# Chapter 6
# Dutch Didactical Approaches in Primary School Mathematics as Reflected in Two Centuries of Textbooks

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**Abstract** This chapter contains an overview of the most important textbook series used in the Netherlands from 1800 to 2010. We distinguish five time periods, and for each period we highlight the textbook series that are most characteristic. To describe the textbooks that were in fashion in the successive periods we distinguish three categories of textbooks: procedural, conceptual, and dual textbooks. The dual textbooks have elements of the first two. For the procedural textbook series, which are also referred to as 'mechanistic', memorisation of mathematical facts, automatisation on of operational procedures and recognising types of problems are the primary interest. Application is only considered at the very end of the teaching trajectory, and then rarely. Smart, flexible (mental) calculations and estimating are not part of the program. The conceptual textbook series have an opposite approach. In learning mathematical facts and procedures, understanding is highly valued, and applications are included from the start as the basis for this. Number sense, flexible (mental) calculation, and estimation are central, next to algorithmic calculation. Students can design their own problems, develop solution methods and work on their own level. As expected, using different textbook series with different content and teaching methods results in pursuing different goals in mathematics education, which in turn results in different learning outcomes, as has been shown by national evaluations of progress in educational achievement.

This chapter is a revised version of Chap. 7 of Adri Treffers' book *Weg van het Cijferen* (*A-Way With Arithmetic*) (Treffers, 2015).

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## 6.1 Introduction

Knowing Dutch mathematics textbooks means knowing a lot about Dutch mathematics education. Especially in primary school, mathematics textbooks in the Netherlands are not just books with exercises for students. They go together with extensive teacher guides. In fact, the textbooks determine largely what mathematics is taught and how it is taught. They can be seen as the potentially implemented curriculum (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Of course, in what way mathematics education comes to life in classrooms can differ between teachers and can also not be the same for all types of students, but in general what is in the textbooks is also found in classrooms. Therefore, textbooks are a good source to gain knowledge about the Dutch didactic tradition. They even give us a window to mathematics education in the past that we cannot witness anymore, and for which no video recordings are available.

In the chapter, a tour is made along textbooks for primary school mathematics education from 1800 to 2010. The aim of this tour is to reveal the various approaches towards mathematics education that were in fashion during this period of time, and in this way to learn about how these different approaches evolved and what lessons can be learned from the past for our current mathematics education in primary school. The description of the textbooks that were in fashion in the successive periods is structured by distinguishing three categories of textbooks: procedural, conceptual, and dual textbooks. Because the various views on the teaching mathematics manifest mostly in the domains of flexible calculation, algorithmic calculation and applications, we focus our description on these domains.

### *6.1.1 Procedural Textbook Series*

The procedural textbook series are sometimes also referred to as ‘mechanic’ or ‘mechanistic’. The adage of these textbooks is “First do, then know”. They focus on the mathematical content. Their intention is to line out this content exactly, that is, to split it based on difficulty level and to unravel every part into the tiniest details. In these textbook series, there is a lot of attention for repetition, based on the idea that practise makes perfect, and provides skill. Little value is given to understanding as the basis for practising, for example in the case of tables of multiplication. Memorisation of mathematical facts, automatisation of operational procedures and recognising types of problems are the first and foremost interests. There is a single-track approach, guided by rules, and aiming to achieve an efficient standard method to solve a particular type of problems as quickly as possible. The operations for calculations up to ten, twenty and one hundred are performed according to a fixed rule, starting with splitting at ten for adding up to twenty. Next, the standard recipes for the algorithmic calculations of the four basic operations with whole numbers, decimal numbers and fractions are successively introduced and rehearsed. Application is only considered at the very end of the teaching trajectory, and then only rarely. Smart, flexible (mental) calculations and estimating are not part of the programme.

### 6.1.2 Conceptual Textbook Series

The conceptual textbook series differ from the procedural textbooks series on almost all the aspects mentioned. The mathematical content is not atomised. Understanding is highly valued in learning mathematical facts and procedures. This is one reason that the most shortened forms of the calculation algorithms are not taught immediately, but that teaching starts with transparent predecessors of these algorithms. Applications are included from the beginning of the teaching trajectory, as the basis for understanding. Number sense, flexible (mental) calculation, and estimation have a central place next to algorithmic calculation. Within a framework of carefully formulated goals, students are given the opportunity to design their own problems, develop solution methods and work on their own level. And finally, the relations between the four basic operations and between the sub-domains ratio, fractions, percentages and measurement are firmly anchored.

Within the category of conceptual textbook series three sub-types can be distinguished. There are textbooks with a heuristic, a functional, and a realistic orientation.

The conceptual textbook series with a heuristic orientation strongly emphasise that understanding must come first. They focus on students' insightful, self-inquiry-based way of dealing with numbers in a whole-class setting led by the teacher. Moreover, these textbook series put great importance on applications being given attention from the very start.

The conceptual textbook series with a functional orientation are characterised by stimulating the understanding of students by, for example, involving them actively in discussions about the adequacy of certain solution methods. Applying learned-by-heart tricks should be avoided as much as possible. The functionality of these methods is mainly reflected by letting the students estimate and check their answers, and the connection that is sought to daily-life related problems.

The conceptual textbook series with a realistic orientation, called 'realistic textbook series'—named after the domain-specific instruction theory of Realistic Mathematics Education (RME)—have much in common with the heuristic and functional textbook series, but distinguish themselves through, for example, offering more rich problems and often choosing a thematic approach and a problem-oriented way of teaching. The realistic textbook series also contain new components such as calculations with the aid of a calculator and spatial geometry related to the world around us. A third difference involves the comprehensive use of contexts and models, such as the arithmetic rack, the (empty) number line, the (percentage) bar, and all kinds of diagrams, schemas and tables.

### *6.1.3 Dual Textbook Series*

Dual textbooks series form an intermediate category of textbooks series. They cannot be considered as purely procedural textbook series nor as conceptual, but contain elements of both. This can, for example, imply that a textbook series on the one hand gives no attention to flexible mental calculation, and shows little interest in including applications, but on the other hand deals with algorithmic calculation in an insightful manner. The reverse can as well be the case. Then, conceptual elements of flexible (mental) arithmetic, estimation and their applications go together with the mechanistic approach of a procedural textbook series. Dual textbooks series can also have different approaches for the lower and the higher grades.

