

Chapter 1

A Spotlight on Mathematics Education in the Netherlands and the Central Role of Realistic Mathematics Education



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Abstract In this introductory chapter I give a preview of the landscape of issues concerning mathematics education in the Netherlands and the role of Realistic Mathematics Education (RME) that one can come across in this volume, which contains the reflections of twenty-eight Dutch mathematics didacticians on teaching and learning mathematics in the Netherlands. Although all chapters have their own focus and mostly only discuss one particular aspect, together they provide a rich inside view into what is worth knowing of Dutch mathematics education and RME. The preview highlights some significant topics from these chapters, such as what tasks are preferred in RME to elicit students' mathematical thinking, RME's focus on the usefulness of mathematics, the role of common sense and informal knowledge, changes over time in the content of the mathematics curriculum, aspects of the Dutch educational system, including teacher education and assessment, the implementation of RME, and the context of developing RME.

1.1 Introduction

The 13th International Congress on Mathematical Education (ICME-13) held in Hamburg, Germany, in 2016, and in particular the ICME-13 Thematic Afternoon session "European Didactic Traditions," was a trigger for Dutch mathematics didacticians to reflect on what is typical for mathematics education in their country. In this session, the Dutch approach to teaching and learning mathematics in school, in research, and in development was presented, together with the approaches in France, Italy, and Germany. The aim of the session was to delve into what the four countries

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have in common despite the differences in the cultural, historical, and political circumstances in which their positions and methods regarding mathematics education were developed. The common characteristics that came to the fore and that can be considered as distinctive features of the European didactics of mathematics, were: “a strong connection with mathematics and mathematicians, the key role of theory, the key role of design activities for learning and teaching environments, and a firm basis in empirical research” (Blum et al., 2019, p. 2). These are also the features that recur in the reflections on mathematics education in the Netherlands as described by the twenty-eight Dutch mathematics didacticists in this volume. This places the Dutch didactic tradition inalienably inside the European didactic tradition. Yet within this overarching European framework, Dutch mathematics education and its theoretical grounding have their peculiarities. In the Netherlands, the teaching and learning of mathematics cannot be seen separate from Realistic Mathematics Education (RME), the domain-specific instruction theory that has determined Dutch mathematics education in the last half-century. Therefore, in the reflections presented in this volume, the defining characteristics of RME have a prominent place. In addition to this, ample background information is provided about the educational system in which RME has come into being. In their descriptions, the authors have each their own focus in addressing particular aspects of mathematics education in the Netherlands, and of course, their reflections resonate their own views on RME. They gave their own accentuations and interpretations, which is fully in line with the idea that RME is not a fixed and unified theory of mathematics education.

As an introduction to this multifaceted portrayal of mathematics education in the Netherlands and the central role of Realistic Mathematics Education, in this preview I highlight some of the main thoughts that emerge from the chapters. Underlining these thoughts does not in any way imply that what is characterised as typical for the Dutch approach, is unique in the world of mathematics education. All over the world reforms of mathematics education have taken place and are still happening, and the innovations in the Netherlands have very much in common with those in other countries. In this sense the Dutch reformed ideas on mathematics education are not special.

1.2 The Focus on a Particular Type of Tasks

Several chapters in this volume discuss tasks that should be given to students to elicit mathematical thinking. Preferably, these are tasks that provide students with opportunities to creatively solve unfamiliar open-ended problems, to model, structure and represent problems and solutions, and to work collaboratively and to communicate about mathematics. Tasks that are exemplary for making this happen are described by Wijers and De Haan (Chap. 2). Their experience is that such tasks should be rich, meaning that there is not just one way to come to a solution. Further requirements are that the solutions can vary in mathematical depth, that the tasks build on knowledge students already have and that they offer students opportunities to extend their

knowledge. Also important is that higher-order questions are used which ask *how* and *why*, encouraging reasoning rather than getting an answer. In all these requirements, the very nature of RME is clearly apparent, but what Wijers and De Haan also point out is that these requirements not only apply to problems that are close to the real world, but also to assignments that are situated more within the world of mathematics. Besides tasks in which students, for example, reason about the productivity of workers in a factory in connection with the hours they work without having a break, students also work on tasks in which they have to deal with formulas in a quite abstract context, such as dots moving on a grid. The latter type of tasks can, in RME, also be called context problems.

