversified equally weighted portfolios.

We will construct a portfolio using an equally weighted approach and mean variance approach, choosing from four different asset classes of exchange traded funds: stock, bond, commodity and leveraged. The choosing of assets for the construction of the portfolio will be solely based on the information ratio of respective ETFs.

We will choose 40 exchange traded funds, 10 from each asset class. For each asset class we will choose a corresponding benchmark index. The ETFs will be chosen based on two criteria. Firstly the assets will have to be actively trading at least from January 2010 to January 2020 on the US stock market. Secondly, the ETFs should be highly ranked in the ETF database (ETFdb.com) performance ranking report.

2.CONCEPT

Several concepts are relevant with comparing the performance of an exchange fund portfolio depending on information ratio and index. Some are directly connected and others are somehow interrelated. Some relevant concepts are active and passive investment, systematic and unsystematic risk, active return, tracking error, information ratio, Sharpe ratio, Treynor ratio and so on.

2.1 INVESTMENT

In economic terms, investment is the creation of economic growth in a sustainable way. In investment, money is used to purchase assets in the hope that the asset will generate income over time. Things that naturally lose value over time or with uses are not investment. Investment creates economic activity and helps to grow the economy. Investment employed people for buying and selling of goods and services. Employed people get paid and when they spend money, businesses make more profit. These profits can reinvest in further business activities which can expand the economy. Investment includes real estate, precious objects, art that have potential to increase value over time. Investment can be putting time and effort not only for money but also something with a long-term benefit, such as education.

In finance, an investment is a financial instrument that is bought with an idea that the instrument will provide income (usually in the form of interest or dividend) or later will be sold at a higher price for a profit. In other words, investment is the use of money to make more money. Buying of financial securities for the purpose of making a profit is personal investment. We see different categories of investments and each category has various opportunities which may suit any financial plan. For example, equity related investments such as stocks, options, venture capital, index funds have high return but include high risk. For the low risk taker investors have bonds, saving accounts etc. Cash or cash equivalent investment includes interest bearing saving account or money market funds. A portfolio investment refers to a particular asset collection which can include stocks, bonds, mutual funds, money market funds, exchange traded funds and many more.

One of the most concerns of investment is safety of the principal amount. We know, some investors are more risk tolerant than others and they do not care to lose some of their principal in return if they have some chance of generating a higher profit. The level of risk tolerance differs from investor to investor based on their financial condition, income structure, age, financial needs and so on. The difference between saving and investment is saving is putting money for future use without any risk, while investment is putting money to work for future gain and entails some degree of risk.

Investment is important for management of money. It ensures both present and future financial security. The money which is usually generated from investments can provide financial security and income. Investment is the only way to grow passive income. Technically saving accounts generate 1% interest is an investment, but in the real world, most people consider investment to a higher return like investment in stocks, mutual funds, exchange traded funds etc.

We see different forms of investment depending on the cost of investment usually in the form of fees and level of risk. Active investment and passive investment is the most common two types of investment in finance. Specific type of investment is suitable for specific investors. Now discussion will follow about these two types of investments.

2.1.1 ACTIVE INVESTMENT

Active investment usually outperforms by beating the market compared to an index or benchmark. Active investments are maintained by the professional fund managers usually known as portfolio managers. The managers are expert in making investment decisions and capable of taking advantage of price fluctuations in the financial markets. They spend time and effort to create suitable strategies for the specific investors. The managers create investment policy statements (IPS) based on their expertise combining robust research, databases, analyst reports etc to have the best chance of performing. The return of active investment is usually above from the benchmark. The portfolio managers take full advantage of short term price fluctuations of a particular stock or any other asset. The manager usually oversees a team of analysts who look at both qualitative and quantitative factors and try to determine where and when the price will change.

Risk, like the returns are usually higher in active investment. Though professional portfolio managers are involved in the investment process and are free to buy and sell any investment they think would bring high return, but there is no guarantee. They are great when their decision is right, but terrible when they are wrong. Naturally the active investment is volatile and for that the portfolio manager may not achieve the investment objective in some cases. Active investment requires confidence that whoever is investing, the portfolio manager will know exactly the right time to buy or sell. A successful portfolio manager requires being right more often than wrong.

Cost is another consideration for active investment. Generally, the active investments often carry higher management and performance fees that usually reflect in the cost of active investment. Sometimes, active investment can also underperform the market due to the higher expenses.

We see several types of active investment such as trading of share, hedge funds, commodity trading etc. Active investment is flexible, and the portfolio managers are not required to follow a specific index. They can buy and sell any stocks they believe will perform better. Another advantage of active investment is hedging. Portfolio managers can hedge their bets using various techniques such as short sales or put options. Similarly, they can exit specific stocks or sectors when the risk seems too big. Active investment also helps tax management by selling the losing investment.

2.1.2 PASSIVE INVESTMENT

Passive investment is a cost-effective way which limits the amount of buying and selling within the portfolio. It involves less buying and selling and more often follows hold strategy. The aim of passive investment is to perform in line with a market or sector and the goal is to track the index more closely with low costs. Sometimes it is considered as a forget strategy when passive investors limit the amount of buying and selling. The passive investment fund does not involve stock picking and extensive research which is treated as cost efficient. Types of passive investment can include exchange traded funds and index funds. Investing in an exchange traded fund is considered a type of passive investment when the fund tracks an index such as Russell 3000. Exchange traded funds are a combination of assets bundled together to form a single financial product that trades in the stock exchange same as stocks and the value will go up and down in line with the index they are tracking.

An example of passive investment is to buy an index fund that follows the S&P 500 i.e., top 500 of the US public companies. Typically, the index fund will buy all the 500 companies stocks in the same proportion as they appear in the S&P 500 index. Passive investment owns small pieces of thousands of stocks; it earns return simply from the corporate profits over time through the overall stock markets. Passive investors ignore the short-term downturns and keep their eyes on the long-term returns.

Passive investments need ultra low fees as there is nobody to pick stocks and simply it follows the index as the benchmark. It is also transparent as the investors can see which assets are included in the index fund. It is also tax efficient as buy and hold strategy usually results in a small capital gain tax for the year. The main drawback of passive investment is investors are locked into the holdings and no matter what happens in the market. It also usually generates small returns compared with active investment.

Empirical study over decades' shows disappointing results on active investment. Passive investment works best for most investors and only a small percentage of actively managed mutual funds ever do better than passive index funds. The active and passive investments are just two sides of the same coin and they exist together in the financial market. Combination of two investments can further diversify a portfolio and can help to manage overall risk more efficiently.

2.2 EXCHANGE TRADED FUND (ETF)

In general, an exchange traded fund is a type of fund that holds a bunch of securities like stocks, commodities, gold bars, foreign currencies or oil futures and it trades just like stock. Exchange Traded Funds are called ETF as it is traded on an exchange. The fund has its ownership divided into shares and that shares are traded in stock exchanges. An exchange fund is a public security and serves a proxy for the group. Investors can buy and sell it whenever they want during trading hours. Just like stocks, each of this fund has a ticker symbol and intraday price data that can be easily obtained during the trading day. The fund holders are entitled to a portion of the profit, such as earned interest or dividend paid and they may get a residual value, in case the fund is liquidated. Like the stock, the price of the fund fluctuates when they are bought and sold. The ownership of the fund can easily be bought, sold or transferred in much the same way as shares and bonds. Unlike the company stock, the number of shares outstanding for an exchange fund can be changed daily, because of the continuous creation of new shares and the redemption of existing shares. Though it traded like a stock in public stock exchange, an exchange fund does not have its net asset value calculated once at the end of every day (Jeff Desjardins, 2018), like a mutual fund does.

The idea of Exchange Traded Fund was born in 1989 (Gastineau, 2010, P:109). For the first time, in 1990 (Gastineau, 2002, P:32), Toronto Index Participation Fund was introduced in Canada to track the largest stock. In 1993 (Gastineau, 2010, P:200), the Standard & Poor Depositary Receipts (SPDR) was born in North America and marked as the true beginning of the exchange funds industry. Initially, the exchange traded funds aimed at replicating broad-based stock indices; new funds extended their fields to sectors, international markets, fixed-income instruments, and, lately, commodities. During the first several years the funds represented a small friction and growth rapidly average 132% annually from 1995 to 2001 (Gastineau, 2002). The launch of Cubes in 1999 (Deville, 2008) were a spectacular growth in trading volume and have made the major exchange funds as actively traded equity securities in the US stock exchanges. Since then the markets have continued to grow both in number and variety of products as well as asset and market values .

Exchange traded fund markets have begun featuring from 1999 (Marszk & Kato, 2019) and have got introduced in Europe in 2001 (Detlef Glow, 2020). In 2002, there were only 246 exchange funds (StocksToTrade, 2018) available in the world and after that these funds increased rapidly. By the end of 2005, a total of 453 exchange funds were listed worldwide for asset value of US dollar 343 billion and only in the US, overall asset value were 296.02 billion while mutual funds value were 8.9 trillion (Deville, 2008).In 2009, more than 1000 funds were traded in the US exchanges, whereas in 2014, over 2 trillion assets were traded through 1500+ exchange funds in the US and in 2016, with a total amount of 3 trillion assets were traded globally through 4779 funds (StocksToTrade, 2018). These assets were spread across more than 270 global providers and were listed on 64 separate exchanges throughout 51 countries worldwide. The industry now has over 4 trillion (Jeff Desjardins, 2018) of assets under management (AUM) globally and by the year 2021, this industry is expected to surpass the 7 trillion mark for AUM.

An exchange fund tracks an index and passively managed fund, so typically the fund has low expenses and less time intensive. These funds are an easy way to diversify a portfolio, but some funds are traded very thinly and leave them vulnerable to price swings. For example, if an exchange fund tracks the S&P 500 index, it typically contains all 500 stocks from the S&P. But it is not always the case and there are some exceptions.

Exchange funds are cost saving for the investor as brokerage commission is lesser than buying all the stocks held in a fund portfolio individually. Investors need to execute only one transaction to buy, similarly only one transaction during sell. Brokers typically charge a commission for each trade and in case of this fund, the number of transactions is less, and for that investors lead to fewer broker commissions. Even some brokers sometimes offer no commissions trading on certain low cost funds.

There are many views out there on exchange funds, but it is generally accepted that funds provide an inexpensive, transparent and convenient way to get access to many different asset classes. Some of the main pros of exchange funds include: it can buy, sell or transfer at any time of business hour; no sales load, but lower brokerage commissions; more tax efficient; unlike mutual fund this fund can place a variety of types of orders such as limit orders, stop-loss orders, buy on margin; can move in and out of market quickly, hoping to catch shorter term savings like a hedge fund, can invest on assets using stock and bond and adjust the allocation according to risk tolerance goals; can add alternative assets like gold, commodities or the emerging stock market; access to many stocks across various industries, low expense ratios; risk management through diversification.

Despite this growth and a wide range of benefits, the funds do have some drawbacks. Some funds are very thinly traded and provide wide bid/ask spread and lower liquidity. Sometimes technical issues like tracking error can cause a performance gap between the exchange fund and the index it tracks. There are also some counterparty risks. For example, even if an investor owns physically backed gold through the SPDR Gold Trust, there is still a chance that in extreme situations the investor may not actually get to see the benefit of that gold. The counterparty risk stems from the possibility of a party failing to deliver on their promises and is quite common to see with other types of assets, as well (Jeff Desjardins, 2018). Some other notable cons include: sales are not settled for 2 days following a transaction and for that the funds from a sale are not available to reinvest for 2 days; actively-managed exchange funds have higher fees; single industry focus funds limit diversification; lack of liquidity hinders transactions etc.

There are thousands of exchange funds available in the stock exchanges. Some of the popular funds are SPDR S&P 500, iShare Russell 2000, Invesco QQQ, oil sector fund, energy sector fund, financial sector fund, crude oil fund, natural gas fund etc. For this paper we will use 40 exchange funds from four different sectors. We will take best performing 10 from stock, 10 from bond, 10 from commodity and 10 from leveraged. In the next discussion about different types of exchange funds that will be used for our work.

2.3 TYPES OF ETF

There are different types of exchange funds available in stock exchanges. The funds have revolutionized the investing industry by making it easier than ever for investors to get exposure to the wide possible range of investments. With thousands of funds available worldwide, it can be very challenging for investors to figure out which types of funds are best for their portfolios (Dan Caplinger, 2017). Some of the most common exchange traded funds are: actively managed, bond, broad market, commodity, currency, dividend, exchange traded notes, foreign market , index, industry, inverse, leveraged, sector, stock and style exchange traded funds.

In this paper we will work on four different types of exchange funds: stock, bond, commodity and leveraged. At the same time we will consider four benchmark indexes for the four asset classes. Russell 3000 will be considered as the benchmark of stock funds. For the bond, we will use Vanguard Total Bond Market Index Fund ETF Shares as a benchmark. S&P-GSCI Commodity Index Future and S&P 500 will be considered as the benchmark of commodity and leveraged respectively.

Stock ETF have some internal mechanics that make them very different from a typical stock. The most glaring is the fact that the funds have what is called continuous issuance of shares via the creation and redemption mechanism. This feature enables rapid expansion or redemption of shares outstanding in a fund and is the main facilitating feature that has enabled fund volumes and assets to grow. It is the creation and redemption functionality that unlocks all of the underlying liquidity in a stock fund, making it accessible to every investor. An exchange fund trade is different. A large proportion of fund trades take place between a bullish or bearish investor and a liquidity provider. So instead of buying from another investor with an opposing viewpoint, the investor typically is trading versus a liquidity provider. (Abner, D. J., 2016). Stock exchange funds track a particular set of stocks similar to an index. Stock funds can track a single industry such as energy. It can also track an entire index of stocks like S&P 500 or Russell 3000.

Bond ETF is designed to provide exposure to the bond market, virtually every type of bond available, the US treasury, corporate, municipal, international, high yield and more. The investors can find broad-market bond funds that cover the entire market, or bond sector funds that focus on types of bonds. With bond funds, the investors need to know whether the fund focuses on a maturity or rolls over maturing bonds to buy new ones. The interest rate risks are different for each of those two types of funds, so matching up the exchange funds to investors' needs is paramount (Dan Caplinger, 2017). Bond funds are more liquid than individual bonds and mutual funds. Bond funds also pay out interest through a monthly dividend and a capital gain is paid out through an annual dividend. Bond exchange funds are available on a global basis.

