Results in Physics 19 (2020) 103302

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

Analysis of quantum coherence for localized fermionic systems in an accelerated motion

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ARTICLE INFO

Keywords:
Quantum coherence
Fermionic cavity modes
Relativistic quantum information
Quantum entanglement

ABSTRACT

Although quantum coherence is a well known phenomenon in quantum information theory and quantum optics, it has been investigated from the resource theory perspective only recently. Furthermore, quantum coherence has important implications in relativistic quantum information where the degradation of entanglement can be attributed to decoherence. In this paper, we investigate the quantum coherence of (1 + 1) Dirac field modes localized in a cavity as observed by two relatively accelerated observers. The acceleration is assigned very small values and its effects are investigated in a perturbative regime. For this purpose, we use α parameterized two-qubit pure entangled state and a Werner state. We find that coherence shows a periodic degradation due to accelerated motion. However, this degradation can be balanced by adjusting the durations of uniform and accelerated motion. Moreover, it is found that dynamics of quantum coherence closely resembles that of entanglement under the same settings. This similarity confirms the recent attempts to relate the resource theories of coherence and entanglement in a relativistic regime.

Introduction

Quantum coherence, emerging from quantum superposition principle, plays a pivotal role in quantum mechanics applications that are considered impossible within the realm of classical mechanics [1,2]. However, significance of coherence as a useful resource like entanglement and quantum discord has been realized only recently. In analogy to quantification of entanglement resource, Baumgratz et. al. [3], provided a rigorous framework for the quantification of quantum coherence. By introducing the notions of incoherent states, incoherent operations Baumgratz et. al. defined necessary conditions which should be satisfied by any measure of coherence. For instance, measures like l_1 norm and relative entropy with respect to a certain basis are found to be suitable candidates which satisfy these necessary conditions [3]. This study further triggered the research for finding other suitable measures of coherence and identifying conditions to manipulate the coherence [4–8]. Also, the interrelation of coherence has been recently studied with other quantum information resources like entanglement and quantum discord [9,10]. Furthermore, following the concept of the local operations and classical communication (LOCC) employed for entanglement distillation, a class of local incoherent operations and classical communication (LICC) has been proposed in [11] for coherence distillation. However, because of the basis dependent characteristic of coherence, the quantification of coherence departs from those of the other information resources. More recently, Yao et al. [10] have developed a hierarchical structure of quantum entanglement, quantum discord and quantum coherence for multipartite system. Using this structure they have introduced the basis-free coherence measure and have shown that basis-free coherence is equivalent to quantum discord.

Quite recently, quantum coherence has been envisaged in an innovative way for detecting topological edge states [12]. These topological states have key contribution towards topologically protected manipulation and processing of quantum information. Therefore, detection of such states is of great importance. To this end, Zaimi et al. [12] utilized the concept of decoherence rate of a qubit attached to one end of a topological system. The topological system, on the other end, is connected to a standard tight-binding hopping Hamiltonian which serves as the environment. Subsequently, evaluation of decoherence rate of the attached qubit is rigorously exploited to investigate and classify the nature of topological states as edge states.

In addition to information theoretic investigation of quantum resources at microscopic scale, the dynamics of these resources has also been explored in the relativistic regime. In fact, relativistic quantum

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https://doi.org/10.1016/j.rinp.2020.103302
Received 15 June 2020; Received in revised form 28 July 2020; Accepted 2 August 2020
Available online 14 August 2020

information has emerged as a vibrant field of research which envisages the general relativistic effects on quantum resources like quantum entanglement, quantum discord, fidelity of teleportation and Bell nonlocality [13–22]. Moreover, relation between coherence and entanglement has also been investigated in relativistic regimes. For instance, the acceleration can be termed as an environmental decoherence [23–25] which causes the degradation of entanglement resource and affects the efficacy of quantum information processes, negatively.

More recently, the quantification of quantum coherence in non-inertial or accelerated frames has been studied [26] for free Bosonic and fermionic field modes. In addition to free field modes, the dynamics of field modes confined in cavities has received much attraction for reliable implementation of certain quantum information tasks [27–31]. For instance, loophole-free violation of a Bell inequality has been recently investigated using entangled electron spin separated by a distance of 1.3 km [27]. Moreover, localized eigenstates have also been successfully utilized for stability analysis of classically chaotic quantum systems [32]. Furthermore, over long distance, relativistic signatures for quantum information processes may also become significant [25,13,31,20,33,21,22]. Therefore, it is quite intriguing to investigate the quantum coherence in such situations. With this motivation, we explore the quantum coherence for the fermionic field modes in a cavity observed by two relatively accelerated observers. More precisely, we follow the Dirac field analysis proposed in [21] where the modes of massless Dirac field are confined to a cavity where one of the observers remains inertial while the other one undergoes the segments of inertial and non-inertial motion with uniform acceleration. We restrict the uniform acceleration to be very small ($h \ll 1$) and use perturbation theory to observe the effects of the accelerated motion on the quantification of coherence for the confined modes with respect to the weight parameter α and dimensionless acceleration parameter h. Before proceeding further, it seems reasonable to quickly remind the basic notions and conditions for coherence measures as proposed in [3].