The broad meaning of context problems is clarified in full detail by Vos (Chap. 3). In her fine-grained categorisation of tasks, she distinguishes, apart from bare tasks (tasks without contexts), tasks with mathematical contexts (e.g., matchstick pattern problems), dressed-up tasks (tasks with a pointless question behind which a mathematical question is hidden), tasks with realistic contexts (which are experientially real or imaginable for the students) and tasks with authentic contexts (which use photos, data, and situations from the real world). What the two last types of tasks have in common is that the context justifies the questions that are asked and that the answers to these questions are useful within the described context. For Vos the ‘usefulness’ of tasks means that they lead to developing the competence and understanding required for using and applying mathematics in future practices as professional or as citizen.

1.3 Usefulness as a Key Concept

The idea of teaching mathematics to be useful was and is a strong driving force for developing mathematics education in the Netherlands. Even before there was RME, Freudenthal made a strong plea for this idea in his article “Why to Teach Mathematics So As to Be Useful” published in 1968 in *Educational Studies in Mathematics*. As De Lange (Chap. 17) underlines, at the time of the rise of New Math—which was around the late 1960s—this was a very relevant question. Yet putting usefulness in the centre of our thinking on mathematics education was not new. The culture of usefulness of mathematics as a curricular emphasis has already existed in the Netherlands for five hundred years, and may, according to Vos (Chap. 3), have created a fertile ground for RME.

Concrete examples of the propensity to adhere to the usefulness aspect of mathematics and instances of the deep historical roots of this tendency are presented by Kool (Chap. 7). Her chapter goes back to Dutch arithmetic education in the 16th century. In that century, calculations were initially made with coins and a counting board, but as the result of the more complex trading methods that entered the market then, this cumbersome way of calculating was gradually replaced by a more advanced written calculation method. Many manuscripts and books were published to teach this new method to future merchants, moneychangers, bankers, bookkeepers, and craftsmen. By means of many tasks about all kinds of commercial transaction and

other calculations to be done in various workplace situations, students could learn to solve arithmetical problems of their future profession. This was the main goal of arithmetic education in those days, which was accompanied by devoting much attention to memorising rules and recipes, tables of multiplication and other number relations. When comparing this approach to mathematics education with the current Dutch approach, Kool concludes that teachers of the 16th and the 21st century both want to teach their students the arithmetic they need in daily life and their future profession. As in the 16th century, today's students in the Netherlands need to have knowledge about number relations and arithmetical rules, but different is that they have to learn to apply this knowledge in a flexible way, whereas in the 16th century it was all about using ready-made solution methods.

The relation between mathematics and its usefulness in real-world situations is also shown in the teaching experiment on measurement carried out by Van Gulik-Gulikers, Krüger, and Van Maanen (Chap. 13). What is more, the tasks they have designed for teaching this topic to eight- and ninth-grade students demonstrate that the contexts can also date from three centuries ago. The teaching material they used for this experiment is based on the professional context of a Dutch land surveyor in the 18th century measuring the height of buildings and the width of rivers. Comparable to the surveyors in those times, the students had to use the theory of similar triangles. Of course, nowadays it is common in such situations to use GPS, from which the students can learn as well, but the experiment showed that using the history of mathematics as a didactical tool had a positive effect on the students' motivation and on their conceptual understanding. In particular, the authors found that the transparency of this old-fashioned measurement method made discussions about mathematics accessible.