### *6.1.4 Textbooks Series in Use Over Five Time Periods*

When describing the most important textbook series that were in use from 1800 to 2010 we make a distinction in five time periods. For each of them we highlight the textbook series that were characteristic for that time (see Fig. 6.1). The first period, labelled as 'Procedural didactics and semi-textbook use', running from 1800 to 1875, is a kind of pre-stage of textbook use. From 1875 to 1900 was the period in which the use of complete textbooks started. Then, the first type of conceptual textbook series was used, namely those with a heuristic orientation. In the next half century, from 1900 to 1950, the dual textbook series were in use. The subsequent time period, from 1950 to 1985, is both characterised by the use of procedural textbook series as well as by the use of conceptual textbook series with a functional orientation. The time between 1985 and 1990 is taken up by the conceptualisation of a new curriculum for mathematics education in primary school, published in the *Proeve*[1] (Treffers, De Moor, & Feijs, 1989). This programme was meant to get more coherence in what and how mathematics is taught. In the time period from 1990 to 2010, the divide in procedural and conceptual textbook series was basically over. The large majority of the textbook series which are now in use belong to the conceptual textbook series and have a 'realistic' signature or have at least a number of RME characteristics (Treffers, 2015, pp. 130–135; Van den Heuvel-Panhuizen & Drijvers, 2014).

[1]The full title is *Proeve van een Nationaal Programma voor het Reken-wiskundeonderwijs op de Basisschool.*

| | | | |
|---|---|---|---|
| 1800 | **Procedural didactics** & **Semi-textbook use** | *Lower grades* No textbooks; mathematics taught on the blackboard | |
| | | *Upper grades* textbook series for individual use | 1835: Hemkes<br>1850: Boeser |
| 1875 | **CONCEPTUAL Heuristic** textbook series | | 1875: Versluys<br>1878: Van Pelt |
| 1900 | **DUAL** textbook series | | 1918: Bouman & Van Zelm<br>1935: Diels & Nauta |
| 1950 | **PROCEDURAL** textbook series & **CONCEPTUAL Functional** textbook series | | *Procedural textbook series*<br>1950: Naar Zelfstandig Rekenen<br>1970: Niveaucursus Rekenen<br>*Functional textbook series*<br>1949: Geef Acht!<br>1958: Functioneel Rekenen<br>1969: Nieuw Rekenen |
| 1985-1990 | *Proeve*: Towards a National Programme for mathematics education in primary school | | |
| 1990 | **CONCEPTUAL Realistic** textbook series | | 1981: De Wereld in Getallen<br>1983: Rekenen en Wiskunde<br>2001: Pluspunt<br>2001: Rekenrijk<br>2002: Alles Telt<br>2009: Wizwijs |
| 2010 | **PROCEDURAL** textbook series | | 2009 Reken Zeker |

**Fig. 6.1** Overview of two centuries of textbook series for primary school mathematics in the Netherlands

## 6.2 The Period 1800–1875: Procedural Didactics and Semi-textbook Use

### *6.2.1 Teaching Mathematics on the Blackboard and No Complete Textbook Series Available*

In Article 4 of the first Dutch Education Law launched in 1806, mathematics was considered a compulsory school subject alongside reading and writing. In 1810 the government compiled an official list of recommended mathematics textbooks series. These textbooks were not used for whole-class instruction, but to teach students individually. Moreover, they were only meant for the upper grades of primary school. Therefore, the period 1800–1875 can be referred to as semi-textbook use, since no complete textbook series for all grades of primary school were available.

In the lower grades mathematics was taught without textbooks. This involved mechanistic teaching of mathematics on the blackboard. The approach was clearly procedural. An assistant teacher wrote the problems on the blackboard and the students then had to calculate them on their slate. Beforehand, drill-and-practise activities were done with the whole class to prepare carrying out algorithmic calculations. The approach was purely procedural—there was no space for flexible calculations. Once students had mastered the standard algorithms, then in the upper grades they were given a textbook for the first time and had to work through the book individually under their teacher's guidance.

### *6.2.2 The Textbook Series by Hemkes*

The so-called 'ten cent' textbook series by Hemkes (1846) were the most used textbooks during the period 1800–1875. In 1835 Hemkes published the textbook *De Kleine Rekenaar* (The little calculator), meant for beginners. In this textbook each of the basic operations starts with problems with bare numbers, followed by word problems with named numbers. Then follow assignments for practising with mixed problems for all operations that have been dealt with so far, addition, multiplication, subtraction and division respectively. This is followed by a repetition chapter with these problems—all in all nearly 400 problems. What is remarkable, is the large variation in difficulty between the problems within one category. Take, for example, the difference in difficulty level between the first two of the following problems and also notice in the third problem the often occurring elaborated and informal style of the word problems.

1. Count together: 3, 2, 8 and 9.
2. How much does the sum of all numbers that you can make with the numbers 5, 6 and 9 amount to?

3. Hans could not work out how much money one would have to pay for 5 cows if one cow cost 85 guilders. Oh, do give the fool a hand.

The other 'ten cent' textbooks, consecutively dealing with decimal fractions, money, measurements and weights, the 'rule of three' and proper fractions, follow the same structure. Each time the textbook begins with bare number problems, followed by word problems with named numbers for each operation in the previously mentioned order.

The Hemkes textbooks were rather popular. Only in the second half of the 19th century they were gradually replaced by Boeser's mathematics textbooks.

### *6.2.3 Boeser's Mathematics Textbooks*

The textbook series by Boeser (1850) address the same content topics as Hemkes' textbooks, but Boeser's word problems, contrary to those of Hemkes, were quite factual and brief in their formulation. In the foreword of Boeser's *Eerste Rekenboekje*, which he published in 1850, he wrote:

> Yet, according to me, some of the books for beginning reckoners are too extensive, too tedious; others contain too few imaginable situations derived from the children's world and daily life, whereas most of these books (I give my own opinion here) suffer from the evil of the mechanistic approach. Not only do these books contain a series of dry problems which children have already done while working on the blackboard, the books also offer a number of problems with the assignment 'add together', 'subtract', and so on, which can be done without developing or practising the children's ability to reason.