Let $\mathscr{H}$ be a D-dimensional Hilbert space with a given basis $\{|i\rangle\}$. Using this basis, we can define $\hat{\delta} = \sum_i \delta_i |i\rangle\langle i|$ with an arbitrary set of non-negative probabilities $\{\delta_i\}$ as an incoherent state. Further, let $\mathscr{I} \subset \mathscr{H}$ be a set of consisting of all such incoherent states. As described in [3], any suitable measure of quantum coherence $C(\hat{\rho})$ must satisfy the following conditions:

1. $C(\hat{\rho}) \geq 0$ for $\forall\ \hat{\rho} \in \mathscr{H}$ and $C(\hat{\delta}) = 0$ iff $\hat{\delta} \in \mathscr{I}$
2.
   (a) $C(\hat{\rho})$ is monotonic under all the incoherent completely positive and trace-preserving (ICPTP) maps $\Phi_{ICPTP}(\cdot)$: i.e., $C(\hat{\rho}) \geqslant C(\Phi_{ICPTP}(\hat{\rho}))$, where $\Phi_{ICPTP}(\rho) \equiv \sum_n \widehat{K}_n \rho \widehat{K}_n^{\dagger}$ and $\{\widehat{K}_n\}$ denotes the set of Kraus operators (incoherent operations) which satisfy $\sum_n \widehat{K}_n^{\dagger} \widehat{K}_n =$ and $\widehat{K}_n \mathscr{I} \widehat{K}_n^{\dagger} \subset \mathscr{I}$.
   (b) Monotonic average coherence under the subselection based on measurement outcomes: $C(\hat{\rho}) \geqslant \sum_n p_n C(\hat{\rho}_n)$, where $\hat{\rho}_n = \widehat{K}_n \rho \widehat{K}_n^{\dagger}/p_n$, $p_n = \mathrm{Tr}(\widehat{K}_n \rho \widehat{K}_n^{\dagger})$ and $\widehat{K}_n$ satisfies the conditions defined in 2. (a).
3. Convexity under mixing of quantum states: $\sum_n p_n C(\hat{\rho}_n) \geqslant C(\sum_n p_n \hat{\rho}_n)$.

We now recall known measures of coherence as discussed in [3]. The l_1 norm is intuitively proposed to quantify the coherence by considering the off-diagonal elements of $\hat{\rho}$ in the specified basis [3] and it is given by

$$C_{l_1}(\hat{\rho}) = \sum_{i \neq j} |\rho_{ij}|, \tag{1}$$

where $\hat{\rho} = \sum_{i,j} \rho_{i,j} \left|i\right\rangle \left\langle j\right|$. Following the distance based argument for quantification of quantum resource, the relative entropy of coherence is also proposed in [3] which is defined as

$$C_{rel.ent.}(\hat{\rho}) = S(\hat{\rho}^D) - S(\hat{\rho}), \tag{2}$$

where $\hat{\rho}^D = \sum_i \rho_{ii} |i\rangle\langle i|$ and $S(\hat{\rho}) = -\mathrm{Tr}(\hat{\rho}\log\hat{\rho})$. The rest of the paper is organized as follows. The perturbed Bogoliubov coefficients and Fock space quantization for vacuum and one charged particle Fermionic states in a cavity are described in Section "Bogoliubov Transformation For Inertial and Non-inertial Segments". The quantum coherence (C) for the two mode Fermionic Fock state shared between Alice and Rob is discussed in Section "Coherence for two-mode states". In Section "Coherence for the Werner state", we extend our investigation to Werner state [34]. In Section "Conclusion and discussion", we discuss and conclude our results.

Bogoliubov transformation for inertial and non-inertial segments

In order to find the unitary transformation of cavity's transitions between the inertial and accelerated (non-inertial) segments of motion, we setup the cavities with the observers referred to as Alice and Rob, respectively. Both the observers are initially inertial and completely overlap at $t = 0$. Here we assume that the walls of the cavity are located at $x = a$ and $x = b$ with $0 < a < b$. Rob then moves with uniform acceleration to the right along the time-like killing vector ∂_η for duration $\eta = 0$ to $\eta = \eta_1$ in the Rindler co-ordinates. Duration of the acceleration, $\frac{2}{a+b}$, measured using proper time at the cavity's center is thus $\tau_1 = \frac{a+b}{2}\eta_1$. Finally, Rob again becomes inertial with respect to its rest frame. Alice cavity remains inertial throughout Rob's trip. Therefore, three segments of Rob's trajectories can be identified as regions I, II and III.

This grafting process of Rob's motion is explained here to make this paper sufficiently self contained. Earlier the same process is introduced and exploited by [35,21] to analyze entanglement for Bosonic and fermionic cavity modes.

The Dirac field representation in three regions is

$$\text{I:} \quad \psi = \sum_{n \geq 0} a_n \psi_n + \sum_{n<0} b_n^{\dagger} \psi_n, \tag{3a}$$

$$\text{II:} \quad \psi = \sum_{n \geq 0} \hat{a}_n \hat{\psi}_n + \sum_{n<0} \hat{b}_n^{\dagger} \hat{\psi}_n, \tag{3b}$$

$$\text{III:} \quad \psi = \sum_{n \geq 0} \tilde{a}_n \tilde{\psi}_n + \sum_{n<0} \tilde{b}_n^{\dagger} \tilde{\psi}_n, \tag{3c}$$

with the respective non vanishing anticommutators

$$\text{I:} \quad \{a_m, a_n^{\dagger}\} = \{b_m, b_n^{\dagger}\} = \delta_{mn}, \tag{4a}$$

$$\text{II:} \quad \left\{\hat{a}_m, \hat{a}_n^{\dagger}\right\} = \left\{\hat{b}_m, \hat{b}_n^{\dagger}\right\} = \delta_{mn}, \tag{4b}$$

$$\text{III:} \quad \{\tilde{a}_m, \tilde{a}_n^{\dagger}\} = \{\tilde{b}_m, \tilde{b}_n^{\dagger}\} = \delta_{mn}. \tag{4c}$$