1.4 Common Sense and Informal Knowledge

The RME characteristic of connecting mathematics education to reality is closely related to the reinforcement of the role of common sense and using informal mathematical knowledge from daily-life experiences as a starting point for teaching. Dekker (Chap. 4) calls this 'the Dutch school' and describes a silent revolution that has taken place at this point in the Netherlands. There is a large difference between what she remembers from the start of her first mathematics lesson as a secondary school student and what students often hear nowadays. Then it was 'forget what you know, here you will learn all sorts of new things', whereas now the motto is 'use your common sense'. Students acquire a lot of mathematical knowledge in the realistic context of their life, and education should make use of this informal knowledge. In this respect, Dekker refers to the pioneering work of Ehrenfest-Afanassjewa, a Russian mathematician who worked in the Netherlands and in 1932 published a course on geometry based on the idea that students have already developed intuitive geometrical notions in reality. These intuitive notions were taken as the starting point of this course. Dekker describes that many people involved in mathematics education were shocked by Ehrenfest's radical ideas. However, this was not true

for Freudenthal who was impressed by her revolutionary approach, and stimulated developers of instructional materials to take over these ideas. Also, several other chapters make a point of this shift in teaching geometry, and mention the important role of Ehrenfest-Afanassjewa for Dutch geometry education (see Chaps. 5, 9, 11 and 15).

A question that is inevitable here and asks for discussion is where these intuitive notions and informal knowledge come from, or what common sense is. De Lange (Chap. 17) gives a first-hand peek into Freudenthal's thoughts about this, when he describes a discussion that took place at the Freudenthal Institute between Freudenthal and a number of staff members. Freudenthal was writing a new article meant for what would become his last book. According to *the professor* mathematics is rooted in common sense; for example, your common sense reasons that $2 + 3$ is 5 and that the area of a rectangle is $h \times b$. After he said this, the discussion continued. Someone questioned whether it is really true that ' $2 + 3 = 5$ ' and 'area is length \times width' are common sense. Finally, it was concluded: common sense is local, both in time and place, and it includes reasoning. Freudenthal mumbled something, not audible for the others, and decided that he would rewrite his draft.

1.5 Mathematical Content Domains Subject to Innovation

As a constituent of the reform that took place in the Netherlands, the content of the mathematics curriculum changed in many respects. Several chapters pay attention to these changes. For example, Doorman, Van den Heuvel-Panhuizen, and Goddijn (Chap. 15) shed light on the change that happened in geometry education. Here an axiomatic approach to teaching geometry was gradually superseded by an intuitive and meaningful approach focussed on spatial reasoning. Supported by Freudenthal—who was in his turn inspired by Ehrenfest-Afanassjewa and Van Hiele-Geldof—from the 1970s on, experiments were carried out within a new content domain, called 'vision geometry'. Characteristic of this RME-based geometry education is that, together with the introduction of this new content, the structure of the geometry trajectory was also changed. Traditionally, the structure in a teaching-learning trajectory for geometry was provided by a deductive system starting with formal definitions and basic axioms. This 'anti-didactical inversion' of the learning sequence, as Freudenthal called it, means that the final state of the work of mathematicians is taken as a starting point for mathematics education. In RME, the reverse order is followed, in which geometry education starts with offering students geometrical experiences based on observing phenomena in reality. Through explorative activities, geometrical intuitions develop further, and mathematisation is elicited, resulting in the development of situation models like vision lines, which eventually bring the students from informal to more formal geometry. The concepts and reasoning schemes that emerge from this 'local organisation'—again a term introduced by Freudenthal—have the potential to create, for students in the more advanced levels of secondary education, the need for axioms, definitions and mathematics as a logic-deductive system.