Here Boeser has a point; when application problems have been placed within a particular category of problems, for example in a section about multiplication problems, the students automatically know that they can find the solution by doing a multiplication. And a further point is: why should a mathematics textbook contain bare number problems if the students previously practised these problems through doing whole-class calculations on the blackboard? Hence, it is no surprise that Boeser's textbooks particularly contain mixed application problems. An example follows now.

> A man worked daily for 12 hours; he spent 2 hours on eating and 3 hours on leisure activities. The remaining time he spent on sleeping. How many weeks did he spend sleeping in a year?[2]

Boeser's books with the mixed applications were particularly popular and they were reprinted until the 20th century.

[2] All quotations from textbooks are translated from Dutch by the authors.

## 6.3 The Period 1875–1900: Conceptual Textbook Series of a Heuristic Orientation

### *6.3.1 Influence from Germany*

At the end of the 19th century the conceptual approach to teaching mathematics made its appearance. This was also the first time that a complete textbook series was used. This new approach was launched with the publication in 1865 of Rijkens' translation of a handbook of the well-known German mathematics didactician Hentschel (Hentschel & Rijkens, 1865), who is called the 'father of the new arithmetic in public schools'. Partly due to this publication, a renewed interest arose in the work on early mathematics education done by Grube, who was another important German mathematics didactician, whose *Leitfaden für das Rechnen* (Guide for arithmetic) was published in 1842. In the Netherlands, Grube's publication became known through the slightly adapted version that was published by Brugsma in 1847 (see also Brugsma, 1872).

### *6.3.2 Versluys*

Versluys, the founding father of the Dutch didactics of mathematics, was heavily inspired by the German mathematics didacticians Hentschel and Grube when writing his textbook series *Rekenonderwijs ten Dienste van de Lagere School* which was published in 1875 (see Versluys, 1875). He characterised his conceptual textbook series as 'heuristic', which means that the focus was on insightful, self-inquiry-based learning of mathematics within a whole-class setting guided by the teacher.

Yet, in Versluys' textbook for the first year of primary school not much can be recognised of what is usual in today's mathematics education. The main reason for this is the monographic method that Versluys applied. Here, he followed Grube. Typical for this method is the 'operational' way the numbers 1–20 are handled. Every time a number is built up, split up, and 'measured' with preceding numbers. This means that from the very beginning the four basic operations are involved in the teaching of mathematics, but first through oral assignments and word problems and not in formal symbolic notations. In the teacher guide, for each number under ten about one hundred problems are provided for making calculations with present or absent objects and amounts, and with unnamed numbers. For the numbers from ten to twenty the quantity of provided problems has been reduced to less than fifty.

However, the way Versluys treats calculations up to one hundred has a lot in common with how we do it today. Additions and subtractions are not done in a prescribed manner, but flexibly. At the same time the multiplication (and division) tables are memorised more or less automatically, following a heuristic approach, structured

| | | | |
|---|---|---|---|
| 4 | 4 | 4 | 4 |
| 23 x | 23 x | 23 x | 23 x |
| 80 | 12 | 12 cent | 12 enen |
| 12 | 80 | 8. dubb | 8. tienen |
| 92 | 92 | 92 cent | 92 |

**Fig. 6.2** Structure of teaching multiplication by Versluys (1875) (cent = cent; dubb = dime; enen = ones; tienen = tens)

through doubling and reversing, and by means of products with five and ten. The properties of the multiplication operation which are at stake here can be used later for mental arithmetic. Here, too, there is again a lot of attention for applications. With problems such as 9 × 13 and 13 × 9 the impetus is given for flexible (mental) calculation, whole-number-based written calculation and insightful algorithmic (digit-based) written calculation, plus the relationships between them. For all these, a rectangular model serves as a visual basis.

Algorithmic multiplication is structured ingeniously. On the one hand, the problem 4 × 23 ("4 children each get 23 cent") makes a connection with repeated addition, and thus with the shortened algorithmic method the students learned earlier for addition. On the other hand, through 23 × 4 ("23 children get each 4 cent") students learn the zero rule of algorithmic calculation that is applied in the case of multipliers with multi-digit numbers (Fig. 6.2).

The digit-based algorithm of long division is prepared through a whole-number repeated subtraction approach. Again, an elementary word problem serves as a concrete basis: "How many times can 4 guilders be taken from 936 guilders?" First a chunk of 4 guilders is taken away 200 times, then from the remainder 30 times a chunk of 4 and finally 4 times a chunk of 4 guilders; altogether this makes 234 times.

Versluys assigns as much value to flexible mental arithmetic as to algorithmic calculation. For both mathematical domains, he starts with numbers up to one hundred. What is noticeable, is the large amount of word problems and the rather small number of bare number problems—for Versluys arithmetic is in the first place applied arithmetic. Only in the upper grades of primary school does this change: then complex algorithmic calculations appear which are in Dutch called 'vormsommen' (form problems). Figure 6.3 shows an example of such a problem.

$$19 \times \frac{6\frac{3}{4} - (5{,}79 - 3\frac{1}{2})}{31\frac{2}{3} : 10} + \frac{256 \times 19}{475} - \left(\frac{(22{,}2}{1{,}2} - 0{,}46 \times 2\frac{4}{23}\right) : 0{,}5.$$

**Fig. 6.3** Example of a 'vormsom'

### 6.3.3 Van Pelt

Van Pelt, who published *De Nieuwe Rekencursus* in 1878 (Van Pelt, 1878, 1896, 1903), also makes use of the monographic method for calculations up to twenty. However, for calculations up to hundred and thousand, his approach differs from that of Versluys. In Van Pelt's approach, there is no room for algorithmic calculation in the first three years of learning. It is mental arithmetic all the way. Furthermore, Van Pelt makes it very clear that for him the final goal is that the multiplication tables are known by heart, and that on the way to that goal the students have to learn to understand a variety of properties that they will be able to utilise in mental arithmetic in future. Van Pelt (1903, p. 15–17) states the approach in his textbook series as follows:

> No sensible teacher will ask here whether this is the fastest way for students to learn the tables. He will understand that the main goal of our teaching has to be development, development by doing and searching on your own. He will quickly notice that this way of working has such an influence on the students that later on they will choose this approach themselves, especially in mental arithmetic. (…). The author of the arithmetic course expresses his annoyance that at this time so many still learn their tables by memory work: yet, every teacher knows that arithmetic education should be purely *heuristic*.