Commodity ETF is designed to track the price of commodities such as gold, oil or corn. Commodity funds offer exposure to commodity markets, which can provide uncorrelated returns that aren't necessarily linked to those of the stock market. That can be useful for investors looking for true diversification across asset classes. Most commodity funds use futures or other derivatives to get their exposure to their respective markets, while a few actually buy the physical commodity itself, with each share representing a corresponding amount of the commodity. Because of their use of derivatives, many commodity funds can require being familiar with vagaries of the future markets in order to avoid traps (Dan Caplinger, 2017). Those that own physical commodities incur costs that slowly erode their value. Yet for many, those downsides are worth it to get a different type of asset exposure.

Leveraged ETF is a marketable security that uses financial derivatives and debt to amplify the returns of an underlying index. While a traditional exchange fund typically tracks the securities in its underlying index on a one to one basis, leveraged funds may aim for a 2:1 or 3:1 ratio. Leveraged funds that track the S&P 500 might use financial products and debts that magnify each 1% gain in the S&P to a 2% or 3% gain. The extent of the gain is contingent on the amount of leveraged used in the fund. Leveraging is an investing strategy that uses borrowed funds to buy options and futures to increase the impact of price movements. In reverse, leveraged funds can work in the opposite direction as well and lead to losses to investors. If the underline index falls by 1%, the loss is magnified by the leveraged. Leveraged is a double-edged sword meaning it can lead to significant gains, but it can also lead to significant losses (James Chen, 2019). Investors should be aware of the risks to leveraged funds since the risk of losses is far higher than those from traditional investments. Leveraged funds are available for most indexes, such as NASDAQ 100 and the Dow Jones Industrial Average.

Index ETF is designed to track a particular benchmark index like the S&P 500, NASDAQ 100 or Dow Jones Industrial Average. Index funds are increasingly popular as they provide investors with low-cost access to diversified, passive indexed strategies. With an index fund investors can gain exposure to numerous securities in a single transaction. For this paper we will consider four different benchmark indexes for four different asset classes. We will discuss the four benchmark indexes in the benchmark part. Finally we will create a single benchmark to calculate the information ratio, Sharpe ratio, Treynor ratio and others calculations.

2.4 RETURN

Return is the amount of money that an investor receives from the investment. In other way, return is the change in price of an asset or investment over time and it is the main financial goal of any financial investment. Return can be positive or negative. Return tells an investor whether money is made or lost on the investment over some period. A positive return represents a profit while a negative return marks a loss. The return earned during a period of one month is known as monthly return while a yearly return is called annual return.

Amount of return depends on a lot of different things and risk is the main driving force. Empirically high-risk investment generates a higher rate of return and low risk investment generates relatively lower rate of return.

There are several ways to measure investment return. A return is often calculated as a percentage or ratio of the original investment. Some investors like the total return of a portfolio including distribution and contribution while others prefer the net result (after fee, taxes and inflation). Annual returns are usually used for comparison purposes and a holding period return is calculated to see the total gain/ loss during the entire period. Annual return calculates the price change from today to that of the same date one year ago.

In this paper we will use monthly returns of 40 different exchange funds and four benchmarks for the duration of 10 years. The most widely used formula of calculating monthly return is:

$$\text{Monthly return} = \frac{\text{Closing price of last day of month}}{\text{Closing price of last day of previous month}} - 1 \tag{1}$$

Annual return is very important to get a realistic result of how the investment is performing. In the monthly returns, there are usually some good months and some bad months. If the monthly return is same in each month, then we can calculate the annual return easily:

$$\text{Annual return} = (1 + R)^{12} - 1 \tag{2}$$

Where, R is the monthly return which is the same for 12 month. For example, monthly return is 2%, same for 12 month, then annual return will be: (1+0.02)^12 - 1 = 0.2682 or 26.82%. But usually the monthly returns are not the same in most financial investment and there are often ups and downs. In that case, the formula for annual return is:

$$(1 + r_1)(1 + r_2).......(1 + r_{12}) - 1 \tag{3}$$

Here, r is the monthly return and r will be negative where there will be negative return.

To come up with a single number of returns over multiple holding periods, some form of average is required. The two well recognized methods are geometric vs arithmetic average. But the two methods give different results and the geometric method is the most professional method. To calculate the arithmetic average in our dataset, we would add all the monthly returns and then divide by the number of months (120 months in this case). The arithmetic average does not capture month to month volatility. But the geometric average applies compounding to each month's return. Most returns are reported as an arithmetic average because this is the highest average that can be reported to attract investors but the arithmetic average is not accurate as there is no volatility.

Geometric average is the average return of an investment over time. To calculate the geometric average of an investment we need to multiply the items together and then take the nth root. Mathematically:

$$(\prod_{i=1}^{n} x_i)^{1/n} = \sqrt[n]{x_1 x_2 x_3 .....x_n} = (x_1 x_2 x_3 .....x_n)^{1/n} \tag{4}$$

$$\text{Geometric average} = [\prod_{i=1}^{n} (1 + R_n)]^{1/n} - 1 \tag{5}$$

Where, R_n = Return for the time n and
x_i = return of time i

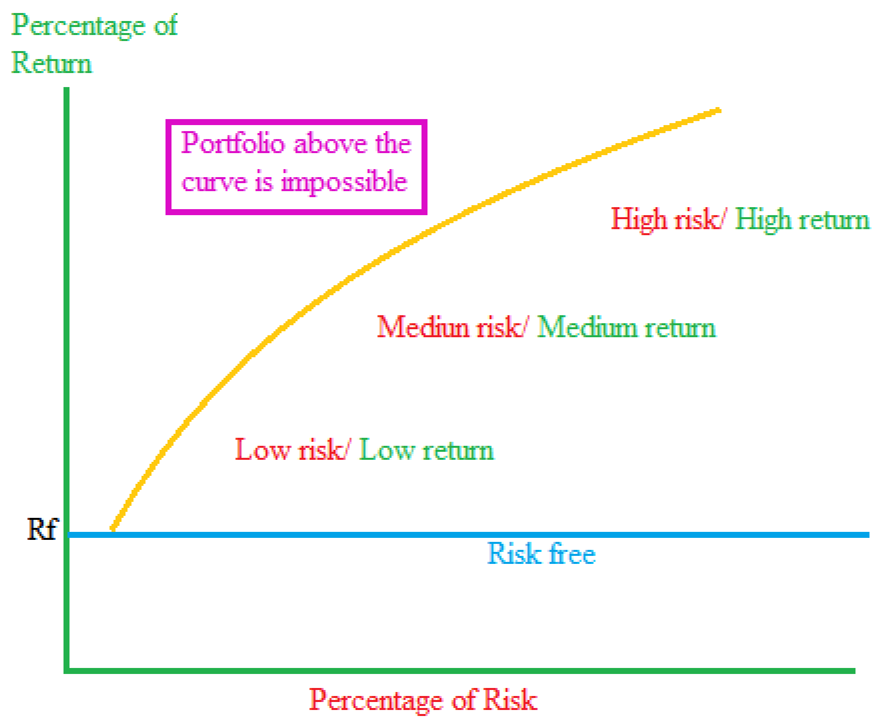

Figure 1: Risk and return relationship

Mathematically, the return on an actively managed portfolio R_P is depended on the returns to the individual exchange funds and the weights of each fund in the portfolio (CFAI, 2020):

$$R_P = \sum_{i=1}^{N} w_{P,i}.R_i \tag{6}$$

Here, $w_{P,i}$ is the portfolio weight of fund i.

2.5 RISK

Risk is the possibilities of occurring loss in investment. In other words, it is the differences between expected result and actual result or the degree of future uncertainties. Investment may lose some or all due to risk. The Capital Asset Pricing Model (CAPM) defines risk as the volatility of returns. Risk involved in the future deviation of uncertainty. All investment involves some degree of risk and for that it is very important to examine and understand the level of risk before any investment.

Involvement of risk is universal with any kind of financial investment in some way. There are different kinds of risk including investment risk, business risk, market risk, inflation risk, stock volatile risk, interest rate risk, liquidity risk and many more. Different kinds of risks originate from different situations. Every investor must weigh the potential reward against the risk. Investment such as in stocks, bonds and mutual funds, each have its own risk profile and understanding the differences can help the investor to diversify the investment more effectively.

Risk and return have horizontal relationships. The traditional rule of thumb is the higher the risk, the higher the potential return. Riskier assets usually have higher volatility and higher expected returns. In general as investment risk rises, investors seek higher return to compensate themselves for taking such risks. We see different risks and returns in every savings and investments. It depends how readily investors can get their money back when needed, how fast the money will grow and how much safe the money is. Risk measures the future uncertainty that an investor is willing to take to realize a gain from an investment. There are many ways to measure risk including standard deviation and variance and risk measurement is different for different investors based on their risk tolerance level. Risk tolerance level depends on some factors like financial condition, income structure, age, duration etc. When an investor chooses to put his money into investments that are riskier than a standard saving, the investor must measure the risk in different areas like possibilities of losing principal part or whole, inflation, required return, management costs, fees etc.

Out of several types there are two main categories of risk, systematic and unsystematic which are most relevant in financial investment. Next will follow a brief discussion of these two types of risks.

2.5.1 SYSTEMATIC RISK

Systematic risks are part of total risks that are usually caused by the macro factors. The causes of systematic risk are out of control of a specific company or individual investor. It involves the uncertainty of the market for external factors that impact all or many companies. When all the investments in the markets are under the systematic risk, there is no way to diversify the individual investment. Systematic risk is a non-diversifiable risk and cannot be diversified away by holding many securities. The causes of systematic risks involve economic, political or social factors which include market risk, interest rate risk, purchasing power risk and exchange rate risk. This risk is unpredictable, and it is not possible to avoid this risk completely. Hedging or correct asset allocation can reduce the consequences of this risk.

Systematic risk can be measured by the sensitivity of a security's return with respect to overall market return.

$$R_P = \alpha + \beta R_M + \epsilon \tag{7}$$

Where $\mathrm{R_P}$ = Portfolio returns
$\mathrm{R_M}$ = Market returns
α = Intercept
β = Beta coefficient
ϵ = Error term

The sensitivity can be calculated by β (beta) coefficient. The β coefficient is calculated by regression the security's return on market return. The value of β can be calculated using the below formula:

$$\beta = Correlation(R_P, R_M)\frac{\alpha_P}{\alpha_M} \tag{8}$$

$$\beta = \frac{Covariance(R_P, R_M)}{Variance(R_M)} \tag{9}$$

Where α_p = Portfolio standard deviation
α_M = Market standard deviation

The β of a stock or portfolio measures the volatility of the instrument compared to the overall market volatility. β is used as a proxy of systematic risk and can be measured how risky a stock is relative to the market risk. In other words β coefficient measures the relative systematic risk of an asset. Assets with β larger than 1.0 have more systematic risk than average. Similarly assets with β smaller than 1.0 have less systematic risk than average. The β value of a portfolio can have the following interpretation:

When β=0, the portfolio is uncorrelated with the market return
When β<0, the portfolio has an inverse correlation with the market return
When β=1, the portfolio return has a perfect correlation with the market portfolio return
When β>1, the portfolio has a positive correlation with the market return (greater volatility)
When $0 < \beta < 1$, the portfolio return is positively correlated with the market return (small volatility)

The β coefficient of publicly traded companies can be on different online investment services such as MSN Money or USA Online Stock Trading. For example if IBM has a β coefficient of 1.05 and Ebay has 1.45. Since IBM has a lower β, the risk can be reduced by placing a greater percentage of the investment in IBM.

2.5.2 UNSYSTEMATIC RISK

Unsystematic risk is associated with a particular investment. It is unique to a particular company or industry and it is asset specific or company specific. Unsystematic risk is also known as 'specific risk', 'diversifiable risk', 'nonsystematic risk' or 'residual risk'.

Unsystematic risk can be of various types, such as a new competitor in the marketplace, a change of regulation, a shift in management, sudden strikes of the employees or natural disaster. Investors may understand some sources of unsystematic risk, but it is impossible to know when and how this might occur. The unsystematic risk is different in each investment of a company. Unsystematic risk can be reduced and even be eliminated through appropriate diversification and increasing the number of total investments.

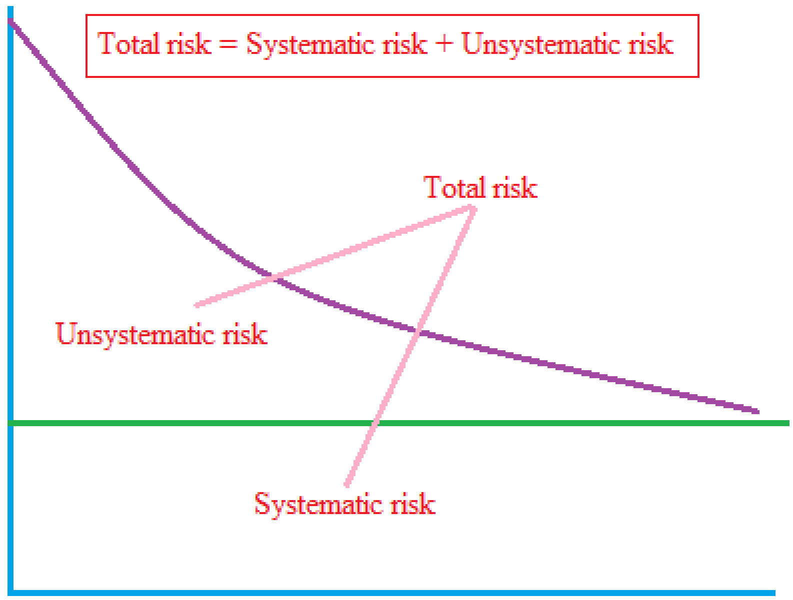

Figure 2: Systematic and unsystematic risk

Unsystematic risk cannot be calculated like the systematic risk as the occurrence is uncertain and there is no formula to calculate the unsystematic risk. It is usually calculated by subtracting the systematic risk from the total risk.

$$\text{Unsystematic risk} = \text{Total risk - Systematic risk} \tag{10}$$

Alternatively, unsystematic risk can be calculated through the below three steps where variance is the risk:

$$\text{Total variance} = \text{Systematic variance} + \text{Unsystematic variance} \tag{11}$$

We can calculate the systematic variance via systematic risk

$$\text{Systematic variance} = (\text{Systematic risk})^2 = (\beta.\sigma.\text{Market})^2 \tag{12}$$

Now we can rearrange and get unsystematic risk:

$$\text{Unsystematic variance} = \text{Total variance - Systematic variance} \tag{13}$$

If the number of risk is standard deviation instead of variance, then unsystematic risk is:

$$\text{Unsystematic risk} = \sqrt{(\text{Total variance-Systematic risk})} \tag{14}$$

2.6 RISK FREE RATE ($\mathbf{R_f}$)

The risk free rate or the risk free rate of return (R_f) is the interest rate that an investor can expect to earn from the investment that carries zero risk. It is one of the most basic terms in finance and used in many theories like capital asset pricing model, modern portfolio theory , Black-Scholes model and some other theories. In this paper we will use the R_f to calculate the Sharpe ratio and Treynor ratio. R_f is an important building block for the mean variance portfolio approach. The R_f is the baseline where the lowest return can be found at the least amount of risk. Theoretically Rf is calculated by:

$$R_f = E(R) - E(R_M) \tag{15}$$

Where R_E = Expected market return
R_M = Expected market risk premium

R_f applies in theories but in reality it is a government security. Typically R_f is represented by the US Treasury bills or T-bills which have zero risk. US T-bill rate is granted by the US government and it has never defaulted on any of its debt obligations. The R_f is commonly considered to be equal to the interest paid on a 3 month government T-bill which is the safest investment that an investor can make. The R_f is a theoretical number and common practice to refer to the T-bill rate as the R_f.