Another content domain that was subject to innovation in the Netherlands was calculus. Kindt (Chap. 14), who takes the reader along the history of how calculus developed over time, characterises this innovation process as balancing between conceptual understanding and knowing algebraic techniques—a process which is in fact indicative for the development of RME as a whole. Starting in the 1960s, attempts have been made to develop calculus courses that start with an introduction that is meaningful for the students. The idea was to give students a broadly oriented entrance to differential calculus by starting with a problem about rate of change in a context that made sense to the students, such as a cheetah and a horse that were both running. The students had to answer the question: Does the cheetah overtake the horse? Later on, this RME approach in which a long conceptual introduction with open tasks precedes the teaching of algebraic rules, did not always appear in the textbooks, which were mostly more structured and less challenging than the experimental units. Nevertheless, the current situation is that important elements of this approach, in which attention is paid to exploring linear and exponential relationships in meaningful contexts with tables and difference diagrams, can still be found in Dutch textbooks.

The implementation of the RME-based reform in lower and pre-vocational secondary education described by Hoogland (Chap. 11) which began in the 1990s, and which was meant to move from mathematics for a few to mathematics for all, also implied many changes in the curriculum. The reform affected all elements of mathematics education in secondary schools, including a new and broader curriculum, alternative ways to approach students, fostering students to develop more and other skills such as problem solving, and using different assessment formats such as contextual and open-ended problems. Within the domain of algebra, the emphasis shifted from algebraic and computational manipulation to reasoning on the relationships between variables and to flexibility in switching between different types of representations of relations. In geometry, there was a change from two-dimensional plane geometry with a strong calculational approach, towards two- and three-dimensional geometry with a focus on ‘vision geometry’. Numeracy was introduced as a new domain in secondary education, as were data handling, and statistics containing data collecting and visualisation to be used in decision making.

Apart from changes in the mathematical content that occur together with a new RME-based thinking about teaching and learning mathematics, changes, or at least prompts to rethink the practice and theory of mathematics education, were also induced by the new technologies that became available for education. This issue is addressed by Drijvers (Chap. 10), who discusses the relationship between mathematics education in the Netherlands and digital tools. He shows what it means to implement new technologies in RME-based education and concludes that the match between the two is not self-evident. Technology puts the teaching of mathematics in another perspective. Among other things, Drijvers points out that the phenomena that in RME form the point of departure for the learning of mathematics may change in a technology-rich classroom. Also, the teaching approach of guided reinvention may be challenged by the often rigid character of the digital tools. And finally, the use of

digital tools for higher-order thinking was found to be more complex than foreseen. According to Drijvers, to realise mathematics education as intended by RME, it is necessary to have a digital mathematics environment that allows the teacher to design open and engaging tasks, and enables students to explore and express mathematical ideas in accessible and natural ways.

The complexity of the issue of what mathematics should be taught, and changing ideas about this are signified by Treffers and Van den Heuvel-Panhuizen (Chap. 15) by retracing the content of the domain of number in two centuries of Dutch primary school mathematics textbooks. In their chapter, in which they cover the period from 1800 to 2010, they describe the longitudinal process featuring seemingly inevitable pendulum movements of procedural versus conceptual textbooks. Generally speaking, in the procedural textbooks the focus is on practising calculation procedures with less attention paid to conceptual understanding of number. Operations have to be carried out in a fixed way. Smart, flexible (mental) calculations and estimating are mostly absent in this approach. Finally, in the main, applications are not used until the very end of the teaching trajectory. The RME-based textbooks that appeared in the 1980s belong to the conceptual textbooks, and are the opposite of the procedural textbooks. Although the distinction between these two textbook types is rather coarse-grained, in most cases, RME-based textbooks start teaching in the domain of numbers and operations with applications and the use of contexts that evolve into models to support the development of calculation strategies. Number sense, number relations, flexible (mental) calculation, and estimation have a central place in the programme next to algorithmic calculations, which are introduced by transparent predecessors of the algorithms. This means, for example, that the digit-based algorithm of long division is prepared through a whole-number-based repeated subtraction approach. Contrary to the commonly held thought that mathematics education of some 100 years ago implies a traditional approach to teaching which focusses on drill-and-practise and fixed rule-governed solution strategies, the analysis of two centuries of mathematics textbooks reveals that this assumption is not correct. Already in 1875, Versluys, a mathematics educator who is considered the founding father of the Dutch didactics of mathematics, published a textbook in which the focus was on insightful, self-inquiry-based learning of mathematics within a whole-class setting guided by the teacher. Also, the way Versluys treated calculations up to one hundred has a lot in common with how this is now dealt with in RME textbooks. Furthermore, to a certain degree similar to RME, Versluys' textbook series contains a large amount of word problems and a rather small number of bare number problems. For Versluys, arithmetic is in the first place applied arithmetic. Again, the deep roots of RME are shown here. What is now considered new in some forums (and is therefore sometimes rejected) is, in some way, in essence not new at all. This is also enlightened by Kool (Chap. 7).