Using Bogoliubov transformation, the Dirac field modes between regions I and II are related as [21]

$$\hat{\psi}_m = \sum_n A_{mn} \psi_n, \tag{5}$$

where ψ_n and $\hat{\psi}_m$ are the Dirac field modes in regions I and II respectively. For the small acceleration case, Friis et al. [21] derived these coefficients A_{mn} in the perturbative regime by introducing the dimensionless parameter $h = \frac{2L}{a+b}$, satisfying $0 < h < 2$. These coefficients preserve the unitarity of the transformation to the order $O(h^2)$ for $s > 0$, $s \to 0_+$ and are given in terms of Maclaurin's series expansion as [21,35]

$$A_{mn} = A_{mn}^{(0)} + A_{mn}^{(1)} + A_{mn}^{(2)} + O(h^3), \tag{6}$$

where the superscripts reflect the order of perturbation with respect to parameter h. During the non-inertial trajectory, modes $\hat{\psi}_m$ with respect to Rob remain independent and do not interact. Hence, these modes can only develop some phases during the non-inertial duration $0 \leqslant \eta \leqslant \eta_1$. This change in the modes can be balanced by introducing a diagonal matrix $G(\eta_1)$ whose diagonal entries are [21]

$$G_{nn}(\eta_1) = \exp(i\Omega_n\eta_1). \tag{7}$$

For $\eta \geqslant \eta_1$, the transformation from region II to region III can be obtained by simply using the inverse transformation $A^\dagger = A^{-1}$. The evolution of the Dirac field mode from region I to region III in Rob's frame can then be expressed by the Bogoliubov transformation matrix

$$\mathscr{A} = A^\dagger G(\eta_1)A. \tag{8}$$

Thus the Bogoliubov transformation for the Dirac field modes between regions I and III reads

$$\widetilde{\psi}_m = \sum_n \mathscr{A}_{mn}\psi_n. \tag{9}$$

It can be noticed that $\mathscr{A}$, being the composition of unitary matrices, is also a unitary matrix to the order h^2. Similarly, the Bogoliubov transformation for the Dirac field mode operators can also be expressed as [35,21]

$$k>0: \qquad a_k = \sum_{l\geq 0}\widetilde{a}_l\mathscr{A}_{lk} + \sum_{l<0}\widetilde{b}_l^\dagger\mathscr{A}_{lk}, \tag{10a}$$

$$k<0: \qquad b_k = \sum_{l\geq 0}\widetilde{a}_l^\dagger\mathscr{A}_{lk} + \sum_{l<0}\widetilde{b}_l\mathscr{A}_{lk}. \tag{10b}$$

The relation between the Fock vacua in regions I and III denoted by $|0\rangle$ and $|\widetilde{0}\rangle$ is [36,37,21]

$$|0\rangle = Ne^W|\widetilde{0}\rangle, \tag{11}$$

where

$$W = \sum_{p\geq 0, q<0} V_{pq}\widetilde{a}_p^\dagger\widetilde{b}_q^\dagger. \tag{12}$$

The coefficient matrix V and the normalization constant N are the unknowns to be evaluated. Using (3a), (3b), and (9), the coefficient matrix is given by

$$V = V^{(0)} + V^{(1)} + O(h^2) = V^{(1)} + O(h^2), \tag{13}$$

with

$$V_{pq}^{(1)} = \mathscr{A}_{pq}^{(1)*}G_q = -\mathscr{A}_{qp}^{(1)}G_p^* \tag{14}$$

The relation between Fock vacua in regions I and III given by (11) yields [21]

$$\left|0\right\rangle = \left(1 - \frac{1}{2}\sum_{p\geq 0,q<0}\left|V_{pq}\right|^2\right)\left|\widetilde{0}\right\rangle + \sum_{p,q}V_{pq}\left|\widetilde{1}_p\right\rangle^+\left|\widetilde{1}_q\right\rangle^- - \frac{1}{2}\sum_{p,q}\sum_{i,j}V_{pq}V_{ij}\phi_{p,i}\phi_{q,j}\left|\widetilde{1}_p\right\rangle^+\left|\widetilde{1}_i\right\rangle^+\left|\widetilde{1}_q\right\rangle^-\left|\widetilde{1}_j\right\rangle^- + O(h^3). \tag{15}$$

where $|\widetilde{1}_p\rangle^+ := \widetilde{a}_p^\dagger|\widetilde{0}\rangle^+$ and $|\widetilde{1}_q\rangle^- := \widetilde{b}_q^\dagger|\widetilde{0}\rangle^-$ represent the single-particle Fock states for modes $p \geq 0$ and $q < 0$, respectively. Here, sign $\pm$ in the superscript denotes the sign of the charge. Further, the term $\phi_{pi} := 1 - \delta_{p,i}$ is introduced to incorporate the Pauli-exclusion principle for the single particle states with same charge sign. Also, the ordering of the single-particle ket states reflect the corresponding ordering of the fermionic creation operators rather than the fermionic modes [35]. Similarly, the charged single particle states in region III are [21]

$$k>0: \quad \left|1_k\right\rangle^+ = -\sum_{p,q}V_{pq}\mathscr{A}_{qk}^*\left|\widetilde{1}_p\right\rangle^+ + \sum_{m\geq 0}\mathscr{A}_{mk}^*\left\{\left(1 - \frac{1}{2}\sum_{p,q}\left|V_{pq}\right|^2\right)\left|\widetilde{1}_m\right\rangle^+ + \sum_{p,q}V_{pq}\phi_{pm}\left|\widetilde{1}_m\right\rangle^+\left|\widetilde{1}_p\right\rangle^+\left|\widetilde{1}_q\right\rangle^- - \frac{1}{2}\sum_{p,q;i,j}V_{pq}V_{ij}\phi_{pi}\phi_{pm}\phi_{mi}\phi_{qj}\left|\widetilde{1}_m\right\rangle^+\left|\widetilde{1}_p\right\rangle^+\left|\widetilde{1}_i\right\rangle^+\left|\widetilde{1}_q\right\rangle^-\left|\widetilde{1}_j\right\rangle^-\right\} + O(h^3), \tag{16a}$$