And a bit further on, when it is about calculation up to one thousand, next to the smart calculations, Van Pelt places the stylised, whole-number-based written calculation, which is the prelude to insightful algorithmic written arithmetic in the fourth year of primary school. If there is ample attention in mental arithmetic for insightful solutions for problems such as 40 × 55—via 40 ells[3] at 55 cents—then learning the calculation algorithm for 43 × 55 should not be a problem. According to Van Pelt, you first calculate 3 × 55 and 40 × 55 separately and then combine the results from both.

As far as practical applications are concerned, there is no essential difference between Versluys and Van Pelt. They both give word problems a central place in their textbook series, and utilise them from the beginning as a concrete starting point for learning formal calculation procedures.

### 6.3.4 The Adage of the Conceptual Mathematics Textbook Series with a Heuristic Orientation

The didactic adage of the conceptual mathematics textbook series with a heuristic orientation is "First know, then do, first think, then do"—exactly the reverse of the procedural motto!

However, this adage can be easily misunderstood. Because knowing and thinking do not only refer to understanding of the calculation rules and procedures, but

[3] An 'ell' is an old length measure. In the Netherlands, an ell was 69.4 cm.

also to comprehending their applicability. And according to the heuristic point of view, this applicability can be guaranteed only when applications are part of the teaching trajectories from the very start. This even goes so far that initially the calculation procedures are adapted for the sake of applicability! So, it may happen that students learn two versions of the division algorithm: whole-number-based division with repeated subtraction involving quotative division ("How many weeks are there in 364 days?") and digit-based long division for problems that imply partitive division ("Fairly divide the amount of 364 guilders among 7 people. How much does each of them receive?"). Only after some time, these two forms of division are brought together and combined into the common form of long division. Also, the gradual, whole-number-based start of the algorithmic calculations for the other basic operations is inspired partly by the criterion of applicability.

From the above it can be concluded that 'conceptual' is considered as 'first knowing why' and only then 'knowing how' is not correct. In fact, the why and the how are intertwined here. The teaching of the multiplication tables, as Versluys and Van Pelt advocate, is an eloquent example of this.

## 6.4 The Period 1900–1950: Dual Textbook Series

In the first two decades after 1900, new textbook series were published, which can be called 'dual', since they combine characteristics of both procedural and conceptual textbook series. They are characterised by the fact that they:

- Abandon the monographic treatment of the numbers up to twenty, and the teaching of multiplication and division in the first year of primary school
- Restore the prominent place of counting in the initial phase of education
- Do no longer give priority to word problems, but start with bare number problems
- Assign a larger relevance to algorithmic calculations than to mental arithmetic
- Put more emphasis on skills than on insightful calculation.

The textbook series by Bouman and Van Zelm (1918) was one of these new dual textbook series. This textbook series gained greatly in popularity from around 1920. Considered from the viewpoint of the very beginning of learning arithmetic, Bouman and Van Zelm's (1918) textbook has a one-sided focus on counting—one-sided because this textbook series rejects number images and all kinds of visual means that elicit grouping procedures. Numbers should remain unnamed; they are mathematical conceptions, and as such cannot be related to objects and figures.

In the teacher guide it says: "Mathematics education should put the emphasis on calculation with unnamed numbers." Yet, the authors permit that the students can start by drawing dots and circles as long as they have no specific meaning.

Aside from elementary mental calculation, the textbook series of Bouman and Van Zelm does not give attention to flexible mental calculation outside the area of numbers up to one hundred. Only in 1934 do the authors revise their deviating approach regarding flexible mental calculation.

Algorithmic calculation is taught in an insightful manner through whole-number-based calculations, but the authors remain true to their beliefs: the use of concrete objects and named numbers such as cents, ten cents and guilders to provide insight into the calculation procedures within the decimal system, is discouraged strongly. For example, considering the ten as 'equal' to ten cents does not make the understanding and applicability of the concept simpler, according to Bouman and Van Zelm.

Furthermore, it is curious how little interest they have in the issue of applicability. Take for example the topic of division. In the Booklets 5 and 6 (meant for Grade 3), there are 2500 bare number problems and only fifty application problems for quotative and partitive division. Applications only appear after Booklet 9 (meant for Grade 5), so around the point that students who will not go to secondary education are about to stop.

Because of neglecting flexible (mental) calculation and disregarding the inclusion of application problems in algorithmic calculation in the lower and middle primary school grades, the original version of the textbook series by Bouman and Van Zelm is closer to procedural textbooks than to conceptual textbooks. The great attention that this textbook series devotes to complex thinking problems does not affect this conclusion, since learning to solve these problems is mostly an issue of training to recognise the type of problems for which the solution methods have been practised. If you have not been trained to solve these problems, it will be impossible or at best very hard to solve them correctly—consider, for example, the following percentage problem from Bouman and Van Zelm.

> A pays *ƒ*160,38 for a bale of coffee. If he enjoyed 1% for cash payment and had 10% tare and had to pay 45 cents per half kilogram, what was the gross weight of the bale?

In 1935, Diels and Nauta published their textbook series *Fundamenteel Rekenen* (see Diels & Nauta, 1939, 1944), which can also be characterised as a dual textbook series. Diels and Nauta were strongly against this type of thinking problems. Nevertheless, despite their critique and resistance from primary school teachers and thinking psychologists, they had to include them in their textbook series, because these problems remained part of the secondary school entry examinations. In addition, the textbook series by Diels and Nauta differs not only at the end, but also at the start of mathematics education from (the first version of) the textbook series by Bouman and Van Zelm. In arithmetic up to twenty, in Niels and Nauta's textbook, together with counting, the use of number images is involved. In addition, these authors do not abandon problems with named numbers, although these problems only appear little by little at the end of the teaching trajectory for arithmetic up to twenty.

From the second school year on, Diels and Nauta reserve ample time in each lesson for mental calculation with word problems and oral calculation exercises with bare numbers. Some examples of the word problems from Booklet 9 (meant for Grade 5) of *Fundamenteel Rekenen* published in 1944.

(1) An airplane is at a height of 3600 m and suddenly descends 735 m. How many m is it still above the earth?