1.6 The Systemic Context of Dutch Education

To comprehend the nature of a country's mathematics education, it is necessary to view this education in its national context and have knowledge about how that country's school system is structured, how teachers are educated, how assessments and evaluations are organised, what the role is of the government and the institutions that deliver support services to schools, what the contribution is of teacher associations and what the position is of the publishers of educational material. It goes beyond this volume to give a complete picture of the Netherlands for all these systemic issues, but two issues which are specifically addressed are teacher education (in Chap. 8 by Oonk et al. and in Chap. 9 by Daemen et al.), and assessment in mathematics education (in Chap. 16 by Scheltens et al.). Furthermore, spread out across the volume other aspects of how education is organised in the Netherlands are also discussed. Without being exhaustive, it can be mentioned that information is provided about: the school system of the Netherlands (in Chap. 9 by Daemen et al. and in Chap. 11 by Hoogland), the different mathematics curricula for different school levels (in Chap. 2 by Wijers et al., Chap. 3 by Vos, and Chap. 11 by Hoogland), examination in secondary education (in Chap. 2 by Wijers et al. and in Chap. 14 by Kindt), the textbooks that are used (in Chap. 3 by Vos and in Chap. 6 by Treffers et al.), and about governmental committees and teacher associations (in Chap. 5 by Smid).

If we look at teacher education, we see a dynamic relationship between the approach to educating teachers and the reform movement in the Netherlands. This particularly applies to the primary school level of mathematics education, because primary school teacher educators were heavily involved in the development of the reform. Therefore, parallel to the changes in primary mathematics education, the curricula of primary mathematics teacher education have drastically changed since the 1970s. What this change means is thoroughly outlined by Oonk, Keijzer, and Van Zanten (Chap. 8). They point out that, with respect to mathematics, primary school teacher education, where students are educated to teach all subjects in primary school, can be characterised as including both a focus on the interconnection between mathematics and didactics, and on the integration of theory and practice. What is more, the developed teacher education theory for primary school mathematics teacher education is largely in line with the RME theory for teaching students in school. This parallelism comes to the fore in the approach to teaching teacher students and teaching students in primary school. For both, concrete mathematical situations are taken as a starting point. For primary school students it means to activate their intuitive notions and start with informal procedures, which, under the guidance of the teacher, can evolve to more formal mathematics. The teacher students start their learning to teach mathematics by carrying out mathematical activities at their own level. Subsequently, their own experiences in learning mathematics are combined with reflections on the learning processes of students. Together, these give them a basis for teaching mathematics. By analysing and discussing real teaching practices

and describing their own reflections on these practices, student teachers are prompted to use theoretical ideas and terminology from the didactics of mathematics, and teach mathematics in a professional way. As a result, practical knowledge can develop into so-called ‘theory-enriched practical knowledge’.