There are limited options to use alternatives other than the US T-bill as the T-bill has a grasp of other areas of risk that can have indirect effects on R_f assumptions. There have been attempts to use other options, but the US T-bills remains the best option for its closeness to investment both in theory and in reality. The security of R_f differs from investor to investor and the rule of thumb is to consider the most stable government body offering T-bills in a certain currency. An investor investing in the US securities should use the US T-bill rate while an investor investing in securities that trade in Euros of Francs should use a Swiss or German T-bill. Investors in Norwegian market should follow the T-bill rate of Norges Bank. In this paper we will collect data from yahoo finance and all the data are based on the US dollar, we will use the average 10 years (January 2010 to December 2019) US T-bill rate for our calculations.

2.7 BENCHMARK

The benchmark such as S&P 500 or Russell 3000 is a standard of measurement to analyze allocation, risk and return of a given portfolio. A variety of benchmarks can be used to understand how a portfolio performs against various market segments. Benchmark may have a number of qualities to serve as a relevant comparison for active management (CFAI, 2020). The investor usually selects assets from the benchmark indexes.

Benchmarks are treated as passive portfolios and weights of benchmarks are verifiable. An available security market as an example NY Stock Exchange index is usually used as the benchmark portfolio S&P 500 index. Benchmark weight can be determined in different approaches. The most common way to determine the benchmark weight is by dividing the market value with the portfolio value. The total market value is called market capitalization. Market capitalization and weight plays a dominant role in the development of capital market theory. Market capitalization index is self-rebalancing. Many investors can simultaneously hold the assets. Float adjusted market capitalization is considered a better way of calculating market capitalization as it provides a more accurate reflection of market movements and assets actively available for trading in the market. It only counts those assets that are available to the investors and excludes those assets that are held by the government or other companies. The non-float adjusted indexes count all the assets which are available with the investors, with the government plus with other companies. When all assets are included in the market, the value added from active management becomes a zero-sum game with respect to the market. Here lies the importance for using a float adjusted capitalization weighted market index as benchmark. Active investors cannot outperform in non-float adjusted index as the active management is a zero-sum game. In the zero-sum game, average performances are achieved by all investors (CFAI, 2020). Benchmark has a narrower definition compared with total market and active investors may invest in the assets outside the benchmark index.

Mathematically, the return on the benchmark portfolio R_B is depended on the returns to the individual funds and the weights of each fund in the portfolio (CFAI, 2020):

$$R_B = \sum_{i=1}^{N} w_{B,i}.R_i \tag{16}$$

Here, R_i is the return on fund i,
$W_{B,i}$ is the benchmark weight of fund i,
N is the number of funds

We will use four benchmarks for four different asset classes. Russell 3000 for stock, Vanguard Total Bond Market Index Fund ETF Shares for bond, S&P-GSCI Commodity Index Future for commodity and S&P 500 for leveraged exchange traded funds.

Russell 3000 (^RUA) tracks performance of 3000 largest US traded stocks which represent almost 98% of all the US incorporated equities. This index also serves as the large cap Russell 1000 and small cap Russell 2000 index. The 1000 largest stocks of Russell 3000 constitute the Russell 1000 large cap and the rest 2000 build the Russell 2000 small cap. Russell 3000 employs a fully passive strategy and does not attempt to outperform a benchmark. Russel 3000 is an equity index that provides exposure to the entire US stock market.

Vanguard Total Bond Market Index Fund ETF Shares (BND) is the proxy benchmark index selected for Bond ETFs asset class and provides broad exposure to the US investment grade bonds. It offers relatively high potential for investment income. It is more appropriate for medium or long-term goals where investors are looking for a reliable income stream. It is passively managed and provides moderate current income with high credit quality. All its investments are selected through the sampling process and at least 80% of its assets are invested in bonds held in the index.

S&P-GSCI Commodity Index Future (GD=F) is made up of 24 exchange traded futures contracts and covers physical commodities spanning five sectors. The sectors in 2019 are energy, industrial metals, precious metals, agriculture and livestock.This index is a composite index of commodities and measures the performance of the commodity market. This index is equivalent to S&P 500 and Dow Jones stock indexes. Investing in this index provides a broadly diversified, unleveraged and long-term position in commodity futures. This index is also designed to track performance of exchange fund products. This index is valuable as an economic indicator and a commodities market benchmark.

S&P 500 (ˆGSPC) is a capitalization weighted index of 500 stocks. It is a popular index and it is used to measure the performance of the large cap US stock market. The leading 500 companies are selected for this index. Portfolio managers often index their portfolios to match or beat the S&P 500 index. Many investors believe that tracking a popular index for passive investment is good to ensure long term returns with minimal costs. Many exchange funds use S&P 500 as a benchmark to track the performance.

All exchange funds of the benchmark are not part of the actively managed portfolio. Empirically, the weight of the excluded fund will be zero (CFAI, 2020). Similarly an investor might include funds in the active portfolio that is not available in the benchmark. In that case the benchmark weight would be also zero.

The performance of a portfolio is often measured against a benchmark index. A benchmark is often a broad index of assets like S&P 500 and Russell 2000. The role of the created portfolio is then to outperform the index without adding too much risk. How much risk is too much depends on the type of fund and the capabilities of the investor. A created portfolio usually outperforms compared with a benchmark index (CFAI, 2020). The outperformance can be measured by a higher expected return with the same level of risk or earn the same return with a low level of risk compared with the benchmark.

2.8 ACTIVE RETURN

The active return is also known as value added and passive investing. The active return of a portfolio is the difference between the portfolio return and benchmark return.

$$R_A = R_P - R_B \tag{17}$$

The active return can be positive or negative. If the $R_P > R_B$ then the active returns R_A will be positive and vice versa.

The main objective of creating a portfolio is to add value so that the ultimate return on an active portfolio could be higher than the benchmark return. Active return is the relative performance compared with portfolio performance and benchmark performance. This is also called the active return of passive investment. The created portfolio can outperform or underperform compared with a benchmark portfolio. When the created portfolio outperforms ($R_P > R_B$) the benchmark portfolio, active return is positive. If the portfolio underperformed the benchmark portfolio ($R_P < R_B$), active return is negative. In the later case when ($R_P < R_B$), the investor can simply hold the benchmark portfolio at least to avoid net fees and expenses of the active portfolio.

The active return can be measured by portfolio's risk relative to the benchmark, usually denoted by the portfolio's beta. Mathematically:

$$\alpha_P = R_P - \beta_P.R_P \tag{18}$$

The term alpha (α) is also often used to refer to active return, which usually assumes that the beta of the managed portfolio relative to the benchmark is 1.

From the equation (18) - (17), we can write the difference of portfolio weight and benchmark weight as the difference of active weight:

$$\Delta w_i = w_{P,i} - w_{B,i} \tag{19}$$

Here the symbol Δ (delta) denotes the difference of portfolio weight from the benchmark weights. These values are known as the active weights Δw_i of the managed portfolio.

Combining the equations (17) and (18) and considering this definition for active weights returns, we get the active return $\mathrm{R_A}$ as the sum product of active weight Δw_i and security returns $\mathrm{R_i}$.

$$R_A = \sum_{i=1}^{N} \Delta w_i.R_i \tag{20}$$

We have mentioned that the sum of the active weights Δw_i is zero. In that case, we can write the active return $\mathrm{R_A}$ as the sum product of active weights Δw_i and active security returns $\mathrm{R_{A,i}}$:

$$R_A = \sum_{i=1}^{N} \Delta w_i . R_{A,i} \tag{21}$$

Where, $R_{A,i} = R_i - R_B$

The above equation indicates that positive active return will generate when the securities which have higher returns than the benchmark returns ($R_i > R_B$) are over weighted. Similarly the securities which have lower return than the benchmark return ($R_i < R_B$) are underweighted.

We can simply illustrate the concept of active return through a numerical example. Suppose that we have a composite portfolio that has stock, bond, commodity and leveraged exchange funds and benchmark weight is 25% each. We believe that leveraged and commodities will outperform compared with stock and bond in next year and we hold a portfolio where weight of leveraged & commodities are 75% and bond & stocks are 25%. In this situation the managed portfolio is called to be overweight by leveraged & commodities (75%) and underweighted by bonds and stocks (25%). The active return is 25% on leveraged & commodities and (-)25% on stock & bond.

2.9 TRACKING ERROR (TE)

Tracking error (TE) can be defined commonly as the time series standard deviation of the differences between a securities return ($R_{security,t}$) and its benchmark index return ($R_{index,t}$) (Grinold & Kahn: 1999). In this paper, securities are four types of exchange traded fund ($R_{etf,t}$). Mathematically:

$$TE = \sigma[R_{etf,t} - R_{index,t}] \tag{22}$$

In general, a created portfolio typically aims for an expected return higher than the benchmark return and at the same time to have a low TE volatility to minimize the risk of significantly underperforming the index. Roll (1992) and Jorion (2003) stated that mean variance analysis in the excess return framework is a standard tool of active managers.

The common definition of TE effectively assumes a beta (β) equal to 1 (one) with respect to the benchmark index and any deviation from β of 1 will generate TE. But Cremers & Petajisto (2009) adopted a slightly modified definition of TE, obtained by regressing excess fund returns on excess index returns.

$$TE = R_{etf,t} - R_{f,t} = \alpha_{etf} + \beta_{etf}(R_{index,t} - R_{f,t}) + \epsilon_{etf,t} \tag{23}$$

$$TE = \sigma[\epsilon_{etf,t}] \tag{24}$$

Following from the above definition, any persistence allocation to cash or high β or low β fund will not contribute to the measure of TE.

For the TE, the calculated expected return may be negative. The main causes of negative TE may be that higher expenses can lead to lower returns; or higher degree of index replication typically lead to lower TE; or fair value pricing can be an influential driver of TE.

TE measures the consistency of excess return. TE is important to determine how active the strategy of creating the portfolio is, the lower the TE, the closer the benchmark. In contrast, the higher the TE, the more deviates from the benchmark.

2.10 PORTFOLIO WEIGHT

Portfolio weight means the percentage of an investment portfolio and the most basic way to determine the weight of an asset is by dividing the value of the asset by the total value of the portfolio. This approach gives a more accurate picture of the weight of various assets in the portfolio. A portfolio is created with weights such as 40% stock, 20% bond, 30% commodity and 10% leveraged asset. The value of the entire portfolio changes with the movement of the market. It is very important to keep a sharp eye on the weights in the portfolio and need to adjust them periodically to get better returns. For example, a portfolio is designed 50% commodity and 50% bond funds. Say one or two bond funds soar in price and the weight becomes 65% in bond and 35% in commodity assets. In that situation, the investor may sell some of the high performing bonds which will bring the weight back to the 50-50 and in addition the investor can earn some profit. Market values change frequently and for that portfolio weight depending on market value are like fluid. In this approach the portfolio weight must be rebalanced frequently to maintain the desire weighting.

$$\text{Portfolio weight} = \frac{\text{Value of the asset}}{\text{Total value of the portfolio}} \tag{25}$$

There is another approach to measure the portfolio weight where the number of assets is the unit of measurement. Here the investor needs to divide the number of units of a given asset by the total number of assets in that portfolio. Portfolio weight should be determined depending on investment strategy.

$$\text{Portfolio weight} = \frac{\text{Number of the asset}}{\text{Total number of asset in the portfolio}} \tag{26}$$

In both the approaches, portfolio weight is measured in percentages. In other words, total portfolio weights are considered equal to 1 (one) and calculate the portion of an asset as a percentage of total portfolio either in values or in numbers.

Portfolio weight can be calculated in terms of sectors, geographical resign, index exposure, short or long position, type of securities, small cap, large cap or any other factors. For this paper we will use this number of unit approaches to create the portfolio. But to analyze the performance, we will use the returns to measure different ratios like information ratio, Sharpe ratio and Treynor ratio.

2.11 INFORMATION RATIO (IR)

The information ratio (IR) measures the risk adjusted returns of an asset or portfolio relative to a certain benchmark. The aim of IR is to show the excess returns relative to the corresponding benchmark. The IR shows the consistency of generating excess returns which is measured by the tracking error (TE). The TE is usually calculated by taking the standard deviation (σ) of difference between the portfolio returns and the index returns. σ helps to measure the level of risks or volatility associated with an investment. A high σ means there is more volatility and less consistency. The IR tells the investor how much active return has been earned or expected to be earned incurring the level of active risk.

The selection of benchmarks is subjective. The most commonly used benchmarks are the returns of government issued bonds (such as the US Treasury Bills) or a major equity index (such as S&P 500) or an index that represents the market or a particular sector or industry.