(2) A troop of soldiers covers 23 km in 4 h. Now they still need to cover 11½ km. How long does that journey take, if they rest for two hours along the way?

(3) Michiel de Ruyter was born in 1607 and died 1676. How old did he become? How many years ago did he die?

(4) A gentleman loses a wallet with ƒ4000. He will give 2½% as a reward to the finder. How much reward will the finder receive?

The oral calculation exercises (according to the teacher guide, these exercises mean that “the teacher reads out the problem, the students write down the answer”) usually consist of bare number problems (Fig. 6.4).

With this approach, Diels and Nauta firmly followed the 1936 school inspection guideline that states: “Mental arithmetic should take up a prominent position; when it is done with small numbers, it does not merely have practical value, but also fosters understanding.”

The textbook series *Fundamenteel Rekenenl* is the first one that has put estimation on the educational programme. This is motivated as follows: “It was attempted to further discourage the purely mechanical work by repeatedly having the result estimated before calculating it.”

Considered so far, this textbook series by Diels and Nauta is a purely conceptual textbook series. However, this changes when we consider algorithmic calculation. As can already be seen from the quotation on estimation, the authors are critical, not to say negative, about algorithmic calculation. In this context, they speak about training and mechanisation which lead to thoughtlessness and would obscure understanding of the number system. The amount of algorithmic problems is therefore considerably smaller than with Bouman and Van Zelm—for some parts it is only a quarter of it. Another noteworthy point is that in the Diels and Nauta textbook series the algorithmic procedures are not taught in a comprehensive way, not even for relatively small numbers such as 364 ÷ 7, where whole-number-based division by means of repeated subtraction would be an obvious approach. Also, the applications of algorithmic calculation with large numbers including decimal numbers are only treated in the higher grades. In this respect, this textbook series shows all in all a typical procedural approach. In other words, along with the innovative, conceptual elements of flexible (mental) arithmetic, estimation and their application, the textbook series of Diels and Nauta does also contain elements that are in line with the procedural textbook series. Therefore, this often-used textbook series, just as the textbook series

Which problems have the wrong answer?

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4,8 | + 5,3 | = 8,1 | | 4,3 | + 0,7 | = 5 | | 160 | + 59 | = 209 | |
| 2,7 | + 6,4 | = 9,1 | | 10 | x 2,15 | = 215 | | 12 | x 4¾ | = 57 | |
| 27 | : 6 | = 4½ | | 300 | x ¼ | = 75 | | 3,6 | : 9 | = 0,04 | |

**Fig. 6.4** Oral calculation exercise from Diels and Nauta (1944)

of Bouman and Van Zelm, has to be characterised as dual, even if the Diels and Nauta textbook series is closer to the conceptual approach than the latter.

It was only in the course of the 1950s that these two textbook series lost their leading position in the market.

## 6.5 The Period 1950–1985: Procedural Textbook Series and Conceptual Textbook Series with a Functional Orientation

Considered quantitatively, the procedural textbook series dominate in the period from 1950 to 1985. Five of such textbook series managed to acquire a substantial market share, with *Naar Zelfstandig Rekenen* (Zandvoort, Venekamp & Kuipers, 1955/1970) for many years the largest. With respect to content, this textbook series fully meets the general characterisation of the procedural approach. Nevertheless, what makes the series special is the practical organisation of independent, individual work in a looser classroom setting, which is made possible by the particular setup of the booklets. Alongside every page with problems there is a page with suggestions and worked examples that can, if necessary, be further explained by the teacher. The learning content is systematically divided into small units which have been organised on the basis of increasing complexity. Based on this information about the textbook series, it can only be concluded that this procedural textbook series is proficiently assembled: the structure of the learning content is well thought out, and the instructions for the students are adequate. Partly because of this, the textbook series is fairly easy to use in everyday teaching. The other procedural textbook series with a large market share, *Niveaucursus Rekenen* (Vossen et al., 1970), is also based on a more loosely organised classroom setting and the approach and content of this textbook series largely corresponds with *Naar Zelfstandig Rekenen*.

Halfway through the 20th century, parallel to the procedural textbook series, a new type of conceptual textbook series emerged, which can be called ‘functional’. The ideas about mathematics education laid down in these textbook series were as innovative as the approaches reflected in the conceptual textbook series with a heuristic orientation that were in use in the 1870s. For the new conceptual textbook series with a functional orientation, new educational avenues were followed inspired by the German Psychology of Thought and its didactical application by the Amsterdam school of Kohnstamm (1952).

The best known functional textbook series is *Functioneel Rekenen* written by Reijnders and Snijders (1958). However, this textbook series was not the first of its type. The honour for being the first to develop a new conceptual textbook series goes to Rombouts and his textbook series *Geef acht!* (Rombouts, 1948). This textbook series is the counterpoint to the pure recipe-based procedural textbooks series. Finally, the third functional textbook series was *Nieuw Rekenen* (Bruinsma, 1969), that advertised itself as a functional textbook series and was, based on its market share, the most successful one of these three series.

The textbook series *Functioneel Rekenen* (Reijnders & Snijders, 1958) emphasises:

– The importance of a whole-class discussion about the various solution methods for bare number problems and context problems
– Own productions of problems in various forms, for example, making up problems that go with a given answer, or finding a suitable word problem that fits to a given bare number problem, or formulating a question for a problem without a question
– The power of visual models such as the number line, tables and bars, in solving problems
– A lot of attention for flexible mental calculation
– The value of estimation for roughly determining or checking the outcome of a calculation, but especially as a didactical tool for teaching precise calculation.

The latter point is an entirely new idea that is explained further using the example of 52 + 29. First the outcome is given a lower (70) and an upper (90) limit, and then this outcome is made more precise in various ways using questions such as “How much more than 70?” and “How much less than 90?” The explanation provided in *Functioneel Rekenen* finishes with: “These kinds of solution methods should emerge constantly in whole-class or group discussions. The numbers are handled in an entirely different way than in algorithmic calculation.” About algorithmic calculation the following is said: “Throughout the whole textbook series the goal has been to avoid mechanical work with numbers—that is, applying tricks that have been learned by heart—as much as possible.”