$$k<0: \quad \left|1_k\right\rangle^- = \sum_{p,q}V_{pq}\mathscr{A}_{pk}\left|\widetilde{1}_q\right\rangle^- + \sum_{m<0}\mathscr{A}_{mk}\left\{\left(1 - \frac{1}{2}\sum_{p,q}\left|V_{pq}\right|^2\right)\left|\widetilde{1}_m\right\rangle^- + \sum_{p,q}V_{pq}\phi_{qm}\left|\widetilde{1}_p\right\rangle^+\left|\widetilde{1}_q\right\rangle^-\left|\widetilde{1}_m\right\rangle^- - \frac{1}{2}\sum_{p,q;i,j}V_{pq}V_{ij}\phi_{pi}\phi_{qm}\phi_{qj}\phi_{mj}\left|\widetilde{1}_p\right\rangle^+\left|\widetilde{1}_i\right\rangle^+\left|\widetilde{1}_q\right\rangle^-\left|\widetilde{1}_j\right\rangle^-\left|\widetilde{1}_m\right\rangle^-\right\} + O(h^3), \tag{16b}$$

where the one particle states $|1_k\rangle^\pm$ in region I are

$$k \geq 0: \qquad |1_k\rangle^+ = a_k^\dagger|0\rangle, \tag{17a}$$

$$k < 0: \qquad |1_k\rangle^- = b_k^\dagger|0\rangle. \tag{17b}$$

Coherence for two-mode states

Here we compute the quantum coherence for a complete trip of a two mode entangled states from region I to region III in perturbative regime to the second order perturbation, h^2, and study the effects of uniform acceleration (h) and weight parameter (α) on the coherence of the evolved fermionic modes confined to a cavity. We consider a bipartite two qubit pure state parameterized by weight parameter α in region I. The state consists of two Dirac field modes in a cavity. The initial parameterized state is

$$|\psi_{init}\rangle_\nu = \alpha|0\rangle_A|0\rangle_R + \sqrt{1-\alpha^2}|1_m\rangle_A^\mu|1_k\rangle_R^\nu, \tag{18}$$

where the subscripts A and R refer to the observers, Alice and Rob, respectively. The superscripts μ and ν reflect positive of negative frequency of cavity modes, so that $\mu(\nu) = +$ when $m(k) \geqslant 0$ and $\mu(\nu) = -$ when $m(k) \leqslant 0$. Following the procedure given in [21], the initial state (18) is represented in two mode Hilbert space by using the two-particle basis, with one excitation for each of the modes m and k in Alice's and Rob's cavities, respectively. The corresponding density matrix in the region I is written as

$$\rho_\nu = \alpha^2|0\rangle_A\langle 0| \otimes |0\rangle_R\langle 0| + (1-\alpha^2)|1_m\rangle_A^{\mu\mu}\langle 1_m| \otimes |1_k\rangle_R^{\nu\nu}\langle 1_k| + (\alpha\sqrt{1-\alpha^2}|0\rangle_A^\mu\langle 1_m| \otimes |0\rangle_R^\nu\langle 1_k| + H.c). \tag{19}$$

It should be noted that all the modes, except the reference mode in the Rob's cavity are related to the environment. Therefore, a partial trace is taken over all of modes in the Rob's cavity except mode k. By exploiting the unitarity of the perturbed Bogoliubov transformation (8) up to the second order perturbation and using the inside out partial tracing approach [21], we obtain the following reduced density matrix in region III

$$\mathrm{Tr}_{\neg k}\rho_\nu \equiv \rho_{\nu,k} = \alpha^2|0\rangle_A\langle 0| \otimes \{(1-f_k^{-\nu}h^2)|\widetilde{0}\rangle_{III}\langle\widetilde{0}| + f_k^{-\nu}h^2|\widetilde{1}_k\rangle_{III} \langle\widetilde{1}_k|\} + \alpha\sqrt{1-\alpha^2}\{(G_k + \mathscr{A}_{kk}^{(2)}h^2)\times|0\rangle_A^{\mu}\langle 1_m| \otimes |\widetilde{0}\rangle_{III}^{\nu} \langle\widetilde{1}_k| + H.c\} + (1-\alpha^2)|1_m\rangle_A^{\mu\mu}\langle 1_m| \otimes \{(1-f_k^{\nu}h^2)|\widetilde{1}_k\rangle_{III}^{\nu\nu} \langle\widetilde{1}_k| + f_k^{\nu}h^2|\widetilde{0}\rangle_{III}\langle\widetilde{0}|\}, \tag{20}$$

where f_k^{ν} and $f_k^{-\nu}$ are defined as

$$\nu > 0: \qquad f_k^{\nu} = \sum_{p\geq 0}\left|\mathscr{A}_{pk}^{(1)}\right|^2, \tag{21a}$$

$$\nu < 0: \qquad f_k^{\nu} = \sum_{q<0}\left|\mathscr{A}_{qk}^{(1)}\right|^2. \tag{21b}$$

The density matrix can be re-written as

$$\rho_{\nu,k} = \rho_{\nu,k}^{(0)} + \rho_{\nu,k}^{(2)}h^2, \tag{22}$$

where $\rho_{\nu,k}^{(0)}$ and $\rho_{\nu,k}^{(2)}$ denote the unperturbed and perturbed matrix components, respectively. In order to evaluate the the quantum coherence given by (2), we need to compute the eigenvalues of the density matrices $\rho_{\nu,k}$ and $\rho_{\nu,k}^{D}$. Here, $\rho_{\nu,k}^{D}$ is a diagonal matrix which contains the diagonal elements of $\rho_{\nu,k}$. The eigenvalues of the unperturbed part of the evolved density matrix $\rho_{\nu,k}^{(0)}$are $\{\lambda_i^{(0)}\} = \{1, 0, 0, 0\}$. Note that the eigenvalues 1 and 0 denote the non-degenerate and degenerate case, respectively. Before proceeding further to find second order corrections to the unperturbed eigenvalues, it is pertinent to seek the source of degenerate eigenvalues. It can be noticed by meticulous comparison of (19) and (20) that the leading order terms in (20) remain invariant under Bogoliubov transformation. Thus, the degeneracy of unperturbed eigenvalues may be attributed to invariance or symmetry of leading order terms under Bogoliubov transformation. Interestingly, it is worth noticing that the symmetry is broken in perturbative regime when second order terms are also considered. Therefore, the broken symmetry may be attributed to second order perturbative contribution of the accelerated motion which indeed led to non-degenerate eigenvalues given by (23). We, now, compute the second order corrections to the non-degenerate unperturbed eigenvalue $\lambda_1^{(0)} = 1$using standard perturbation procedure However, in case of the triply degenerate eigenvalue $\lambda_{2,3,4}^{(0)} = 0$, the standard perturbation method is not valid and is needed to be replaced by the degenerate case. The second order corrections to the degenerate eigenvalue can be obtained by finding the eigenvalues of its degenerate subspace matrix Mas described in [38]. Consequently, the eigenvalues of the perturbed density matrix $\rho_{\nu,k}$ are obtained as