The IR is calculated using the below formula:

$$IR = \frac{R_P - R_B}{\sigma(R_P - R_B)} = \frac{R_A}{\sigma(R_A)} \tag{27}$$

Where $\mathrm{R_P}$: return of portfolio,
$\mathrm{R_B}$: return of benchmark
$\mathrm{R_A}$: Excess return of portfolio over benchmark, i.e., active return
$\sigma(R_A)$: Tracking error, i.e. σ of active return

A high IR can be achieved by having a high return in the portfolio as compared to a lower return in the benchmark and a low TE. A high ratio means that better returns are produced consistently compared to the benchmark index. The IR can be ex post or ex ante. To calculate the ex post or historical IR, realized average active returns and the realized sample σ of the active return are used. Ex post IR could be negative if the active return is negative. In reverse, ex ante or expected IR can be calculated by using the ex ante or forecasted active returns and forecasted sample σ of the forecasted active return. If any investors do not expect that the IR to be positive, the investors should simply invest in the benchmark. In that ex ante case, equation (17) would be replaced by the expected returns:

$$E(R_A) = E(R_P) - E(R_B) \tag{28}$$

The IR is usually used as a performance measure by fund managers. It is also frequently used to compare the skills and abilities of fund managers to generate excess returns relative to a benchmark with similar investment strategies. Higher IR indicates a desired level of consistency and low IR indicates the opposite. Some hedge and mutual funds also use IR to calculate performance fees that they charge their clients. Many investors use the IR to select assets or mutual funds based on their preferred risk profiles. Though past performance is not an indicator of future results, the IR is used to determine the performance of a portfolio, whether it is exceeding a benchmark index or not. IR calculation provides a quantitative result of how well the fund is managed. IR identifies the consistency of the performance by incorporating TE or σ components. The TE identifies the level of consistency in which a portfolio tracks the performance of an index. A low TE means the portfolio is beating the index consistently over time. In reverse, a high TE means the portfolio returns are more volatile over time and not as consistent in exceeding the benchmark.

The IR and Sharpe ratio (SR) are similar to some extent. Both ratios determine the risk adjusted return of a security or a portfolio. IR measures the risk adjusted returns relative to certain benchmarks and SR compares the risk adjusted returns to the risk free rate (R_f). Like SR, IR is affected by the addition of cash or the use of leveraged. Unlike SR, if an investor adds cash to a portfolio of risky assets, the IR for the combined portfolio will shrink and the IR is unaffected by the aggressiveness of active weight. If each active security weight (w_i) of a portfolio is multiplied by some constant (c), the IR will remain unchanged and the active return on the altered portfolio R_C will:

$$R_C = \sum_{i=1}^{n} c\Delta w_i R_{A,i} = c\sum_{i=1}^{n} \Delta w_i R_{A,i} = cR_{A,i} \tag{29}$$

Similarly, the active risk of the altered portfolio is $c\sigma(R_A)$, so the IR of the altered portfolio will:

$$IR_C = \frac{cR_A}{c\sigma(R_A)} = IR \tag{30}$$

Both IR and SR have their usefulness, but the index comparison makes the IR more attractive to investors since the index fund is usually higher than the $\mathrm{R_f}$ return. IR also has some drawbacks. Each investor has a different risk tolerance level and depending on factors such as age, occupation, financial situation, income might have different investment goals. So, the IR needs to interpret differently for each investor. Moreover, comparing multiple funds against a benchmark is difficult to interpret.

2.12 SHARPE RATIO (SR)

Sharpe ratio (SR) is the measure of risk adjusted return of a financial portfolio. SR uses standard deviation (σ) to measure a fund's risk adjusted return. SR is used to compare the portfolio return in excess of risk-free (R_f) rate with the volatility of portfolio's excess return. Higher SR is better for higher return relative to the amount of investment risk it has taken. The ratio provides a measure of how much is receiving in excess of a R_f rate for assuming the risk of the portfolio. SR measures the excess portfolio return over the R_f rate relative to its σ. Usually, the 90 days Treasury bill is taken as the proxy for R_f rate. To calculate the SR, subtract the return of the 90 days Treasury bill from the funds return and then divide that figure by the excess fund's σ. If an exchange fund produced a return of 18% and excess fund's σ is 10% and T bill return is 3%, the SR would be $\frac{0.18-0.03}{0.05} = 1.5$. The formula of calculating SR either actively managed portfolio or a benchmark is:

$$SR_P = \frac{(R_P - R_f)}{\sigma_{R_A}} \tag{31}$$

Where, SR_P: Sharpe ratio of portfolio
R_P: Return of portfolio
R_f: Risk free rate of return
σ_{R_A}: Standard deviation of the portfolio's excess return, i.e. the tracking error of the portfolio

σ of the portfolio's excess return is often called volatility or total risk. σ is the denominator, so the higher the σ, the lower the SR. Funds with lower σ can produce a higher SR if the return is consistent. SR indicates a better historical risk adjusted performance and does not indicate a lower volatility fund. Higher SR just means the risk and return relationship is more proportional and optimal. If two assets offer similar returns, the one with higher σ usually has a lower SR. To compensate for the higher σ, the asset needs to generate a higher return to maintain a higher SR. SR shows how much additional return an investor earns by taking additional risk. SR of a R_f asset is zero.

SR can be used for both ex-post and ex-ante. Ex-post is the realized performance of a portfolio over some time period and historical realized return is used to calculate ex-post SR. On the other hand, expected return is used to calculate ex-ante SR. In the ex-ante situation, equation 31 would have the expected portfolio return $E(R_P)$, minus the risk-free rate (R_f) in the numerator. The denominator would have the forecast of volatility $E(\sigma_{R_A})$. The expected return and σ of return will usually vary among different investors. But in the case of ex-post and applied to multiple time periods, the numerator in equation 31 is the difference between the average realized portfolio return (R_P) and the average R_f rate and the denominator is the sample excess σ. Annual data is usually used to calculate ex-post SR. As we have used the monthly fund returns in this paper, we will annualize both the portfolio average return and portfolio σ. The average monthly return can be multiplied by 12 to get the average annual return. But to get the average annual σ, we need to multiply the monthly σ with a square root of 12. The logic for multiplying the σ by the square root of 12 is that variance increases proportionally, not compounded.

The SR is unaffected by the addition of cash or leveraged in a portfolio. Portfolio diversification usually has low to negative correlation tends to reduce the overall portfolio risk and as a result SR increases. If we consider a combined portfolio with a weight of w_P and a weight of $(1 - w_P)$ on risk free. The return on the combined portfolio (R_C) and the σ of combined portfolio will be $[(1 - w_P)R_f]$. Mathematically:

$$R_C = wP.R_P + (1 - wP)R_f \tag{32}$$

σ of combined portfolio:

$$\sigma(R_C) = wP.\sigma(R_A) \tag{33}$$

Applying these two relationships (31 and 32) gives the SR of the combined portfolio as:

$$SR_C = \frac{R_C - R_f}{\sigma(R_c)} = \frac{wP.(R_P - R_f)}{wP.\sigma(R_A)} = SR_P \tag{34}$$

The above equation is the same as equation 32. The weight in the combined portfolio (wP) could be greater than 1. So, (1-wP) could be negative. This means leveraged could be created by borrowing risk free cash and investing in risky assets does not affect the SR of the portfolio.

The investors could form portfolios using two funds according to their preference: one of which is the risk-free asset and the other is the risky asset portfolio with the highest SR. If the expected σ of a risky asset portfolio is higher than the investor's preference, the σ could be reduced by holding more cash and less of the risky portfolio. In the reverse case where expected σ of a risky portfolio is lower than the investor's desire, the σ and expected return can be increased by leverage.

For example, an exchange fund portfolio has one stock, one bond, one commodity and one leveraged asset that means the weight is equally distributed, 25% in each asset. Considering the previous example of SR of 1.5 where R_P was 18%, we get the portfolio return: *0.25x18% + 0.25x18% + 0.25x18% +0.25x18% = 18%*. Similarly, the σ of portfolio's excess return would be 10%. If the risk-free rate is 3%, in this case the SR will be $\frac{18-3}{10} = 1.50$. If another asset is added in the portfolio, let's consider one stock (20%), one bond (20%), one commodity (20%) and two leveraged (40%). The expected return and the σ of the new portfolio will not change. But after this addition the portfolio returns will rise to 23%. σ and R_f rate remain the same. In this situation, SR will be $\frac{23-3}{10} = 2.0$. This shows that the addition of one new asset can give a change to the overall portfolio return without adding any risk.

The SR is a relative measure of risk adjusted return. It does not provide much information about the performance of the fund. Moreover, the measure considers σ which is a symmetrical distribution of returns. For asymmetrical return distribution with a skewness greater or lesser than zero and kurtosis greater or lesser than 3, the SR may not be a good measure of performance. Furthermore, σ takes into account both the positive and negative deviation in return from the mean, so σ does not accurately measure the downside risk. Measures like sorting can consider negative deviation from the mean return and can reduce the limitation of SR to some context.

2.13 TREYNOR RATIO (TR)

Treynor ratio (TR) is a performance metric for determining the excess return generating for each unit of risk taken by the portfolio. Excess return in the sense that the return earned above the return that could have been earned in a risk-free investment similar to Sharpe ratio. This ratio is used to adjust the portfolio returns for systematic risk.TR was developed by Jack Treynor in 1965 and this ratio is also called reward-to-volatility ratio.

TR is similar to the SR. The difference is SR uses portfolio's TE to adjust portfolio's return while TR uses portfolio's beta (β_P). TR is calculated by dividing excess return of the portfolio by the β_P.Mathematically:

$$TR_P = \frac{(R_P - R_f)}{\beta_P} \tag{35}$$

Where, $\mathrm{TR_P}$: Treynor ratio of portfolio
$\mathrm{R_P}$: Return of portfolio
$\mathrm{R_f}$: Risk free rate of return
β_P: The sensitivity of the portfolio towards the movement of the underlying benchmark or market index

The β_P indicates the sensitivity of the portfolio return to the changes in the underlying benchmark or market index.The β of a portfolio invested in a highly volatile asset class will be higher than the portfolio invested in a lower volatile asset class. The higher the β, the higher the investment risks and sensitivity of the portfolio returns and the lower the β, the lower the investment risks and sensitivity of the portfolio returns.

A higher TR means the portfolio is more suitable for investment.The portfolio with a higherTreynor value indicates that the portfolio has a higher risk-adjusted performance compared to a portfolio consisting of a low TR value. When comparing two portfolios it is assumed that the portfolio with higher TR is better at compensating for risk-taking investment as compared with portfolios with lower TR.

2.14 THE PORTFOLIO PERSPECTIVE & MEAN VARIANCE PORTFOLIO

The perspective of a portfolio is to maximize expected rate of return for a given level of risk or minimize the level of risk for a given expected rate of return. In other words, the portfolio perspective is the sum of all the investments and return of the portfolio and the perspective underlies the portfolio management process and investment policy statement. The average returns of many assets are associated with average risks and similarly the risk associated with one asset's returns are usually related to risk associated with other assets' returns. If we only evaluate the prospect of each asset separately and ignore the relationship among the assets, then we will most probably misunderstand the risk and return prospects of the investment position.

The portfolio perspective was introduced by Nobel laureate Harry Markowitz (1952). The author divided the portfolio selection process into two stages. The first stage starts with observation and experience and ends with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio. To develop the theory of portfolio choice, the author began with the perspective of investing for a single period. Later on subsequent researchers such as Treynor (1961) and Sharpe (1966) founded the field of mean variance portfolio theory (MVPT). MVPT is the analysis of rational portfolio choices based on the efficient use of risk and after introducing the MVPT, investment management was felt in the field of revolution. At the beginning professional investment practice began for recognizing the importance of the portfolio perspective as well as for achieving investment goals. Secondly, MVPT helped to spread the knowledge for using quantitative methods in the field of portfolio management. Nobel laureate Merton (1973) explored the dynamics of portfolio choice in a multi period setting. These subsequent contributions make the MVPT content a high level of enrichment. Today portfolio management is more developed and frequently uses both qualitative and quantitative methods.

The main objective of funds or portfolio managers is to construct a diversified asset to minimize risk and maximize expected return. Mean variance portfolio theory has revolutionized the financial world by assisting fund managers to quantify investment risk and expected return. This theory enables investors to construct a portfolio that optimizes expected return based on risk. In recent years, the focus on individual assets risk has shifted to entire portfolio risk. One important innovation of MVPT is redefining the measurement of risks, not in terms of single security, but how the risks of different securities relate in a portfolio (Chernoff, 2002). Markowitz MVPT quantifies the definition of risks in investment to provide means of calculating the cost of risk. Additional risks that must be borne have to be compensated by increasing in the expected return of the portfolio (Markowitz, 1991).

$$E(R_P) = \sum_{i=1}^{n} w_i.E(r_i) \tag{36}$$

According to Fabozzi and Markowitz 2002, appropriate diversification should enable an investor to maximize expected return and minimize risks or maximize return at the same level of risks or minimize risks at the same level of return. In MVPT the sum of the weight of expected return of securities in the portfolio is the expected return of the portfolio. Efficient frontier is the optimal combination of input (risks) per unit of output (expected return) and a diversified portfolio is called an efficient portfolio (Markowitz, 1991). According to MVPT the computation of each security expected return is based on the underlying benchmark, the security track that measures the performance of the market.

$$\alpha_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i.w_j Cov(r_i, r_j) \tag{37}$$

$$\alpha_P = \sqrt{\alpha_P^2} = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i.w_j Cov(r_i, r_j)} \tag{38}$$

where, α_P^2 = The variance of the expected return during the period
Cov(r_i, r_j) = The covariance between the assets
w_i = Weights of asset i and
w_j = Weights of asset j

Markowitz, Merton and some other researchers created the supply for portfolio perspective and the investment community created demand for the perspectives. The three main demands of portfolio perspectives are -

i) The first development related to institutional investment which has increased and is still increasing worldwide. The increased investment is playing a dominant role in financial markets. So, measuring and controlling the risk is very important for the institutions. If the institution fails to measure and tackle the risk effectively, it will be thrown away from the market.

ii) The second development was related to Information Technology (IT), the availability of low cost computers, internet, program, software creates unlimited communication possibilities. As a result for this implementation of MVPT portfolio concepts become easy and populous.

iii) The third related development was related to the professionalization of the financial and investment management arena. This finance and investment professionalization has been crucial and spread worldwide. CFA (Chartered Financial Analyst) is playing a major role in creating financial professionals. The CFA professional knows the utility of the MVPT and thus they are creating unlimited demand worldwide.