Although the title of the textbook series emphasises the functional aspect, this principle is not applied to the teaching of algorithmic multiplication. As with Diels and Nauta the standard procedures for whole numbers, decimal numbers and fractions are taught in a purely recipe approach. Yet, for the upper grades of primary school the textbook series does contain applications of various forms: problems without questions, free productions, closed problems in which the students have to find the right operations, and also elementary mental calculation and estimation problems.

The textbook series *Nieuw Rekenen* assigns an important role to mental calculation. This mainly involves calculations done while using your head, rather than done in the head. The general introduction of the textbook series *Nieuw Rekenen* (Bruinsma, 1969, p. 12) states this as follows:

> Mental calculation is always arithmetic that is functional and based on understanding. What it is not: algorithmic calculation done in the head or applying tricks. Mental calculation always demands understanding of the structure of the numbers. Mental calculation certainly does not imply that paper may never be used. It is often advisable to give especially those children with a less good memory the opportunity to write down intermediate answers; this puts the child at ease and can strengthen confidence. Real life demands the ability to perform simple mental calculations quickly; the child must be able to quickly understand relationships and know which calculations to perform; mental calculation increases understanding of the number system and encourages discovering the many possibilities that lead to the same answer.

In short, flexible (mental) calculations are not just a goal in themselves, but also function as a didactical tool to foster number sense, insightful arithmetic and applicability. Five examples from Booklet 4b for Grade 4 on multiplication show how this basic concept is made concrete.

1. Calculate:
   $5 \times 98 = 5 \times 90 + 5 \times 8 =$
   but also $5 \times 100 - 5 \times 2 =$
   and half of $10 \times 98 =$
   Calculate in different ways.
   $7 \times 98$ $\quad 4 \times 98$ $\quad 12 \times 25$

2. Calculate in the simplest way.
   $6 \times 94$ $\quad 8 \times 97$ $\quad 28 \times 29$

3. Make your own problems.
   There is a fence around a meadow.
   The meadow is long 120 m, wide 80 m.

4. Make 5 multiplications.
   The outcome is always 450.

5. Estimate first!
   $9 \times f3.75 =$ $\quad 85 \times f0.97 =$

Problem 1 puts the students on the trail of smart calculation that can be applied in Problem 2. In Problem 3, algorithmic calculation comes into view with $28 \times 29$.

Moreover, the textbook series *Nieuw Rekenen* makes an insightful transition from calculation by splitting and whole-number-based calculation to algorithmic calculation. For multiplication, this transition could take place in a few lessons and could go as follows (Fig. 6.5).

In (a) and (b) the whole-number-based calculation is shown. In (c) the transition to calculating with digits is made. Earlier, this approach is also followed for addition

7 x 98 = 7 x 90 + 7 x 8 = 630 + 56 = 686.

| (a) | (b) | (c) |
|---|---|---|
| | | 5 |
| 98 | 98 | 98 |
| 7 x | 7 x | 7 x |
| 630 | 56 | 686 |
| 56 | 630 | |
| 686 | 686 | |

**Fig. 6.5** Structure of multiplication in *Nieuw Rekenen*

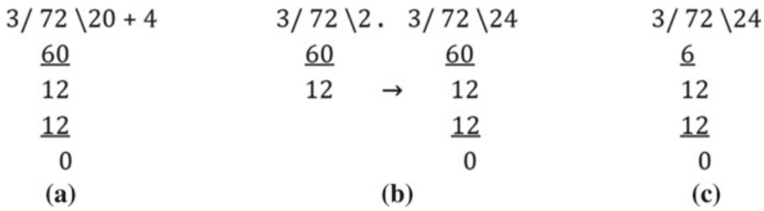


**Fig. 6.6** Structure of long division in *Nieuw Rekenen*

and subtraction, after first showing the position values of the ones, tens and hundreds using coins (cents, dimes, and guilders).

Long division (Fig. 6.6) is introduced using a problem such as 72 ÷ 3 with the question: "Into how many groups of 3 can I divide 72?"

The students can start with (a) for a few problems, but soon they will have to switch to (b). Using the shorter notation, from '20' to '2.' to '2' takes place in one lesson. The transition from (b) to (c) is made in more difficult problems.

This phased approach can also be found in the conceptual textbook series with a heuristic orientation by Versluys and Van Pelt and in the dual textbook series by Bouman and Van Zelm. The main difference with these previous textbook series is that in *Nieuw Rekenen* the transition from (a) via (b) to (c) happens in a few lessons and the description in the textbook is apparently mostly aimed at the teacher who can use this to give an insightful explanation of the shortened standard procedure. In *Nieuw Rekenen*, fractions, ratios and percentages are also taught in an insightful manner in the sense that the teacher, following the textbook series, can explain the concepts and operations as clearly as possible on the basis of models and schemes.

Moreover, *Nieuw Rekenen* contains many and varied application problems, that are sometimes placed together in a thematic series, for example, a series about shopping, sales receipts, foreign money, train journeys and distances in Europa. The closed, half-open and open assignments match well with what is being taught at the time; they are applications of what students have previously learned in a purely numerical way.

In this respect, this textbook series distinguishes itself from the textbook series *Geef Acht!* In the teacher guide Rombouts (1959) says:

> The problem, the genuine mathematics problem, is both the start and the end. It is used to give context to the calculations, and it is returned to over and over again, since everything has to have a purpose for the students as well. Not start with 'mathematics' and later on 'applied mathematics', but together, connected.

This textbook series is further characterised by the emphasis on flexible mental calculation, own productions and a reduction of the learning content. The latter was, with respect to the thinking problems and problems about the metric system, was only made possible when at the beginning of the 1970s, the comprehensive entry examination for secondary education was replaced by the less broad Cito End of Primary School Test.[4]

## 6.6 The Period 1985–1990: Towards a National Programme for Primary School Mathematics

The time period between 1985 and 1990 is characterised by the many efforts that have been made to get a new programme for mathematics education in primary school. The start of working on this programme goes back to the beginning of the 1970s when the large-scale Wiskobas[5] project was launched. From 1971 to 1981 the Wiskobas team developed, together with the educational field and in school practice, a large collection of rich problems and themes for various topics within arithmetic, measurement and geometry, and they developed initiatives for new textbooks. In this period, also the foundation was laid for what later became known as Realistic Mathematics Education (RME).