Compared to primary school teacher education, teacher education for secondary mathematics teachers is far more complex. In this respect, the overview given by Daemen, Konings, and Van den Bogaart (Chap. 9) speaks volumes. Although in one respect secondary teacher education is less complicated than teacher education for primary school, because the focus can be on one subject, the complicating factor comes with the situation that in secondary education there are different school levels and different types of schools, including general education and all kinds of vocational education. This means that there are different routes for qualifying as a secondary education mathematics teacher. For the highest levels of secondary education student teachers go to university. For the other levels they go—like most student teachers for primary school—to colleges for higher vocational education, nowadays called universities for applied sciences. All school levels have their own teacher education programme, which has to prepare student teachers for teaching secondary school students of different capability levels and teaching, to a certain degree, different mathematical content. To prevent the learning process to be too fragmented, much effort is put into working with profession-related tasks which follow a ‘whole-task’ model. Such a task could include, for example, designing a lesson or a test, or designing a lesson series that one has to carry out. Through these profession-related tasks, the aim is to achieve coherence between theoretical courses and practice-oriented activities.

A determining element of the systemic context of Dutch education is the system of assessment and evaluation. This is highlighted by Scheltens, Hollenberg, Limpens, and Stolwijk (Chap. 16), who are affiliated to Cito, the Netherlands national institute for educational measurement. In their chapter, they provide an outline of the tools that are available in the Netherlands for informing schools, teachers, and students about the learning achievements in mathematics for both formative and summative purposes. They describe the content and goals of the various national primary and secondary standardised tests, and illustrate their descriptions with samples of test items. Moreover, they also include examples of examination tasks, for which they also offer the marking guidelines. The overview shows that the picture of official assessment in the Netherlands—that means the assessment commissioned by the government—looks rather diverse. The tests and examinations contain context-based open tasks, but also multiple-choice tasks and bare mathematical tasks. Similarly to what can be seen in the textbooks, the reality of assessment shows a quite moderate version of the big ideas of RME. This, again, is an act of balancing between different approaches to mathematics education and between different interpretations of RME.

1.7 The Implementation of RME

Although the government to a certain degree facilitated the development of RME by establishing institutions and commissions and by giving grants for projects for doing research on mathematics education, developing new instructional materials, and organising professional development for teachers, the reform cannot be labelled as a government-instigated enterprise. This is at least not the case for primary school mathematics education. In secondary education, there was more government interference in connection with decisions made about the central examinations at the end of secondary school.

A major government-paid implementation project in lower and pre-vocational secondary education taking place in the 1980s and 1990s is described by Hoogland (Chap. 11). In this project, the Ministry of Education made funds available for pilot schools and the development of experimental teaching materials, as well as making possible the change of the formal curriculum and the final examinations for secondary vocational education in the examination year 1996, which they did with broad support from parliament. For the teachers in the pilot schools, the most common way to communicate the curriculum changes was through discussing exemplary tasks of the final examinations and comparing ‘old’ tasks with ‘new’ tasks. Characteristic of the whole implementation project was the broad involvement of all relevant stakeholders. In addition to teachers, students, parents, editors, curriculum and assessment developers, teacher educators, publishers, media and policy makers were also part of it, and a continuous and extensive dialogue took place among them. Also, the spirit of that time was an important factor in this implementation process. In education and society there was a general feeling that change was necessary. There was an agreed focus on equity and basic education for all, including mathematics, and at the same time there was a commitment not to waste the human potential in mathematics, in particular not that of girls. Another factor that contributed to the implementation was the use of so-called ‘advocate teachers’. These were teachers at the pilot schools who acted as advocates for the reform and had an important role in the professional development activities. Other important change agents were the in-service and pre-service teacher education institutions, the publishers, and the education inspectorate, who all supported the chosen vision or were at least benevolent to the change. As Hoogland indicates, the intended changes have proven to be quite sustainable, since the current mathematics textbook series and final examinations still reflect essential tenets of the original vision. At the same time, however, he makes it clear that the change is very vulnerable, by referring to the debate and the framing in social media that started in the first decade of this century, which claim that the educational change in the 1990s is to blame for the alleged low level of mathematics of today’s students.