2.15 DIVERSIFICATION OF PORTFOLIO

Diversification is the risk management tool that mixes different financial products within an investment vehicle. A constructed diversified portfolio contains a mixture of distinct type assets from a different assets class. The class of assets contained in the diversity portfolio must exhibit perfect negative correlation so that the assets price will move in opposite directions. The rationale is to reduce the risk exposure of the investment vehicle to the assets or risk and on the average, yield higher long-term return and lower the total risks of the individual security. The unsystematic risk events are reduced in a diversified portfolio, so the positive performance of some financial assets offset the negative performance of others.

The primary objective of diversification in portfolio management is to maximize return and limit the volatility on a portfolio. There are different ways of diversification that must be considered when constructing a portfolio.

Individual company diversification is the simplest method of diversification assets by mixing individual stocks of companies through a large group of different exchange traded or mutual funds. This diversification reduces the portfolio risk drastically when additional units of stock are added to the portfolio even though the individual stocks are all of equal risk (Markowitz, 1952). Another research of Longboard Asset Management revealed that over the period from 1983-2006 nearly 2 in 5 stocks actually lost money, almost 1 in 5 lost their value, 2 in 3 underperformed the Russell 3000 index and 1 of 4 stocks over the period accounted for nearly all gains (Issac Presley,2016)

Industry diversification sticks mostly to the stock market like the individual company diversification. This diversification spreads the investment throughout a wider range and have a balanced assets allocation in multiple industries to reduce the risk exposure of only investing in a single industry.An industry diversification portfolio filled with energy stocks would also have investing in food companies or some technical stocks. Most investors tend to invest more in the industry that they are part of and that might avoid diversification.

Asset class diversification is when the fund manager invests in different classes of assets and determines the percentage of portfolio to be allocated to each asset class. Assets may perform differently in various economic conditions. For instance, stocks are likely to perform well during a bull economic condition and bond during a bear economic condition. These assets class include stock, bond, Treasury bill, commodity, forex, leveraged, real estate, certificate of deposit etc.

Geographical diversification create opportunities to reap more diversification benefit by investing in domestic and foreign markets. Foreign securities may tend to be less correlated to domestic securities and may provide opportunity for exposure not offered by the domestic securities. The home phenomena bias may result in investors preferring the stocks of companies based in their countries to the detriment of international diversification to mitigate market volatility.

Time diversification is the continued contribution to the investment account and this diversification may reduce the exposure to the responsibility of poor timing decisions by helping from short-term guessing to long-term planning. This diversification is also known as dollar cost averaging (Carlson, 2015). The investor can disburse the funds equally and invest periodically such as weekly or monthly or quarterly in selected assets weight over the year.

2.16 ROLLING WINDOWS

In the time series analysis rolling window is an important concept. In finance it is usually considered that recent data has more predicted power than the old data. Rolling windows usually consist of the selection of a specific window sized data. Depending on the duration of data, rolling windows can be done in different window sizes such as 6, 12, 24 or 36. For example if the best window size is 12 months, the first window will consist of the first 12 months data. The second window will depend on data from 2 to 13 and the third window will consist of the data from 3 to 14 and so forth.

Rolling window is one of the formal ways to forecast the model. In time series forecasting, we often choose a few potential models and drop some data to see how each model performs. If we do only one or just a few forecasting tests, the next forecast result may change dramatically. Alternatively, if we take half of the sample to forecast the other half, this forecast is absolutely better than one or few but does not account for possible change. One of the most accurate ways to compare models is rolling windows. Rolling window is used to calculate root mean square error (RMSE), mean absolute error (MAE) or mean square error (MSE) and formal tests such as Diebold & Marianno.

$$MAE = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2 \tag{39}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2} \tag{40}$$

Here, n is the number of observed data
Y_i is the observed values
$\hat{Y}_i$ is the predicted values

In general, a fixed rolling window is comparatively better than the expanding window. If we have two models:

$$\text{Model 1} : Y_t = \beta_1 X_{1,t-1} + \epsilon_t \tag{41}$$

$$\text{Model 2} : Y_t = \beta_1 X_{1,t-1} + \beta_2 X_{2,t-1} + \epsilon_t \tag{42}$$

If the true value of β_2 is zero, then both the models will be the same and it may be difficult to distinguish which model is better to forecast y_t. In theory, model 1 is better than model 2, because model 1 has small samples and it is similar accurate as the large samples. Rolling window is free from these types of problems as the rolling window keeps the sample size fixed.

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Part 2: MOPP in article format will start from the next page

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Compare the out-of-sample performance of mean-variance optimization relative to equally-weighted or naïve 1/N portfolio

Candidate Number 14 & 15

PART 2: MOPP

Abstract

We have evaluated the out-of-sample performance of the mean-variance model relative to the equally-weighted model. To do this, we have proposed an Exchange Traded Funds portfolio and implement it consisting of four distinct asset classes: stocks, bonds, commodities, and leveraged. All are traded in the US during the period of January 2010 to January 2020. We have tested two common ways to portfolio construction; mean-variance optimization and the naïve $\frac{1}{N}$ portfolio. These strategies are carried out to choose four ETFs each from one out of four asset classes in our portfolio construction. The in-sample and out-of-sample performance of the mean-variance assets allocation model consistently outperforms the equal-weighted or naïve portfolio model, in terms of Sharpe ratio, information ratio, active return, and return in terms of all the overlapping rolling window estimation have used in the study. However, the mean-variance underperforms relative to equally-weighted in terms of tracking error and Treynor ratio, which indicates that the mean-variance portfolio exhibits a high level of systematic risks compared to the equally-weighted portfolio. This is surprising, as the mean-variance method is supposed to reduce systematic risks. The result of our study indicates a high level of Sharpe ratio and information ratio when applying the overlapping rolling window method.

Keywords: mean-variance, equally-weighted, naïve portfolio, ETFs, out-of-sample performance, diversification

INTRODUCTION

The surge in the amount of passive and active investment vehicles such as Exchange Traded Funds (ETFs), in the past two decades, can be associated with the bad performance of mutual fund portfolios. Many researchers have studied the performance of actively managed mutual fund investment and concluded that few mutual fund managers outperform their benchmark indexes consistently (Carhart, 1999: Bollen and Busse, 2005). It has been a successful financial innovation for investment in the portfolio market or index trading. Over the past two decades, the estimated total assets held by ETFs has grown to about US$4.3 trillion globally (Lettau and Madhavan, 2018). Such passive investment vehicles that track a benchmark index have become very popular in a relatively short period of time. They are opened-ended investment funds that are traded like a stock in the financial market. ETFs are different from mutual funds, as baskets of securities that are traded in real-time on the stock market at a price determined by the supply and demand (Madhavan, 2011).

Generally, ETFs are designed to mimic or track the market or benchmark index and should manage to stay close to their benchmark indexes with low levels of tracking error. They can hold stocks, bonds, commodities, currencies, gold, treasury, or even a mix of different asset classes under one umbrella unified by specific investment strategies. They are considered to be superior to active or passive mutual funds as they often have lower fees on average and traded on the secondary market like stocks. The first ETF was launched in 1993 was the Standard and Poor's Depositary Receipt (SPDR) which tracked S&P 500 as its benchmark index. SPDR, Vanguard, and iShare are the global dominant organizations in the ETF market. Most ETFs are created like mutual funds and operate as stand-alone funds managed in the financial market except for Vanguard which developed its ETFs as a share class of its index fund. The inflow and outflow occur in exchange transactions, where an investor can receive a share of ETFs worth the equivalent amount of a securities turn over that match the organization portfolio fund. In asset management settings, high transactional costs result in extreme portfolio reallocation that may result in the poor out-of-sample performance of the portfolio strategies (Best and Grauer, 1991).

Diversification in ETFs investment has become one of the major ways to reduce exposure to risk and still get a reasonable return from investments. They are one of the vehicles for indirect international diversification that has been widely discussed in recent years, but no consensus has been reached on its benefits by researchers. However, some research studies have argued that though diversification seems promising and brings some benefits to the investor. According to Berril and Kearney (2010), the majority of investors prefer investing in their domestic market rather than the foreign market due to the home bias phenomena. Some research finding shows that the benefits of indirect international diversification through ETFs have declined in recent times, due to the interdependency or correlation among both developed markets and emerging markets (Bekaert, Hodrick & Zhang, 2009: Jacob & Langlois, 2012). The observation of mean-variance portfolio theory indicates that investing in a diversified index fund like ETFs with low management and the transactional cost is the most efficient investment strategy for superior performance (Malkiel, 2003).

Before introducing ETFs, investors had limited tools for overall risk management such as economic conditions, an industry, and an asset class. These conditions can only be managed through the choice of stock or bond traded in a less liquid market. Therefore, ETFs have become the logical choice for investors who want to buy-and-hold and invest in a distinct asset class portfolio. This financial innovation is often referred to as investing democratization has become more widely adopted because it can be accessed through exchanges and held by the largest and smallest investors. The range of investment portfolios in the ETFs market has brought the average investors which were only available to larger firms two decades ago (Hill, 2016).

The Markowitz (1952) mean-variance assets allocation strategy, in theory, provides the investor with the optimal rule of asset allocation across risky investment when only the mean and variance of a portfolio's return are the relevant parameters to the investors. However, when implementing asset management settings, portfolio managers mostly try to cope with the limitation of mean-variance (MV) strategy by putting constraints on producing extreme portfolio weights that fluctuate frequently over time and underperform poorly out-of-sample returns. These suggestions range from multiple statistical approaches for imposing portfolio constraints (Jagannathan & Ma, 2003; Behr, Guettler, & Miebs, 2013), to "diffuse prior" (Bawa, Brown & Klein, 1979). The Bayesian method of estimation error for the MV sample (Jobson & Korkie, 1980; Jorion, 1985, 1986; Pastor, 2000; Pastor & Stambaugh, 2000)use of factor models (Chan, Karceski, & Lakonishok, 1999) and the relaxation of portfolio weight (Kono, Yatrakis, Simon & Segal, 2007).

Furthermore, to better understand and analyze the MV assets allocation strategies it must be compared with equally-weighted or naïve diversify portfolio strategy (DeMiguel, Garlappi& Uppal, 2009). We have defined the equally-weighted or naïve portfolio rule to the fraction of $\frac{1}{N}$ of assets allocated to each of the N assets class available for the investment portfolio. In this portfolio strategy, the investor does not involve in any estimation or optimization done in the MV strategy. The dataset is completely ignored by investors which implies that expected returns are proportional to total volatility rather than market volatility. However, we wish to emphasize that the use of equally-weighted or naïve portfolio strategy is not to advocate for its use as asset allocation strategy, but merely serve as the benchmark index for assessing the performance of various portfolio strategy in the proposed literature (DeMiguel, Garlappi& Uppal, 2009).

DeMiguel, Garlappi, and Uppal (2009), analyzed the out-of-sample performance of various MV strategies, suggested in the literature, were unable to outperform a naïve portfolio strategy. Their study results show that none of the MV portfolio strategies have consistently outperformed the naïve or equally-weighted portfolio. However, Bessler, Opter, and Wolff (2014), investigated the out-of-sample performance of Black Litterman (BL), MV, and naïve or equally-weighted strategy and report that BL significantly outperforms naïve portfolio strategy but MV insignificantly outperform naïve or equally-weighted strategy. The difference in the result may be because DeMiguel, Garlappi, and Uppal, only used stock datasets while Bessler, Opter and Wolff, used stocks and bonds to construct the portfolios. Building on the model of Bessler, Opter, and Wolff, we have intended to use a range of different asset classes of ETFs to study the risk and return of both portfolio optimization strategies as well as the equally-weighted strategy. The construction of a diversified portfolio with ETF securities using the MV method increases return per unit of risk superior to the market index (Kono, Yatrakis, Simon & Segal, 2015). In addition, this study also indicated portfolios constructed based on the international ETFs had the highest return per unit of risk and highest alpha compared with S&P 500 which may not be an optimal proxy for international market index. Harper, Madura, and Schnusenberg (2005), research shows that ETFs exhibit higher return and higher Sharpe ratio on the average, compared with the corresponding closed-end funds. The international diversification vehicle of ETFs to reduce the risk exposure is insubstantial particularly during bear market periods due to the correlation between international domestic markets (Vermeulen, 2013). Blitz and Huij (2012) find that ETFs exhibit higher tracking error in global emerging markets than developed markets ETFs, which relates to equity return in emerging markets having a larger cross-sectional dispersion structure. The study shows evidence of contagiousness in international ETFs securities during the great financial crisis, as an increase in correlation during the crisis as compared to stable periods (Lee & Kim, 1987:

King & Wadhwani, 1990). We find it important to use ETFs in measuring the out-of-sample performance of MV and equally weighted (EW) or naïve portfolio strategies.

For this study, we have collected a large sample of exchange traded funds from which we have chosen four ETFs from four asset classes. We have chosen one ETF from each asset class with the highest information ratio to construct our portfolio. We have created two portfolios with the four chosen ETFs applying the mean-variance optimization method and naïve $\frac{1}{N}$ or equally-weighted portfolio method.

The main objective of this paper is to understand by empirically testing MV optimal portfolio model performance. To do this, we have analyzed the in-sample and out-of-sample performance of diversified MV portfolio strategy relative to the performance of EW or naïve diversify portfolio strategy. There are a limited number of studies that measure the out-of-sample performance using ETFs datasets. We aim to construct a diversified domestic ETF portfolio and the benchmark index traded in the US market. The diversification of the ETFs in a domestic market by using different asset classes aimed to reduce the risk exposure of the portfolio to the market. In this study, we aim to measure the out-of-sample performance of constructed portfolios compared with our constructed benchmark index. We also intend to compare the in-sample and out-of-sample performance of the metrics used in this study.

DATA

The data used in this study were monthly ETFs prices for the period from January 2010 to January 2020 collected from Yahoo Finance and ETFs database. All financial instruments traded in developed and developing countries can be sourced from yahoo finance. We use the closing price and classify them in per the asset classes. All the ETFs considered must be ten years or more and continue to trade as when the data were collected to avoid survivorship bias. We selected four asset classes of ETFs namely stock, bond, commodity, leveraged, and their respective benchmark or market index. We selected ten ETFs and their ten years historical closing price for each asset class. Russell 300 (RUA) was the index for stock ETFs, S&P500 (GDF) index for commodity ETFs, S&P500 (GSPC) index for leveraged ETFs, Vanguard Total Bond (BND) as a proxy market index for bond ETFs and the mean of US Treasury bill rate for the past ten years is used as risk-free rate (R_f). The selections of benchmark index based on what the asset classes mostly track and the correlation between the selected benchmark index and the ETF asset classes.