The Wiskobas publications functioned as a source of inspiration for the new, realistic textbook series, that is, RME-oriented textbook series, that hesitantly appeared on the educational market around 1980—hesitantly, because their content was not a seamless match for the then prevailing mathematics teaching practice and the Cito End of Primary School Test. The effect of all this was a (too) large diversity in textbook series. The need for a (new) national mathematics curriculum with explicitly stated end goals made itself felt. On the initiative of the Nederlandse Vereniging tot Ontwikkeling van het Rekenwiskundeonderwijs (NVORWO; Netherlands association for the development of mathematics education), hundreds of people involved in the field of education were consulted about the future of mathematics education in primary education. In 1987 this resulted in the *Proeve* (Treffers et al., 1989), the first design for a national programme for mathematics education in primary school. A few years later, the mathematics end goals for primary school described in this publication were officially given approval by the government (OCW, 1993) and served as beacons for textbook authors and test developers.

The concrete learning goals in the *Proeve* involve the domains of basic skills, algorithmic calculation, ratio, fractions, percentage, and measurement and geometry. These core goals can be typified as follows.

- For the domain of basic skills much emphasis is put on the understanding of the decimal place value system, mental calculation, estimation, applications, as well as appropriate use of calculators.
- Algorithmic calculation provides room to learn variants of the conventional procedures, such as the ones earlier described in this chapter for long division.

[4]The Cito End of Primary School Test is developed by Cito, the Netherlands national institute for educational measurement.

[5]Wiskunde op de Basisschool (Mathematics in Primary School).

– Because of its broad applicability the domain of ratio is interpreted much more widely than the formal ratios of traditional arithmetic. Practical calculations with percentages are also given a lot of attention—with the focus firmly on understanding the concept of percentage.
– Fractions and decimal numbers, and the relations between them, are given meaning in various ways. The students should be able to compare, order, add, subtract, multiply and divide fractions and decimal numbers in simple application situations. Directly leading students to mastery of the mathematical rules for these four basic operations is rejected.
– More attention than in the past is given to measurement, calculations with common measures and representing measurement data in schemes and graphs. However, there is less attention for practicing the metric system.
– Teaching geometry starts with focussing more on observing (peep dioramas, photos, localising, light and shadow, building block constructions) than on making calculations. Students come across a great diversity of aspects of mathematics from starting with observable reality, such as visualising, using geometrical models, spatial orientation and reasoning, reflecting on one's own actions, applying geometrical knowledge and insights to practical and puzzle-like problems, and all this in relation to topics from the field of arithmetic and measurement.

The *Proeve* does not limit itself to describing the end goals of mathematics education, but also makes statements about the didactics. Because it is a good habit within RME to use examples from teaching practice or from a textbook series for explaining particular approaches, we also will do this now. To illustrate what this didactics means, we have chosen a problem where different core aspects of 'numbers and operations' can be seen. RME-oriented mathematics education sometimes uses newspaper clipping to ask mathematical questions. In this case this is done as well.

> Hard work in the bulb fields
>
> Every year in spring in the Netherlands there is a lot of work to be done in the bulb-growing industry. This is the fourth year that Johan has been doing this work; he has worked both in the fields and in greenhouses. Now he works in the transport department of a company in the auction halls. "I usually load the trucks, that is heavy work. I usually work 220 hours a week. That's good, because you earn money that way", according to Johan.

How would students in Grade 3 and 4 react to this newspaper clipping? It depends on one's expertise and sensitivity regarding the thinking of students whether one can judge students' solutions in advance. But even if one has the experience, children will still come up with surprises. To begin with, there are students who do not (purely) calculate. Here are some examples:

– No, that is impossible because people work 36 or 40 h a week and this is way too much.
– Yes, it is possible because it's very heavy work and that often takes long to do.
– No, because my mother already works 180 h a week. If he worked harder, he'd be working the whole day, that's a bit much.

| | | |
|---|---|---|
| | | 2 |
| 24 | 24 | 24 |
| 7 x | 7 x | 7 x |
| 140 | 28 | 168 |
| 28 | 140 | |
| 168 | 168 | |

**Fig. 6.7** RME structure of algorithmic multiplication

– Yes, because you can load trucks and grow bulb fields at the same time.
– It's not possible, because a week only has 168 h.

Moreover, the calculations that lead to the last conclusion are very diverse:s

– repeated addition: 24, 48, 72 … 168
– repeated doubling: 24, 48, 96, 192; 192 – 24 = 168
– multiplication by splitting: 7 × 24 = 140 + 28 = 168
– algorithmic multiplication: 7 × 24 written 'underneath each other'
– smart multiplication: 7 × 24 = 7 × 25 – 7 = 175 – 7 = 168
– idem, but wrong: 7 × 24 = 7 × 25 – 1 = 175 – 1 = 174 (!)
– estimation: 10 full working days is 240 h, so 220 h would be well over 9 days and a week only has 7 days.

The newspaper clipping about Johan is an interesting problem, because it is located on the crossroads of different content strands. Calculation by splitting, smart calculations, algorithms and estimation can all be included in it. It is clear that the students can learn much from each other as a result of discussing differences in reasoning and calculations. The initial focus will be on the various calculations of 7 × 24, starting with the fairly laborious approach of repeated addition. Next, the approaches of multiplication by splitting and algorithmic multiplication provide an opportunity to show once again that the algorithmic approach to multiplication is in fact a shortened procedure of the splitting approach (see Fig. 6.7).

The smart calculating of 7 × 24 via 7 × 25 requires further explanation. We know that $7 \times 25 - 1 = 174$ results in an incorrect answer. How can we make the right approach insightful?

The teacher has a model at hand to visualise the correct solution method: a stack of 7 boxes (days), filled with 25 units (hours), from which one in each box, so a total of 7, has to be removed: $175 - 7 = 168$. How will the students in fact try to explain this calculation to each other?

Furthermore, extensive attention can be given to the magnificent 'suppose that' reasoning. You can start from 220 h per week and show that you end up with an impossible number of days per week, that is, $220 - 24$ or in other words well over 9 days. Another possibility when faced with that kind of reasoning is to divide 220 by 7, ending up with a day having over 31 h. These last-mentioned argumentations will automatically lead to estimation. At the end of the lesson the question arises: "Is Johan that dumb, or was it simply a slip of the tongue?"