Besides large projects purposely set up to introduce RME in school practice, Wijers and De Haan (Chap. 2) illustrate that extra-curricular mathematics competitions and events, such as the Mathematics A-lympiad, the Mathematics B-day, the Lower-Secondary-Mathematics-Day, and the National Mathematics Day for primary education, can also form a springboard for innovation. For example, when teachers

have to prepare their students for the Mathematics A-lympiad competition by giving them opportunities to get experience in working in groups on rich open-ended unfamiliar problems which require mathematical reasoning and modelling, the influence can also work in the other direction. Experience with these competitions which contain other types of problems than the regular textbook problems can prompt teachers to change their regular teaching of mathematics. This means that in this way these competitions and events can become an implementation instrument.

Speaking about ways to implement RME raises the question of what was accomplished of the ideas of RME in Dutch classrooms. Similar to other questions that can emerge when thinking about mathematics education in the Netherlands, this volume cannot give a full answer. In general, most authors indicate that the ideas of RME are unmistakably recognisable in Dutch mathematics education, but in a number of chapters, there are also clear concerns about deficiencies in the implementation. One thing that is rather often mentioned is the difference between what are considered good tasks to elicit mathematical thinking in students and the tasks which can regularly be found in textbooks, the production of which is left to the market in the Netherlands. As Wijers and De Haan (Chap. 2) describe, if open problems are included in textbooks, these mostly refer to the core content of the lesson or the chapter at hand. This means that students do not need to model the problem situation to find a strategy for solving the problems, because the strategy is the one that has been treated in the chapter. The findings of Vos (Chap. 3) when she analysed a textbook chapter and a sample of examination tasks also highlighted that quite a number of tasks in the textbook were dressed-up tasks offering students training to find formulae. Also, many artificial contexts were used, which contrasted with the finding that the examination tasks contained authentic contexts more often. The difference between what RME stands for and what is offered in textbooks was already clearly brought to the fore in the first decade of this century, when it was found that primary school textbooks mostly contain straightforward calculation problems and that opportunities for real problem solving and mathematical reasoning are almost completely lacking. To the same conclusion Gravemeijer (Chap. 12) came. He observed that advanced conceptual mathematical understandings are not formulated as instructional goals, neither in the textbooks, nor in official curriculum documents, and that textbooks capitalised on procedures that can quickly generate correct answers, instead of investing in the underlying mathematics. It is clear that the ideal situation differs from what is actually realised in reality!

This discrepancy also applies to another essential requirement that should be fulfilled in order to realise RME in practice and bring it to fruition, namely a change in classroom culture. One of the cornerstones of RME is that a learning environment should be created that makes guided reinvention possible, in which students can come up with their own solutions and discuss these with other students. Offering students rich open-ended problems that they can work on collaboratively and through which they have opportunities to express their thinking, only works when there is a classroom atmosphere which really stimulates students to communicate about mathematics. Implementing RME in class requires that justice is done to RME's activity principle (treating students as active participants in the learning process) and

its interactivity principle (using interaction to evoke reflection and bring students to a higher level of understanding).

However, as indicated by some of the authors, this RME classroom culture has not been entirely successfully implemented. Kool (Chap. 7) explains that in practice it has turned out that it is quite challenging to stimulate students to join actively in interactive problem solving and reasoning, and it places high demands on teachers. Providing students with ready-made solution methods will no longer do. Instead, teachers have to ask their students thought-provoking questions such as “Why does this work?” and “Does it always work?”. At the same time however, the teacher should work on a classroom atmosphere in which the students feel confident enough to explain and justify their solutions, to try and understand other students’ reasoning, and to ask questions when they do not understand something, and challenge arguments they do not agree with.

Also, Van Gulik-Gulikers et al. (Chap. 13) experienced in their teaching experiment about the 18th century land surveyor that the students were not used to a situation in which they had to delve deeply into problems that require more fundamental thinking, broader exploration and endurance. According to Van Gulik-Gulikers et al., this unfamiliarity with such problems may be because, in their regular classes, students often work independently in their textbook, which makes that these complex tasks are often skipped or split into a number of small parts that are easy to digest. This kind of practice is not what one would expect when thinking of RME-based teaching.