METHODOLOGY

Construction of portfolio

For the construction of the portfolio, we have chosen the ETF with the highest information ratio in each asset class. When these four ETF have been chosen, we have constructed a portfolio by using the MV method and equally-weighted method of asset allocation strategy. The MV and EW or naive portfolios consist of distinct ETFs that track their respective benchmark or market index. The funds selected for the data analysis were based upon the following criteria:

a) Must be more than ten years of existence
b) Must be in operation as when the data have been collected
c) Must be in the top ten rankings of performing ETFs in the ETF database, only when the first and second criteria are met that the fund can be selected. However, if the criteria are not met the ranking threshold is increased.

PORTFOLIO STRATEGIES

Equally-Weighted Portfolio (EW) or Naïve Portfolio

This portfolio strategy involves holding a portfolio at a certain equal weight in risky assets. The asset allocation will be in a fraction of $\frac{1}{N}$ to each of the N assets class selected for the portfolio construction. Under this portfolio strategy the concepts of optimization, estimating, and data analysis are completely ignored (DeMiguel, Garlappi & Uppal, 2009). The maximum 25% was allowed for each ETF asset class to allow for portfolio diversification. We have chosen the ETFs with the highest information ratio from each asset class for the construction of the portfolio and the benchmark, respectively. However, it must be emphasized that the purpose is not to advocate for naïve portfolio strategy but to use it as the benchmark for comparative out-of-sample performance analysis for the study. For the equally-weighted portfolio, we use

$$W_i = \frac{1}{N} \tag{1}$$

Where w_i = Weight of asset i and N = Number of asset class

Mean-Variance Portfolio (MV)

The mean-variance portfolio strategy proposed by Markowitz (1952) seeks to optimize the tradeoff between the mean and variance of the portfolio's expected return. To implement the model, we have solved the equation below by calculating the mean return and covariance matrix of the asset return of the monthly dataset. The maximum participation of ETFs for each is relaxed from 15% to 50% to optimize the portfolio diversification (Kono, Yatrakis, Simon & Segal, 2007). The ETFs with the highest information ratio is selected from each asset class are used for the MV portfolio construction.

$$\alpha_P^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i.w_j Cov(r_i, r_j) \tag{2}$$

$$\alpha_P = \sqrt{\alpha_P^2} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i.w_j Cov(r_i, r_j)} \tag{3}$$

where α_P^2 represents the variance of the expected return during the period and $\mathrm{Cov}(r_i, r_j)$ is the covariance between the assets i and j. w_i and w_j represent the weights of asset i and j respectively.

Markowitz (1952) study showed that since the risk of the portfolio return is given by the variance of the portfolio return, the optimal portfolio expected return can be derived from the composition of various asset classes.

Performance Measurement

We have analyzed in-sample and out-of-sample performance of MV diversify portfolio strategy relative to EW or naïve diversify portfolio strategy of a monthly return across four asset classes. We have used the following performance evaluation metrics; standard deviation (σ), active return (R_A), Sharpe ratio (SR), tracking error (TE), information ratio (IR), and Treynor ratio (TR) for each portfolio strategy. The aim is to evaluate the out-of-sample performance of the portfolios.

We have employed the overlapping rolling window approach by DeMiguel, Garlappi, and Uppal (2009), to measure the out-of-sample performance of the portfolio strategies. The back-testing dataset of monthly asset returns to analyzed different estimation window lengths of months = 6, months = 12, and months = 24 to ensure the robustness of the performance results.

Information Ratio

Information ratio measures the ability of the portfolio to generate excess returns relative to the benchmark index. The IR shows the consistency of generating excess returns which are measured by the tracking error (TE). The IR is usually calculated by dividing excess return between the portfolio returns and benchmark returns with the standard deviation (σ) of active returns. We have used the below formula to calculate the IR:

$$IR = \frac{R_P - R_B}{\sigma(R_P - R_B)} = \frac{R_A}{\sigma(R_A)} \tag{4}$$

Where $\mathrm{R_P}$ is return of portfolio, $\mathrm{R_B}$ is return of benchmark, $\mathrm{R_A}$ is Excess return of portfolio over benchmark, i.e., active return and $\sigma(\mathrm{R_A})$ is tracking error, i.e., σ of active return.

The benchmark consists of Russell 3000 which is the benchmark for the stock, S&P 500 for the leveraged, S&P 500 Commodity index for the commodity, and Vanguard Total Bond index for the bond. The benchmark would be constructed similarly to the portfolios using MV and EW methods. It is important to clarify that no one asset class of ETFs that perfectly track a specific benchmark index. We have chosen a proxy benchmark index based upon tracking the history source from the ETF database, Yahoo Finance, and the extent of the positive correlation between the ETFs asset class and the chosen benchmark index. The proxy benchmarks indexes are not the true representation of the actual market index of the asset class.

Active Return ($\mathbf{R_A}$)

Active return is the difference between the actual portfolio return and the benchmark return. When the portfolio outperforms the benchmark index, the active return is positive. We measure active return as the out-of-sample return deducted by the out-of-sample benchmark return.

$$R_A = R_P - R_B \tag{5}$$

Where R_A represents active return on assets, R_P is the return on portfolio and, R_B is the return on benchmark index.

Tracking Error (TE)

Tracking error (TE) can be defined commonly as the time series σ of the differences between a portfolio's return ($R_{P,t)}$ and its benchmark index return ($R_{B,t)}$ (Grinold & Kahn: 1999).

$$TE = \sigma[R_{P,t} - R_{B,t}] \tag{6}$$

Roll (1992) and Jorion (2003) stated that mean-variance analysis in the excess return framework is a standard tool of active managers.

Sharpe Ratio (SR)

SR was introduced by William Sharpe (1966) after the launch of the Treynor ratio. SR is the measure of risk-adjusted return of a financial portfolio. SR uses standard deviation (σ) to measure a fund's risk-adjusted return. We have measured the out-of-sample performance SR for the strategy. To do so, we have defined the sample mean of out-of-sample return minus risk-free rate (R_f), divided by their excess sample standard deviation.

$$SR_P = \frac{(R_P - R_f)}{\sigma_{R_A}} \tag{7}$$

Where, SR_P is Sharpe ratio of portfolio, R_P is return of portfolio, R_f is risk free rate of return and σ_{R_A} is standard deviation of the portfolio's excess return.

Standard deviation (σ) of the portfolio's excess return is often called volatility or total risk or tracking error. σ is the denominator, so the higher the σ, the lower the SR.

Treynor Ratio (TR)

This ratio measures the returns earned more than risk-free returns per unit of systematic or market risks. This ratio was developed by Jack Treynor (1965) to measure performance called the Treynor rate. It is calculated by dividing the excess return of the portfolio by the beta of the portfolio. The portfolio's beta (β) indicates the sensitivity of the portfolio return to the changes in the underlying benchmark or market index. To do this, we define the sample mean of out-of-sample excess return minus risk-free rate, divided by their sample market risk.

$$TR_P = \frac{(R_P - R_f)}{\beta_P} \tag{8}$$

Where, TR_P is Treynor ratio of portfolio, R_P is return of portfolio, R_f is risk free rate of return and β_P is the sensitivity of the portfolio towards the movement of the underlying benchmark or market index.

The portfolio with a higher Treynor value indicates that the portfolio has higher risk-adjusted performance compared to a portfolio consisting of a low Treynor value.

EMPIRICAL ANALYSIS

In this section, we have empirically constructed two portfolios using different portfolio strategies and compared their in-sample and out-of-sample performance using various performance tools. We have reported our analysis based on the monthly distribution of return for the two portfolios for ten years. The portfolio construction of mean-variance models developed by Markowitz (1952) and an equally-weighted portfolio or naïve portfolio strategy by DeMiguel, Garlappi & Uppal (2009). The equally-weighted portfolio strategy does not involve any optimization of expected return at a lower risk exposure and it ignores the data completely by implying that expected returns are proportional to total risk rather than market risk. The mean-variance portfolio is the wealth allocation that optimizes the tradeoff between the mean and variance of the portfolio's expected return. To allow for the diversification of the mean-variance portfolio, the maximum participation of ETFs were relaxed (Kono, Yatrakis, Simon & Segal, 2007).

We have begun our analysis by comparing the performance of the 40 selected ETFs from each asset class for portfolio construction. The information ratio is the performance measurement tool based on which the ETFs would be chosen to construct the portfolio using EW and MV portfolio strategies.

Table 1: Annual return, standard deviation, active return, tracking error and information ratio of 40 ETFs

Ticker	Asset class	R_{etf}	σ_{etf}	R_A	TE	IR
SPY	Equity	12.025	10.119	-0.062	1.230	-0.051
IVV	Equity	12.061	10.243	-0.027	1.174	-0.023
VTI	Equity	12.118	10.646	0.030	0.174	0.172
QQQ	Equity	18.379	12.884	6.292	5.215	1.206
VEA	Equity	3.566	12.917	-8.522	5.349	-1.593
VIG	Equity	10.933	8.960	-1.155	3.511	-0.329
EFA	Equity	3.267	12.766	-8.821	5.365	-1.644
VTV	Equity	10.065	10.072	-2.022	2.439	-0.829
IJH	Equity	11.815	13.366	-0.273	6.026	-0.045
IWF	Equity	14.740	11.502	2.652	4.241	0.625
AGG	Bond	0.962	3.193	0.195	0.211	0.925
BLV	Bond	3.634	10.929	2.868	7.904	0.363
LQD	Bond	2.347	5.750	1.580	3.095	0.511
VCIT	Bond	2.159	4.199	1.393	1.716	0.812
MBB	Bond	0.150	2.146	-0.616	1.830	-0.337

Ticker	Asset class	R_{etf}	σ_{etf}	R_A	TE	IR
BSV	Bond	0.129	1.165	-0.638	2.341	-0.273
SHV	Bond	0.037	0.052	-0.729	3.257	-0.224
BIV	Bond	1.107	4.188	0.341	1.173	0.291
HYG	Bond	0.316	6.030	-0.450	6.284	-0.072
TIP	Bond	1.337	5.199	0.570	2.921	0.195
IAU	Commodity	4.966	17.142	4.863	21.675	0.224
DBC	Commodity	-2.713	18.570	-2.816	5.083	-0.554
DJCI	Commodity	-4.176	16.148	-4.280	9.571	-0.447
DJP	Commodity	-4.804	16.539	-4.908	8.994	-0.546
GCC	Commodity	-2.495	15.539	-2.598	12.468	-0.208
GSC	Commodity	-4.033	20.480	-4.137	4.596	-0.900
GSP	Commodity	-5.018	20.746	-5.121	6.030	-0.849
SLV	Commodity	4.134	29.970	4.030	21.139	0.191
UCI	Commodity	-1.416	18.424	-1.520	6.841	-0.222
GLD	Commodity	4.802	17.089	4.698	21.653	0.217
SSO	Leveraged	25.957	23.190	13.905	13.132	1.059
FAS	Leveraged	35.971	50.684	23.918	42.134	0.568
QLD	Leveraged	38.072	30.704	26.019	21.458	1.213
TECL	Leveraged	54.930	64.222	42.877	56.121	0.764
UPRO	Leveraged	39.669	39.128	27.616	29.137	0.948
TNA	Leveraged	30.800	52.140	18.748	44.536	0.421
DDM	Leveraged	25.514	25.687	13.461	17.144	0.785
MVV	Leveraged	23.986	29.770	11.934	22.007	0.542
UWM	Leveraged	22.488	32.509	10.435	25.120	0.415
TQQQ	Leveraged	55.379	51.730	43.326	42.604	1.017

Table 1[1] presents the summary of the statistical analysis of the performance of the 40 selected ETFs from the US financial market spanning for a period of ten years. Column 3 shows the annual average return from the monthly data for the 40 ETFs over a period of 10 years. The leveraged ETFs which track the S&P500 (GDF) exhibit a higher average return on monthly data than the benchmark index (table 2). All leveraged ETFs have a positive active annual return with TQQQ recording the highest with 43% and UWM has the lowest with 10% respectively. leveraged ETFs show the highest level of tracking error (TE) and information ratio (IR) more than the other ETFs asset classes. The tracking error investigates how our sample ETFs are closely managed to stay to their respective benchmark index. The TECL leveraged ETFs record the highest TE of 56% and the lowest TE of 13% by SSO leveraged ETFs. QLD leveraged ETFs have the highest positive IR of 1.213 in the chosen ETFs from the four asset classes. Therefore, the risk-adjusted excess return of leveraged ETFs is higher than

[1] Table 1 provides the return, standard deviation, active return, tracking error, and information ratio for the 40 ETFs dataset from four asset classes that are considered in the empirical analysis. The period covers from the month of January 2010 to January 2020. The first column represents the ETFs tickers, R_{etf} denotes the mean of annual returns, 'σ_{etf}' denotes annual standard deviation, 'R_A' denotes active return, 'TE' denotes tracking error and 'IR' denotes information ratio.

the bond, commodity, and equity ETFs on the average annual return. This result indicates that leveraged outperforms the S&P 500 benchmark index, but the analysis of risk-adjusted performance which is appropriate and we will discuss in detail later in the study.

The overall average annual return of equity ETFs underperforms relative to the Russell 3000 (RUA) market index. All the equity ETFs return on the average is 10.89% and Russel 3000 is higher with an average return of 12.09% on an annual basis. QQQ has the highest average annual return of 18.38% and IJH records the highest σ of 13.37% in the equity asset class. All of the higher average annual return in equity ETFs displays higher risks. Most of the active returns of equity ETFs record a negative figure, and only VTI, QQQ, IWF outperform Russell 3000 annual returns. This indicates that most sample equity ETFs do not track the benchmark index appropriately. The result shows a tracking error ranging from 0.17% to 6.0% based on annual return for a period of 10 years. The QQQ equity ETF exhibits the highest information ratio of 1.21 and the majority of rest record negative information ratio. Nonetheless, the TE and IR level that we have recorded for the leveraged asset class remain adequately higher than the level reported for the equity asset class.