$$\{\lambda_i\} = \{\alpha^2 f_k^{-\nu}h^2, \quad (1-\alpha^2)f_k^{\nu}h^2, \quad 1-(\alpha^2 f_k^{-\nu} + (1-\alpha^2)f_k^{\nu})h^2, \quad 0\}. \tag{23}$$

Since the trace of the perturbed density matrix is $\mathrm{Tr}(\rho_{\nu,k}) = 1$ and all the eigenvalues are non-negative, $\rho_{\nu,k}$ satisfies the density matrix representation. In the similar fashion, eigenvalues of the perturbed diagonal matrix can be computed which are given as

$$\{\lambda_i^{D}\} = \{\alpha^2(1-f_k^{-\nu}h^2), \quad (1-\alpha^2)(1-f_k^{\nu}h^2), \quad \alpha^2 f_k^{-\nu}h^2, \quad (1-\alpha^2)f_k^{\nu}h^2\}. \tag{24}$$

where f_k^{+}, f_k^{-} given by (21) can be re-written as

$$f_k^{+} = \sum_{p\geq 0}^{\infty} |E_1^{k-p}|^2\left|A_{pk}^{(1)}\right|^2, \tag{25a}$$

$$f_k^{-} = \sum_{q<0}^{\infty} |E_1^{k-q}|^2\left|A_{qk}^{(1)}\right|^2, \tag{25b}$$

$$f_k := f_k^{+} + f_k^{-} = \sum_{p=-\infty}^{\infty}\left|\mathscr{A}_{pk}^{(1)}\right|^2, \tag{25c}$$

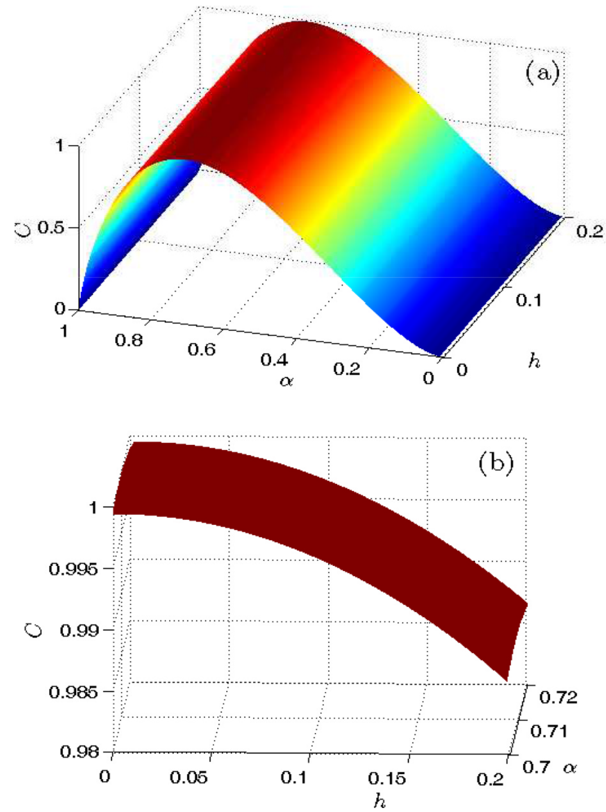

Fig. 1. (a) The plot depicts the variation of relative entropy of coherence C for the evolved state in region III as a function of $h \ll 1$ and α with mode $k = 1$ and $s = \frac{1}{4}$. (b) The plot is shown at magnified scale for greater visibility of coherence degradation as a function of acceleration parameter h.

with

$$E_1 := \exp\left(\frac{i\pi\eta_1}{\ln(b/a)}\right). \tag{26}$$

Next, by plugging the eigenvalues given by the expressions,(23) and (24) in (2), the relative entropy of coherence for the state (20) is given by

$$C_{rel.ent.}(\rho_{\nu,k}) = S(\rho_{\nu,k}^{D}) - S(\rho_{\nu,k}), \tag{27}$$

The variation of relative entropy of coherence as a function of acceleration parameter $0 < h \ll 1$ and weight parameter α is plotted in Figs. 1 and 2. In case of parameter α with fixed value of h, it can be seen in Figs. 1 (a) and 2 (a) that the relative entropy of coherence monotonically increases with the increasing values of α for $0 \leqslant \alpha < \sqrt{1/2}$. Later on, the relative entropy shows monotonically decreasing trend for increasing value of α in the range $\sqrt{1/2} < \alpha \leq 1$. However, for acceleration parameter h, the relative entropy of coherence monotonically decreases with the increase in h for the perturbative regime $0 < h \leq 0.2$. For clear depiction of this degradation, the plots are shown at magnified scale in Figs. 1 (b) and 2 (b). Also, for $h = 0$, the relative entropy of coherence simply reduces to $C_{rel.ent.}(\rho_{\nu,k}) = -\alpha^2 \log_2\alpha^2 - (1-\alpha^2)\log_2(1-\alpha^2)$.