The strongest concern about the implementation of a new classroom culture as one of the core aspects of RME is voiced by Gravemeijer (Chap. 12). Also, he thinks that the innovative point of RME to offer students an inquiry-oriented learning environment with many opportunities for interaction and collaboration did not have a systematic elaboration at classroom level. Based on what recent research has revealed about the instructional practice in the Netherlands, according to Gravemeijer the question can even be asked how RME actually works out in Dutch classrooms. For him the solution is that RME should adopt a socio-constructivist approach.

1.8 The Context of Creating a New Approach to Mathematics Education

In the Netherlands, compared to other countries, the reform of mathematics education that started at the end of the 1960s and eventually resulted in RME was mainly a bottom-up process with low government interference. That this reform happened in this way is in essence a consequence of the Dutch constitutional ‘freedom of education’ that is laid down in the Constitution of 1917. This law was originally meant to give parents the right to found schools in accordance with their religious views. Nowadays, this law implies also that schools can be founded based on particular pedagogical and instructional approaches. Another result of this freedom of education is that the government is rather hesitant in giving instructional prescriptions. In

fact, the Ministry of Education can only prescribe the subject matter content to be taught and not the way in which this content is taught. This means that textbook authors and publishers have much opportunity to include their own views and ideas on teaching mathematics. Moreover, there is no authority which recommends, certifies or approves Dutch textbook series before they are put on the market.

What is also different in the Netherlands than in most other countries is the position of mathematicians. As is clearly underlined by Smid (Chap. 5), Dutch mathematicians have a rather problematic relationship with mathematics education. This means that on this point the Netherlands deviates from what is considered a distinctive feature of the European tradition. Except for Freudenthal, mathematicians did not have a determining role in the mathematics curriculum. From the 1970s on, the role of the mathematicians and their organisations in school mathematics was minimal, and they hardly seemed interested. This changed only in the first decade of this century, when mathematicians discerned a lack of algebraic skills in first-year university students. Moreover, due to unsatisfactory achievement scores of Dutch primary and secondary school students in national and international studies, a public debate emerged about the quality of education, which caused that the government took on more of a steering role. One measure that was taken to assure that all students acquired a certain basic level in mathematics and particularly in arithmetic, was that the government decided that both secondary education students and primary school teacher students had to do a compulsory arithmetic test. Furthermore, recently the Ministry of Education installed a platform and a number of development teams with representatives from primary and secondary education for enacting a society-broad reconsideration of what students should learn in school to equip them for the future society, their later profession and their personal development. Asking people from school practice, along with other experts, to think about the future curriculum is again a kind of bottom-up approach, yet it is different from what begun half a century ago.

The reform that started at the end of the 1960s was in many ways a child of its time. Just as the society of that time was ripe for a change, meaning that the existing values and way of living were turned upside down, the renewal of Dutch mathematics education also showed characteristics of a certain anarchist stance. In the initial period of the reform, this manifested itself for example in the production of texts in which an alternative spelling was used. 'Equivalentie-klassen' (equivalence classes) became 'ekwivalentie-klassen' and 'mate van exactheid' (degree of exactness) became 'mate van eksaktheid', and capitals were left out in names and titles of books and chapters. This atmosphere of wanting to be innovative that was characteristic for IOWO (Institute for the Development of Mathematics Education) and OW&OC (Mathematics Education Research and Educational Computer Centre) has lingered long in the Freudenthal Institute. De Lange's (Chap. 17) reflection unmistakably shows the traces of this ambiance. He characterises the institute as different, sometimes provocative, but often innovative with vision and carefully bombarding the Ministry of Education with an array of novel ideas such as new curricula, new

software, mathematics for all, A-lympiads, cutting edge conferences, and international collaboration. In the words of De Lange, there was never a dull moment. In this way the Freudenthal Institute and its predecessors were for a long time the epicentre of the reform, with Freudenthal as the authority to make it all happen.

Reference

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