Bond ETF's empirical results show a mean annual return of 1.22% on the average, slightly higher than 0.77% recorded by the Vanguard Total Bond Market Index (BND). The BLV bond asset class has the highest average annual return of 3.63% and the highest σ or risks of 10.9% on an annual returns basis. The most active return of the bond asset class is low and negative, and this may be due to the proxy benchmark index used in the analysis. The TE of the bond ETFs is lower as compared with leveraged and equity ETF, with its highest TE at 7.9% on the annual return. The σ and TE of bond ETFs are expected to be very low due to the less risky nature of bonds in the capital market. AGG has recorded the highest IR of 0.93 in the bond assets class.

The worst performing ETF assets class in this study is the commodity ETFs which shows the highest average annual return of 4.97% by IAU, much higher compared with the 0.10% average annual return record by GDF commodity index. Most average return and active return results of the commodity asset class are negative. The commodity ETFs exhibit high σ across all chosen ETFs with the lowest σ of 15.54% by GCC, lower as compared to S&P 500 Commodity Index σ of 22.59%. The high σ and TE recorded by the commodity asset class as compared to their average returns and maybe due to the proxy benchmark index. The highest IR of 0.224 has recorded by IAU commodity ETF.

Generally, all the results from the analysis of the 40 ETFs appear to be consistent with the Markowitz theory of risks-returns trade-off in financial investment. The performance measurement analysis result from Table 1 will assist in choosing the appropriate ETFs from the 40 of the four asset classes to construct the portfolio using MV and EW portfolio strategies.

The four ETFs highlighted in Table 1 from each asset class with the highest Information ratio meet the criteria for selection. There are different metrics for choosing the ETF funds as we have discussed earlier in this study. We have selected the information ratio as the metric for choosing the ETF from each asset class because it is determined to be one of the most significant metrics in assessing managers' skills in managing funds. The result in Table 1 shows QLD leveraged has the highest IR of 1.213, followed by QQQ equity recording 1.206, AGG bond recording 0.925 and IAU commodity reporting 0.224 in that order.

Table 2: Annual return and standard deviation of 4 benchmarks for 4 asset classes

Ticker	Asset class	R_B	σ_B
RUA	Equity	12.088	10.578
BND	Bond	0.767	3.250
GDF	Commodity	0.104	22.595
GSPC	Leveraged	12.053	10.103

Table 2[2] provides a statistical overview sample of exchange traded funds benchmark indexes. The data were sourced from yahoo finance to select the market indexes usually tracked by the ETFs. For each asset class we have chosen a target broad market index that shows a great tracking history and has high positive correlation with the asset classes. It is important to clarify that none of the asset classes have a single market index that they track. We have chosen a proxy benchmark index which may not be the true representation of the actual market index.

Figure 1[3] shows the annual return of the chosen exchange traded funds and the benchmark indexes across different asset classes over a period of 10 years in the US. From the plot all the ETFs and the market indexes have a volatile annual return. The QLD Leveraged recorded the highest annual return that lies above the GSPC index. QLD outperformed the S&P 500 index and exhibited the highest standard deviation (σ) due to the sharp decrease and increase in return over the ten-year period with the highest downward trend in 2018. AGG Bond shows a moderate upward and downward trend and tracks the Vanguard Total Bond Market Index appropriately and records the lowest annual return and σ throughout the period. The equity ETFs QQQ exhibited a high annual return second to QLD, closely tracking and outperforming the Russell 3000 index. IAU commodity shows a volatile annual return and slightly outperforms the S&P GDF Commodity Index. IAU recorded the lowest annual return in 2013 with the GDF benchmark index recording the lowest annual return in 2014. Both QLD and QQQ recorded their lowest annual return in 2016 and 2019, and follow a similar pattern in the plot. The return of the ETF from the four asset classes is generally positive and volatile with the exception of the AGG ETF in the 10 years period. It must be noted that all bond asset classes are usually known not to be very volatile financial assets.

[2] Table 2 provides the benchmark return and standard deviation for the four benchmarks or market indexes in the empirical analysis. The first column represents the tickers of the benchmark index, the 'RUA' denotes Russell 3000 index, 'BND' denotes Vanguard total bond market, 'GDF' denotes S&P commodity index and 'GSPC' denotes S&P 500 leveraged index. The third and fourth column of 'R_B' denotes the benchmark index return, and 'σ_B' denotes the benchmark index standard deviation.

[3] The plot of figure 1 provides the annual mean return of the four chosen ETFs and the benchmarks from 2010 to 2019. For the four asset classes four benchmark indexes have been chosen. 'GSPC' denotes S&P 500 leveraged index, 'RUA' denotes Russell 3000 index, 'BND' denotes Vanguard Total Bond Market index and 'GDF' denotes S&P 500 commodity index. The chosen ETFs with the highest information ratio are 'IAU' denotes the ishare Gold Trust, 'QLD' denotes the Proshare Ultra QQQ, 'QQQ' denotes the Invesco QQQ Trust, and 'AGG' denotes iShare Core US Aggregate Bond.

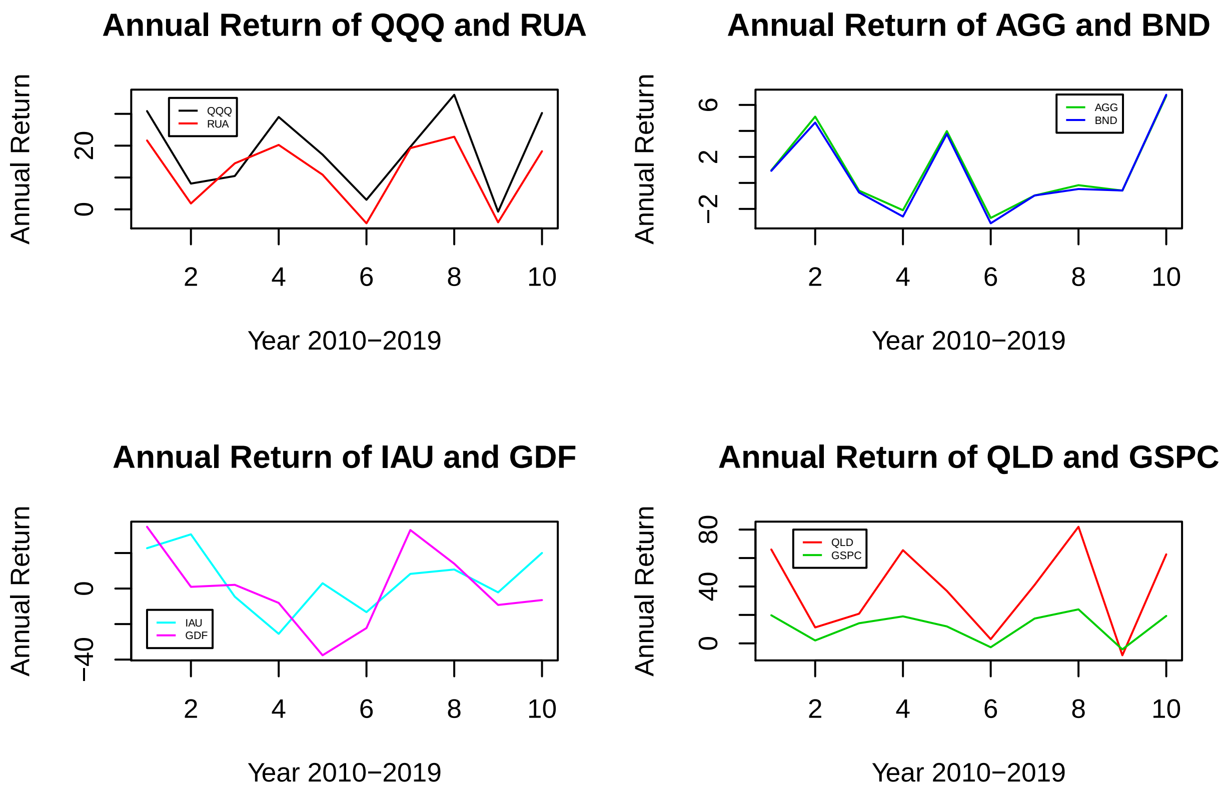

Figure 1: Annual return of ETFs and benchmarks with the highest information ratio

PERFORMANCE MEASUREMENT

We compared the empirical in-sample and out-of-sample performance of the mean-variance (MVP) relative to equally-weighted (EWP) or naïve portfolio assets allocation strategy.

Equally-Weighted or Naïve Portfolio (EWP)

This portfolio strategy ignores optimization, estimation in the construction of the portfolio. The ETFs that reported the highest information ratio in table 1 were used in the construction of the portfolio. The weight is restricted to 1 and equally apportioned among the four asset classes of QLD (25%), QQQ (25%), AGG (25%) and IAU (25%) chosen ETFs. The benchmark index was constructed using the $\frac{1}{N}$ weight to assist in the performance measurement of the equally-weighted or Naïve Portfolio.

Mean-Variance Portfolio (MVP)

The four chosen ETFs with the highest information ratio from each asset class highlighted in Table 1, are used in the construction of mean-variance portfolios by Markowitz (1952). The selected ETFs data were based upon funds in existence as at January 2020 and have satisfied the criteria stated under the methodology section of this study. The QLD of leveraged assets

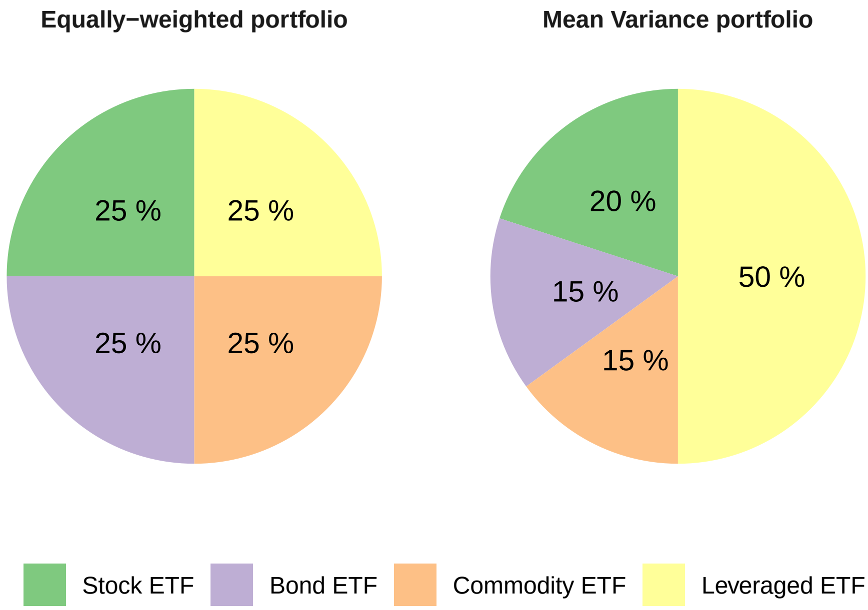

Figure 2: Weight of equally-weighted portfolio and mean variance portfolio

class, QQQ of equity assets class, AGG of bond assets class and IAU of commodity assets class were the chosen ETFs. The optimal mean-variance portfolio was constructed considering the mean, covariance matrix and the participation of the chosen ETFs were restricted to the minimum of 15% to the maximum of 50% to allow for optimal diversification of portfolio. The optimal weight for the diversified mean variance was QLD (50%), QQQ (20%), AGG (15%) and IAU (15%). The benchmark index was calculated using the same weights.

Figure 3[4] plots the annual returns of equally-weighted (EWP) and mean-variance (MVP) ETFs portfolios and the respective equally-weighted (EWB) and mean-variance benchmarks (MVB) index. The MVP outperforms the MVB by annual return average of 15.03% and both show high levels of volatility in revenue over the 10 years period. The MVB is always below the MVP whether there is upward or down trend in return, therefore MVP exhibits higher return and standard deviation (σ) over the period. EWP shows a sharp upward and downward trend which clearly indicates the volatility in return. The EWP outperforms the tracking EWB with an annual return average of 9.34% with a moderate σ compared to EWB. The MVP is theoretically expected to be the dominant return over EWP. As shown in the plot, the MVP outperforms EWP, MVB and EWB throughout the 10 years period. Both

[4] The plot in figure 3 provides the annual mean return for mean-variance and equally-weighted portfolio for a period from 2010 to 2019. The ‘EWBR’ denotes the equally-weighted benchmark return, ‘EWPR’ denotes the equally-weighted portfolio return, ‘MVBR’ denotes the mean-variance benchmark return, and ‘MVPR’ denotes the mean-variance portfolio return. The datasets for the empirical analysis are annual return from January 2010 to January 2020.

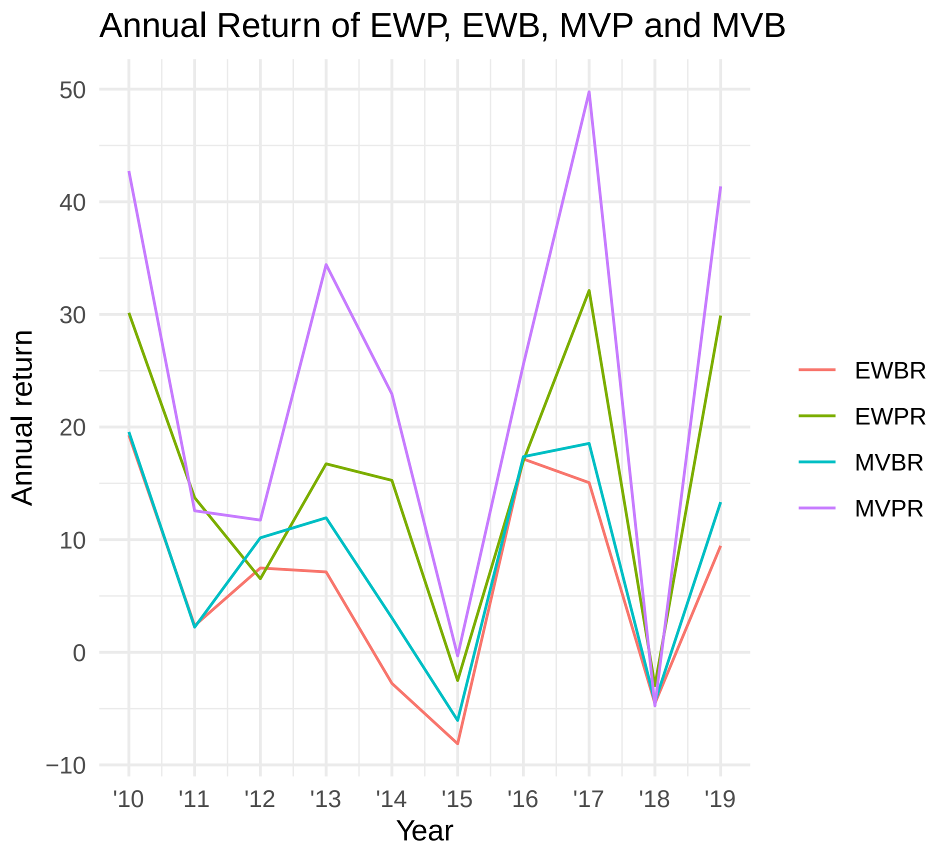

Figure 3: Annual Return of equally-weighted portfolio, equally-weighted benchmark, mean-variance portfolio and mean-variance benchmark

portfolios and their benchmark have a deep downward trend in 2015 and 2018 year.