In addition to the above analysis for parameters α and h, the relative entropy of coherence is also analyzed in Fig. 3 for full duration $(0 \leqslant \tau \leq 1)$ of inertial and non-inertial motion of Rob's cavity. Since, the functions $f_k^{\pm\nu}$ given by the relations (21) are periodic and non-negative over the period τ, the coherence between the modes in non-inertial and inertial cavities is also periodic in the durations of the individual trajectory segments. Moreover, relative entropy of coherence shows higher magnitudes of periodic degradation with the increase in acceleration parameter h. This behavior of coherence degradation due to accelerated

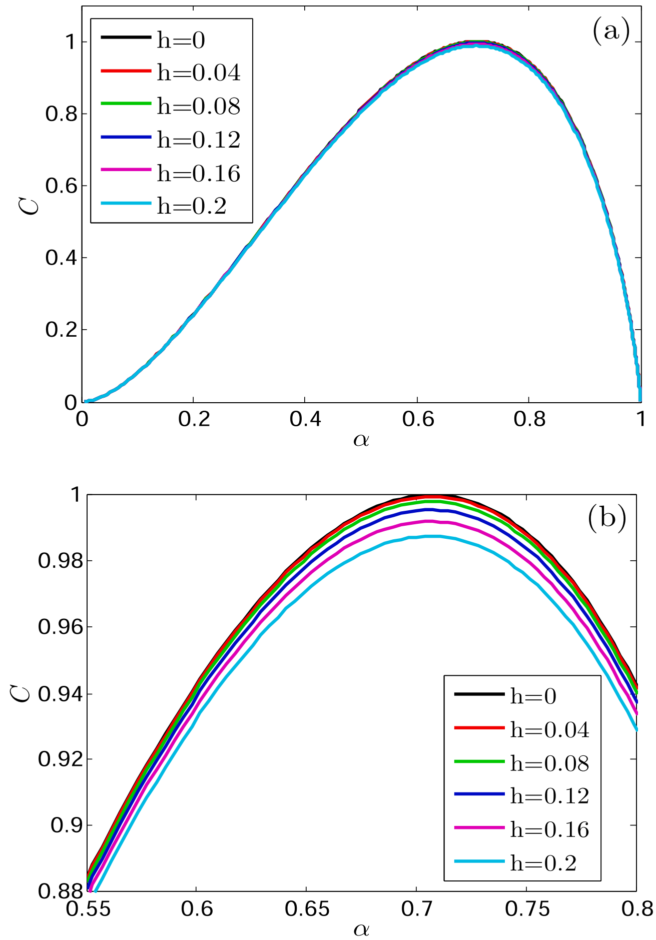

Fig. 2. (a) The plot depicts the variation of relative entropy of coherence C for the evolved state in region III as a function of $h \ll 1$ and α with mode $k = 1$ and $s = \frac{1}{4}$. (b) The plot is shown at magnified scale for greater visibility of coherence degradation as a function of acceleration parameter h.

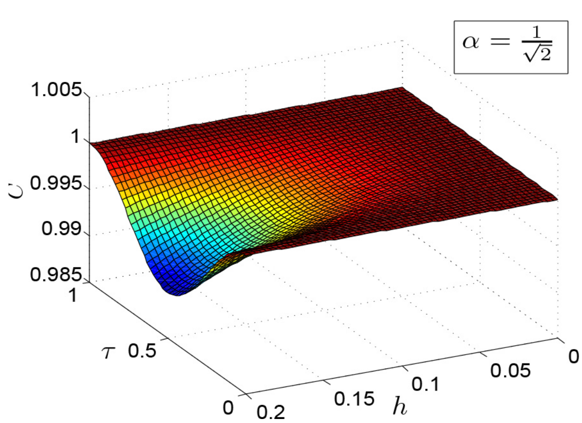

Fig. 3. The plot shows relative entropy of coherence C of the evolved sate in region III (for $k = 1$) as a function of acceleration $h \ll 1$ and $\tau := \frac{1}{2}\eta_1/\ln(b/a)$, over the full period $0 \leqslant \tau \leq 1$.

motion closely resembles the entanglement degradation phenomenon investigated in [21]. That is, the non-inertial motion of an observer in some sense induces a certain environmental decoherence which affects the information resources like entanglement and coherence in a similar fashion. This degradation, in turn, limits the efficacy of certain quantum information theoretic tasks which rely on these pivotal resources. However, this degradation of coherence can be adjusted by fine tuning the duration of inertial and non-inertial motion as described in [21]. Moreover, the interdependence of entanglement and coherence investigated in [23–25] for free field modes has also been confirmed here for cavity field modes.

Coherence for the Werner state

In the previous section, we considered pure state for relative entropy of coherence. However, in realistic situations, probe states are mixed. Therefore, we further investigate relative entropy of coherence by considering evolution of the Werner state [34] from region I to region III. For this purpose, we consider the initial two qubit Werner state in region I given by

$$\rho_{W;\nu} = r(\alpha|0\rangle|0\rangle + \sqrt{1-\alpha^2}\sin\theta|1_m\rangle^{\mu}|1_k\rangle^{\nu})(\alpha\langle 0|\langle 0| + \sqrt{1-\alpha^2}^{\mu}\langle 1_m|^{\nu}\langle|1_k|) + \frac{1-r}{4}, \tag{28}$$

where the parameter r indicates the mixedness of the pure entangled two qubit state and the maximally mixed bipartite state. Considering the perturbed evolution of the state from region I to region III in the similar fashion as discussed earlier, we obtain the transformed density matrix in Region (III) basis to the order h^2 with the help of (15) and (16). Afterwards, by exploiting the unitarity of the Bogoliubov transformation (8) and applying the partial trace as described in [21], the reduced density matrix in the region III is expressed as