Return (R) and Standard deviation (σ)

The results shown in Table 3 report the simple return and standard deviation over a 10 years period for equally-weighted or naïve portfolio strategy and mean-variance portfolio.

Table 3[5] presents the in-sample and out-of-sample return (R) and σ of an average return for EW and MV using m=6, m=12 and m=24 months overlapping rolling estimates. EWP and MVP outperformed their benchmark index and recorded a higher σ than the benchmark index. However, the MVP outperformed the EWP from all the presented results in Table 3. The out-of-sample average return of EWP for overlapping rolling window estimates of m=6 record was 1.138 relative to the EWB of 0.480 and MVP recorded 1.721 compared to the mean-variance benchmark index MVB of 0.672. Both portfolio strategies have shown higher

[5]Table 3 provides the empirical result of the rolling window mean return and standard deviation of an annual return over a ten year period. The first column shows the details of the rolling window of average return and standard deviation. The 'EWP', 'EWB' denotes the equally-weighted portfolio and equally-weighted benchmark index, the 'MVP' and 'MVB' denotes the mean-variance portfolio and mean-variance benchmark index of monthly return dataset from January 2010 to January 2020.

σ than their respective benchmark index. Furthermore, the EWP and MVP out-of-sample average return and standard deviation decrease as the overlapping rolling window months increase.

Table 3: Return and standard deviation of equally-weighted portfolio and mean-variance portfolio and respective benchmarks

	EWP	EWB	MVP	MVB
Annual R	15.595	6.253	23.601	8.574
Monthly R	1.213	0.499	1.816	0.693
6M rolling window R	1.138	0.480	1.721	0.672
12M rolling window R	1.095	0.464	1.683	0.660
24M rolling window R	1.046	0.425	1.655	0.635
Annual σ	12.696	9.417	18.638	9.353
Monthly σ	3.535	2.762	5.326	3.025
6M rolling window σ	0.981	0.975	1.502	0.984
12M rolling window σ	0.641	0.633	0.927	0.605
24M rolling window σ	0.324	0.408	0.470	0.367

Tracking Error (TE) and Active Return ($\mathbf{R_A}$)

The result from Table 4[6] gives the TE and R_A of equally-weighted portfolio (EWP) and mean-variance portfolio (MVP) for the dataset being considered. This performance indicator analyses the volatility of expected return difference between the portfolio (Tracker) and its market index. Most investors' primary measure of replication quality of a tracker relies on the tracking error (TE).

The MVP recorded an annual in-sample TE and R_A of 11.3%, 15.03% relative to the EWP annual in-sample TE and AR of 7.71%, 9.34% and clearly indicates the concept of risk-reward tradeoff. We find the monthly out-of-sample for TE for MVP is slightly higher relative to EWP strategy. The overlapping rolling window of m = 6 out-of-sample TE recorded was 0.705 for EWP relative to MVP TE of 0.882 and m = 12 reduce to 0.461 (-53%) for EWP or naïve portfolio compared to 0.556 (-59%) of MVP. EWP and MVP exhibited the smallest difference in TE in m = 24 overlapping rolling windows of 40 basis points. Furthermore, the monthly level of out-of-sample TE or risks of MVP portfolio reduces faster than EWP over time as reported in the result under overlapping rolling window m = 6, m = 12 and m = 24. The overlapping rolling estimate m = 12 and m = 24 record a slightly similar TE for EWP and MVP. This may be due to the increase in management skills for the chosen ETFs fund managers as the overlapping rolling estimates increase. The study shows the tracking error recorded by mean-variance is higher than naïve diversification approach (Bessler,

[6] Table 4 provides the in-sample and out-of-sample 'TE' and 'R_A' on the monthly datasets from January 2010 to January 2020. 'TE' denotes the tracking error, R_A denotes active return of the portfolio and overlapping rolling window estimated months. The 'EWP' denotes the equally-weighted portfolio and 'MVP' denotes mean-variance portfolio strategy used in the empirical analysis.

Opter and Wolff, 2014) and this finding is consistent with the result of our study. All the portfolio strategies reported positive out-of-sample active returns. This indicates that EWP and MVP strategy outperform their respective benchmark index (EWB & MVB). In all the rolling window estimated months as shown in Table 4, the MV outperforms the EW. The out-of-sample R_A for overlapping rolling estimates m = 12, m = 24 for EWP are 0.63 and 0.62 relative to 1.023 and 1.021 for MVP. Both portfolios out-of-sample R_A slightly decrease when overlapping rolling estimates increase.

Table 4: Tracking error and active return of equally-weighted portfolio and mean-variance portfolio

	EWP	MVP
Annual R_A	9.342	15.026
Monthly R_A	0.713	1.123
6M rolling window R_A	0.658	1.050
12M rolling window R_A	0.630	1.023
24M rolling window R_A	0.621	1.021
Annual TE	7.711	11.299
Monthly TE	2.128	3.035
6M rolling window TE	0.705	0.882
12M rolling window TE	0.461	0.556
24M rolling window TE	0.289	0.293

Information Ratio (IR)

Table 5[7] shows the result from the investigation of the information ratio (IR) for equally-weighted Portfolio and mean-variance Portfolio on a monthly return data of 10 years.

Table 5: Information ratio of equally-weighted portfolio and mean-variance portfolio

	EWP	MVP
Annual IR	1.211	1.330
Monthly IR	0.335	0.370
6M rolling window IR	0.933	1.190
12M rolling window IR	1.365	1.839
24M rolling window IR	2.150	3.487

The result in Table 5 shows the ability to generate excess returns relative to the market index and identifying the consistency of the return over time. Both portfolio strategies

[7]The empirical result in table 5 provides the information ratio (IR) of mean-variance and equally-weighted portfolio strategies. The 'EWP' denotes the equally-weighted portfolio or naïve portfolio and 'MVP' denotes the mean-variance portfolio strategy on monthly return data from January 2010 to January 2020.

recorded positive IR in all estimated overlapping rolling window months. The MVP slightly outperformed EWP by recording the in-sample IR of 0.370 compared to 0.335, which is 350 basis point differences. Annual in-sample IR of MVP outperforms EWP by 1190 basis or 0.119%.

Again, the out-of-sample IR of overlapping rolling estimate m = 6, m = 12 for EWP increased from 0.933 to 1.365 (+46%). MVP results also show an increase from 1.190 for m = 6 to 1.839 for m = 12. The MVP outperformed EWP out-of-sample IR in all the overlapping rolling window estimates, especially m = 24 by 62% on a monthly return data of 10 years. The empirical results show a high information for the overlapping rolling window estimate relative to normal expected market IR. This increment may be due to the reduction in the historical data at each overlapping rolling estimate, for instance from m = 6 to m = 24. It may also be due to the proxy benchmark index use for the portfolio is not the true representation of the ETFs asset class. The results in Table 5 are consistent with the result in Table 4, since TE is key metrics in IR analysis.

Sharpe Ratio (SR)

Table 6[8] shows the analyzed results of Sharpe ratio for the mean-variance portfolio and equally-weighted portfolio strategies. It measures the risk adjusted return of the portfolio strategy based on monthly and annual returns.

Table 6: Sharpe ratio of equally-weighted portfolio and mean-variance portfolio

	EWP	MVP
Annual SR	1.715	1.879
Monthly SR	0.477	0.533
6M rolling window SR	1.334	1.727
12M rolling window SR	1.944	2.672
24M rolling window SR	2.940	4.981

We have observed that Markowitz mean-variance portfolio attains a slightly higher out-of-sample Sharpe ratio (SR) than the equally-weighted portfolio. The EWP recorded an annual in-sample SR of 1.715 compared with 1.879 for MVP strategy. The monthly in-sample SR of 0.477 for EWP underperform by MVR monthly in-sample SR of 0.533. Moreover, the SR for out-of-sample is important to assess the magnitude of the probable gain that investors can realize, and is therefore necessary to analyze the out-of-sample of the portfolio's strategies. The difference between in-sample Sharpe ratio and out-of-sample Sharpe ratio allow us to gauge the estimation error, the MVP and EWP have higher out-of-sample SR than in-sample SR. That is, the indication of low estimation error that could not erode the optimal diversification portfolio gains.

[8]Table 6 provides the empirical result of Sharpe ratio for the mean-variance and equally-weighted or naïve portfolio strategy from the monthly return dataset. The 'EWP' denotes the equally-weighted portfolio and 'MVP' denotes the mean-variance portfolio.

The overlapping rolling window estimate of m = 6 recorded out-of-sample SR of 1.334 for EWP relative to 1.727 for MVP strategy. The m = 12 and M = 24 overlapping rolling window estimates exhibited results of 1.944 (+46%) and 2.940 (+51%) for EWP, and MVP recorded 2.672 (+55%) and 4.981 (+86%). The MVP outperforms the EWP in-sample Sharpe ratio and out-of-sample Sharpe ratio for m=6, m=12, and m=24 in the overlapping rolling window. Our empirical results show a high SR when employing the overlapping rolling window and maybe due to the reduction in historical data use for our analysis. This may also be due to the proxy benchmark index use in the construction of the portfolios benchmark for the performance measurement. These findings are similar to Kono, Yatrakis, Simon and Segal (2007) study results and show that Sharpe ratio of Markowitz portfolio strategy outperforms the S&P 500. The Sharpe ratio of Black Litterman (BL) is significantly larger than naïve portfolio strategy, but the mean-variance portfolio strategy out-of-sample Sharpe ratio is not significantly outperformed naïve (Bessler, Opter & Wolff, 2014). The study investigated stock and bond out-of-sample performance using BL, MV, and EW strategy from developed and emerging markets from Jan 1993 to Dec 2011. However, the data employed in our study were ETFs which operate similarly as stocks but not in the same way. In contrast, our findings differ from DeMiguel, Garlappi and Uppal, (2009) and Murtazashvili and Vozlyublenaia, (2013) who conclude that $\frac{1}{N}$ strategy outperforms all the mean-variance strategies. This may be due to differences in dataset employed and both studies analyze using only stock data while our study analyzes ETFs of different asset classes for broader diversification and to enhance portfolio optimization benefit.

Treynor Ratio (TR)

The result in Table 7[9] shows the empirical result of mean-variance and equally-weighted Treynor ratio which indicate the excess return of a portfolio in relation to the market risk.

Table 7: Treynor ratio of equally-weighted portfolio and mean-variance portfolio

	EWP	MVP
Annual TR	12.322	12.094
Monthly TR	0.993	1.047
6M rolling window TR	1.265	1.206
12M rolling window TR	1.202	1.187
24M rolling window TR	1.505	1.456

The MVP recorded a higher monthly in-sample TR of 1.047 relative to EWP of 0.993, outperforming it by 540 basis points. However, the EWP outperformed MVP in terms of annual in-sample TR of 12.322 against 12.094. The out-of-sample TR for EWP outperformed the MVP in all the estimated overlapping rolling windows. Out-of-sample overlapping rolling

[9] Table 7 provides the in-sample and out-of-sample Treynor ratio for the mean-variance and equally-weighted or naïve portfolio strategy from the monthly return datasets. The 'EWP' denotes the equally-weighted portfolio and 'MVP' denotes the mean-variance portfolio of asset allocation strategy.

window estimates of m = 24 reported 1.505 for EWP relative to 1.456 for MVP. That is, MVP slightly underperformed by 490 basis points compared with EWP in overlapping rolling window m = 12. Both portfolios TR slightly increased with the increase in overlapping rolling window months, but EWP slightly outperformed MVP. This is surprising, as the MV method is expected to reduce the systematic risks of investment. The MVP is more exposed to market or systematic risk as the result of a degree of diversification relative to EWP which is an equally diversified portfolio.

CONCLUSION

We have compared the performance of equally-weighted portfolio and mean-variance portfolio strategy by Markowitz (1952) and the constructed benchmark indexes. The comparison of strategy is undertaken using five performance measurement metrics for the period from January 2010 to January 2020 and the two strategies perform better than their respective benchmark index. We find that the mean-variance portfolio outperforms the equally-weighted portfolio in most of the performance matrix except for tracking error and Treynor ratio. We applied out-of-sample performance analysis suggested by De Miguel, Garlappi and Uppal, (2009).

The empirical findings establish that MVP generated higher in-sample and out-of-sample Sharpe ratios than the EWP or naïve portfolio in all the estimated overlapping rolling window months. These results are confirmed by previous studies, the out-of-sample Sharpe ratio of mean-variance portfolio strategy does not significantly outperform naïve portfolio strategy (Bessler, Opter & Wolff, 2014). The in-sample Sharpe ratio of Markowitz diversify portfolio outperforms the S&P 500 index (Kono, Yatrakis, Simon & Segal, 2007). However, these studies finding contradict our result, they conclude that all the mean-variance portfolio strategies out-of-sample Sharpe ratios underperform relative to naïve or $\frac{1}{N}$ portfolio strategy (DeMiguel, Garlappi & Uppal, 2009; Murtazashvili & Vozlyublenaia, 2013).

Furthermore, the MVP exhibited high in-sample and out-of-sample performance active return, information ratios relative to EWP or naïve portfolio. The out-of-sample performance of TE result recorded in our studies is supported by the findings of Bessler, Opter and Wolff (2014), that shows mean-variance portfolio out-of-sample TE is higher than naïve diversify portfolio. However, from our findings we realized when investing solely in one assets class of ETFs the MVP is outperformed by the diversified EWP or naïve portfolio strategy. A broader set of asset classes offers larger diversification opportunities and higher assets reallocation benefit in the long run.

In summary, we provide empirical evidence that MV diversify portfolio consistently outperforms EW or naïve diversify portfolio for all the analyzed datasets. This will assist ETFs portfolio managers when employing MV in assets allocation strategy.

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