$$\begin{aligned}\rho_{W;\nu,k} &= r[\alpha^2|0\rangle_A\langle 0| \otimes \{(1-f_k^{-\nu}h^2)|\widetilde{0}\rangle_{III}\langle\widetilde{0}| + f_k^{-\nu}h^2|\widetilde{1}_k\rangle_{III} \\ &\langle\widetilde{1}_k|\} + \alpha\sqrt{1-\alpha^2}\{(G_k + \mathscr{A}_{kk}^{(2)}h^2) \times |0\rangle_A^{\mu}\langle 1_m| \otimes |\widetilde{0}\rangle_{III}^{\nu}\langle\widetilde{1}_k| + H.c\} \\ &+ (1-\alpha^2)|1_m\rangle_A^{\mu\mu}\langle 1_m| \otimes \{(1-f_k^{\nu}h^2)|\widetilde{1}_k\rangle_{III}^{\nu\nu}\langle\widetilde{1}_k| + f_k^{\nu}h^2|\widetilde{0}\rangle_{III}\langle\widetilde{0}|\}] \\ &+ r^c[(|0\rangle_A\langle 0| + |1_m\rangle_A^{\mu\mu}\langle 1_m|) \otimes \{(1+g_k^{\nu}h^2)|\widetilde{0}\rangle_{III}\langle\widetilde{0}| + (1+g_k^{-\nu}h^2) \\ &|\widetilde{1}_k\rangle_{III}\langle\widetilde{1}_k|\}],\end{aligned} \tag{29}$$

where r^c and $g_k^{\pm\nu}$ are defined as

$$r^c = \frac{1-r}{4}, \tag{30a}$$

$$g_k^{\pm\nu} = f_k^{\pm\nu} - f_k^{\mp\nu}. \tag{30b}$$

Thus the perturbed density matrix can be expressed in the compact form as

$$\rho_{W;\nu,k} = \rho_{W;\nu,k}^{(0)} + \rho_{W;\nu,k}^{(2)}h^2. \tag{31}$$

The unperturbed eigenvalues corresponding to density matrix $\rho_{W;\nu,k}^{(0)}$ are given as

$$\{\lambda_i^{(0)}\} = \{r + r^c, r^c, r^c, r^c\}. \tag{32}$$

Two explicit cases arise for $r = 0$ and $r = 1$. For $r = 0$, the unperturbed density matrix represents the maximally mixed state, with standard basis and degenerate eigenvalue of $\lambda_{1,2,3,4}^{(0)} = 1$. Here, the relative entropy of coherence θ yields a trivial result, $C = 0$. For $r = 1$, the situation is exactly the same as elucidated in Section "Coherence for two-mode states". Therefore we restrict values of r, in the open interval $0 < r < 1$.

It can be noted that, the eigenvalues $r + r^c$ and r^c of the unperturbed part of the density matrix denote the non-degenerate and triply degenerate cases, respectively. Therefore, following the perturbative procedure prescribed in [38] and used in the previous section for the non-degenerate and degenerate cases, the eigenvalues of the reduced density matrix $\rho_{W;\nu,k}$ in the region III are

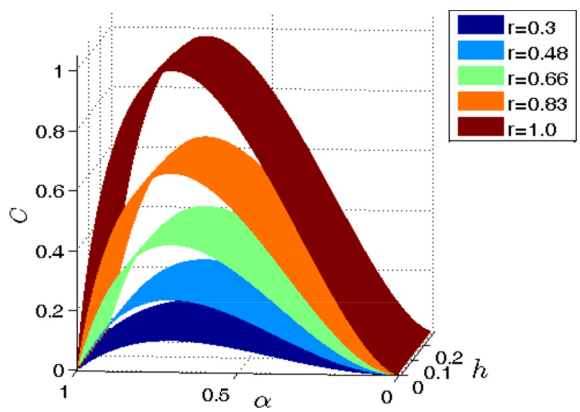

Fig. 4. The plot shows the relative entropy of coherence C of the evolved state for different values of mixedness parameter r as a function of $h \ll 1$ and α with mode $k = 1$ and $s = \frac{1}{4}$.

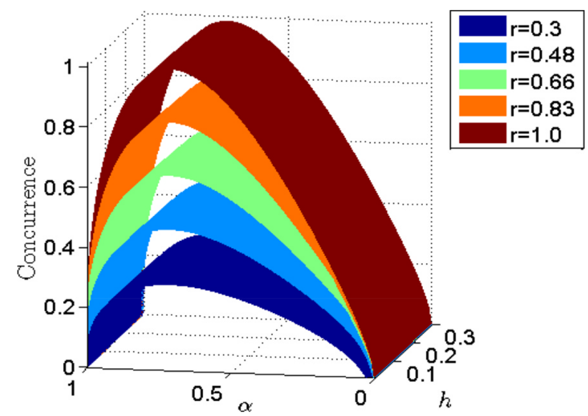

Fig. 5. The plot shows the concurrence $Con(\rho)$ of the evolved state for different values of mixedness parameter r as a function of $h \ll 1$ and α with mode $k = 1$ and $s = \frac{1}{4}$.

$$\{\lambda_i\} = \{r + r^c - r(\alpha^2 f_k^{-\nu} + (1-\alpha^2) f_k^{\nu})h^2 + r^c(\alpha^2 g_k^{\nu} + (1-\alpha^2) g_k^{-\nu})h^2, \\ r^c + r^c((1-\alpha^2) g_k^{\nu} + \alpha^2 g_k^{-\nu})h^2, r^c + r^c g_k^{-\nu} h^2 + r f_k^{-\nu}\alpha^2 h^2, \\ r^c + r^c g_k^{\nu} h^2 + r f_k^{\nu}(1-\alpha^2)h^2\}. \tag{33}$$

From (33), it can be observed that the perturbed density matrix satisfies the density matrix conditions, $\lambda_i \geq 0$;$\forall\ i$ and $\sum_i \lambda_i = 1$. Proceeding in the similar fashion, we find the eigenvalues of the diagonal counterpart $\rho^D_{W;\nu,k}$ of the density matrix $\rho_{W;\nu,k}$ as follows.

$$\{\lambda_i^D\} = \{r\alpha^2 + r^c + (-r\alpha^2 f_k^{-\nu} + r^c g_k^{\nu})h^2, \\ r(1-\alpha^2) + r^c + (-r(1-\alpha^2) f_k^{\nu} + r^c g_k^{-\nu})h^2, r^c + (r\alpha^2 f_k^{-\nu} + r^c g_k^{-\nu})h^2, \\ r^c + (r(1-\alpha^2) f_k^{\nu} + r^c g_k^{\nu})h^2\}. \tag{34}$$

The relative entropy of coherence (C) for the evolved Werner state can be computed using the relation (2). The variation of C as a function of parameter α and mixing parameter r is shown in Fig. 4. As a function of mixing parameter r, the coherence monotonically decreases with the decreasing value of r. This decreasing behavior is justified due to fact that the small values of parameter r reflect the increased inclination towards the maximally mixed state whereas the large values of r reflect inclination towards pure quantum state.

Further, for a given value of r, the coherence with respect to α exhibits first increasing and then decreasing behaviors for $0 \leqslant \alpha < 1/\sqrt{2}$ and $1/\sqrt{2} < \alpha \leq 1$, respectively. The initial increase and then subsequent decline of quantum coherence with respect to superposition parameter α can be justified and explained as follows. The superposition parameter, α, signifies the degree of entanglement via Schmidt decomposition as given by (18). Therefore, as value of α approaches $1/\sqrt{2}$ from either side, the correlation among the quantum states in Alice and Rob regions approaches its maximum value (maximally entangled state). It, therefore, can be observed that increasing trend in entanglement corresponds to increasing trend in quantum coherence as α approaches $1/\sqrt{2}$ from either side. On the other hand, as α moves away from its value $1/\sqrt{2}$, from either side, the correlation decreases which is reflected in the respective decrease in quantum coherence.

In order to investigate the similarity in the dynamics of quantum coherence and quantum entanglement for Werner state, we resort to finding concurrence [39] using the same setting. In contrast to several measures of entanglement such as logarithmic negativity, entropy of entanglement and distillable entanglement, concurrence is more effective measure in operational sense and simplicity, in particular, for two qubit mixed states. The concurrence is also related to entanglement of formation (EoF) which can also be used to quantify entanglement based on the separability criterion [39]. For two qubit mixed state ρ, concurrence is defined as

$$Con(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4), \tag{35}$$

where λ_i $(i = 1, 2, 3, 4)$ denote the eigenvalues of matrix, $M = \sqrt{\sqrt{\rho}\rho^*\sqrt{\rho}}$, arranged in descending order. Here $\rho^* = (\sigma_y \otimes \sigma_y)\rho(\sigma_y \otimes \sigma_y)$ where σ_y is pauli spin matrix. From Figs. 4 and 5, it can be observed that relative entropy of quantum coherence and concurrence for the evolved Werner state exhibit similar behavior. This analysis, therefore, confirms the similarity in the behavior of quantum entanglement and quantum coherence as investigated in [21,23–25]

Loss of quantum coherence due to mixing parameter r can also be explained from the perspective of Halevo bound [40]. Holevo bound quantifies the classical capacity of quantum channel. It has been recently found that the loss in classical channel capacity corresponds to respective loss in quantum coherence due to mixing and vice versa [41]. This correspondence in loss relies on the close resemblance of expressions for measuring quantum coherence and channel capacity based on well-known Shanon entropy. Based on this argument, loss of quantum coherence due to mixing parameter r can be related to channel capacity where the uniform acceleration may be viewed as a cause of noise in the quantum channel defined by Bogoliubov transformation.

Conclusion and discussion

We investigated the effect of accelerated motion on the quantification of quantum coherence of the $(1 + 1)$ of fermionic modes localized in a cavity from the resource theoretic perspective. In our scenario, the Dirac field modes were localized in two cavities with Dirichlets boundary conditions, where one of the observers remained inertial, while the other underwent the segments of inertial and non-inertial motion with uniform acceleration. The acceleration parameter h was assigned very small values and its effects were studied using a perturbative scheme. We considered an α parameterized two-qubit pure entangled state and a Werner state. In former case, with fixed value of α, the coherence shows periodic degradation due to segments of inertial and accelerated motion induced by acceleration parameter h. This behavior of degradation can be justified due to mixing of cavity modes due to accelerated motion. However, by carefully adjusting the duration of accelerated and inertial segments of motion, degradation in coherence can be avoided. We also observed the effect of parameter α with fixed value of parameter h. In this situation, the coherence increases monotonically for $0 \leqslant \alpha < \frac{1}{\sqrt{2}}$ and achieves maximum value at $\alpha = \frac{1}{\sqrt{2}}$ (maximally entangled state). Afterwards, quantum coherence decreases monotonically for $\frac{1}{\sqrt{2}} < \alpha \leq 1$. This behavior of coherence with respect to α confirms the correlation and similarity between quantum coherence and quantum entanglement. In later case (Werner state), quantum coherence is investigated with respect to mixing parameter r, in addition to the parameters α and h. We observed the similar behavior for parameters α and h as discussed in former case. However, for mixing parameter, r, quantum coherence monotonically decreases for $0 \leqslant r \leq 1$. This monotonic degradation can be justified due to the fact that the higher values of mixing parameter r reflect the higher degree of mixedness in quantum states. Consequently, the higher degree of mixedness results in degradation of the quantum coherence.

From the above discussion, we also observed that dynamics of quantum coherence closely resembles that of entanglement under the same settings. This similarity confirms the recent attempts to relate the resource theories of coherence and entanglement in a relativistic regime.

CRediT authorship contribution statement

Zahid Hussain Shamsi: Conceptualization, Methodology, Writing - original draft, Software, Validation, Funding acquisition. **Amna Noreen:** Methodology, Data curation, Visualization, Writing - original draft, Visualization, Investigation, Formal analysis. **Asif Mushtaq:** Supervision, Conceptualization, Writing - review & editing, Software, Project administration, Funding acquisition.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This work was partially supported by University of the Punjab, Lahore-Pakistan, through a research project in the fiscal year 2017–2018 (Z.H.S). Asif Mushtaq would like to thank Mathematics Teaching and Learning, research group within the Department of Mathematics, Bodø, Nord University for the partial support.

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