After the students have discussed the problem in groups of two or three for a few moments, they will collectively conclude that Johan probably made a mistake and meant 220 h a month. This would mean that he works about 55 h per week, that is, 11 h per day in a five-day working week, or 9 h per day in a six-day working week, and that is working hard.

In the procedural approach to arithmetic, which is the dominant approach within traditional mathematics education, the problem $7 \times 24$ appears in three forms:

- As a multiplication problem in horizontal notation; meaning the problem has to be calculated as $7 \times 24 = 140 + 28 = 168$
- As a multiplication problem in vertical notation; meaning the student has to use algorithmic calculation
- As a dressed-up version with the question “How many hours are there in a week?” and a free choice of how to calculate.

In this procedural approach, no attention at all is given to smart calculation of $7 \times 24$ via $7 \times 25 - 7$.

The lesson about the newspaper clipping shows where a conceptual approach with a realistic orientation distinguishes itself from a procedural approach. Moreover, this lesson also makes it clear that the lesson also differs from a conceptual approach with a functional orientation where, in general, such rich problems are not used.

## 6.7 The Period 1990–2010: Realistic Textbook Series

### *6.7.1 An Abundance of Textbook Series*

The revision of the mathematical content and the end goals, along with the didactic repositioning as described in the *Proeve*, did not fail to have an influence on textbook series. Although the realistic textbook series resemble the functional ones, there are differences as well. What both conceptual textbook series have in common is that they in addition to algorithmic calculation with not too large numbers, give attention to insight into numbers, flexible (mental) calculation and estimation, and that they work only with often used fractions and metric measures. New topics in the realistic textbook series involve applied arithmetic using a calculator and geometrical knowledge of the world. The didactic approach of the new, realistic textbook series is broadly along the lines of the functional approach—broadly, since the functional approach falls short for the second half of primary school. Here, there is a difference with the realistic textbook series, in which in fact the entire mathematics curriculum is modernised. Another difference can be found in the more problem–oriented and (often) thematic character of the realistic approach. Yet another difference is the extensive use of contexts and models, such as the arithmetic rack, the (empty) number line, the (percentage) bar and all kinds of diagrams, schemes and tables.

The two most commonly used realistic textbook series after 1990 are *De Wereld in Getallen* (Van de Molengraaf et al., 1981; Huitema et al., 1991, 2001, 2010) and *Pluspunt* (Groen et al., 2001; Van Beusekom et al., 2010). Since 2000, the total market share of these two textbook series is 70%. Other realistic textbook series with a substantial distribution rate are *Rekenrijk* (Bokhove et al., 2001, 2010), *Alles Telt* (Boerema et al., 2002) and to a lesser degree *Wis en Reken* (Buijs et al., 1999; Bergmans et al., 2001), which is the successor of one of the first realistic textbook series *Rekenen en Wiskunde* (Gravemeijer et al., 1983). All these realistic textbooks series are based on the principles of RME, which in general means that students give productive input in an interactive, (semi-)whole-class setting.

In the case of *Pluspunt* students even have so much input that in some parts of this textbook series the leading role of the teacher is marginalised. *Pluspunt* sets three of the five lessons each week for having students to work independently. Partly for this reason, this textbook series is referred to as semi-instructive, as opposed to the instructive *De Wereld in Getallen*. This latter textbook series not only organises four out of the five weekly lessons in a whole-class setting, but also has an approach in which more teacher guidance is provided. The difference between these two realistic textbook series is specifically reflected in the topic of algorithmic calculation (for wich *De Wereld in Getallen* offers a better structure) and the topic of percentage (for wich *De Wereld in Getallen* offers more starting points for teacher guidance). Both its organisation with offering students opportunities to work on their own and its thematic approach make *Pluspunt* an appealing textbook series that manages to acquire a 45% market share in the period 2000–2010, against 25% for *De Wereld in Getallen*. This choice was clearly not inspired by student results, as the first three national evaluations of the progress in educational achievement by Cito, the PPON[6] studies (Wijnstra, 1988; Bokhove, Van der Schoot, & Eggen, 1996; Janssen, Van der Schoot, Hemker, & Verhelst, 1999) that were carried out from 1987 to 1997, were in favour of *De Wereld in Getallen*.

### 6.7.2 The Results from the Cito PPON Studies

If one compares in the first three PPON studies (Wijnstra, 1988; Bokhove et al., 1996; Janssen et al., 1999) the mathematics achievements Cito found for the various textbook series in use, it is clear that the conceptual textbook series, both the functional and realistic ones, perform much better than the procedural textbook series.

In the third study, it was found (see Janssen et al., 1999) that on the 24 mathematical sub-domains that have been investigated, both *Nieuw Rekenen* and *Pluspunt* score on average more than 5% points higher than the procedural textbook series *Naar Zelfstandig Rekenen* and *Niveaucursus Rekenen*. The textbook series *De Wereld in Getallen* finishes on average 10% points higher than these procedural textbook series. For the basic operations with numbers, for estimating, for applied arithmetic with

[6]Periodieke Peiling van het Onderwijsniveau (Periodic Assessment of the Education Level).

the use of a calculator, and for calculations with percentages this difference even increases to around 20% points!

The findings of this Cito study into the effects of various types of textbook series is of lasting relevance. Students perform better when conceptual textbook series are used than when procedural ones are used.

With respect to the sub-domain of operations (mostly algorithmic calculations), which is the main focus of the procedural textbook series, the first three PPON studies show that the differences between conceptual and procedural textbook series are only marginal. Nevertheless, from the results of the 1997 PPON study (Janssen et al., 1999) it is clear that the scores in this mathematical sub-domain are decreasing in comparison to the first study in 1987—a trend that continues in the fourth study carried out in 2004 (Janssen, Van der Schoot & Hemker, 2005), but stops in 2011 (Scheltens, Hemker, & Vermeulen, 2013). Conversely, improved performances are found on other topics, that is, insight into numbers, mental calculation (addition and subtraction), estimation, applied arithmetic with the use of a calculator, calculations with percentages and relations in the contexts of graphs. The performances on all these topics increase on average 15% points—about the same increase as the decrease for algorithmic calculations.

Taken together, these are spectacular research outcomes!

## 6.8 The Future Landscape of Textbook Series in the Netherlands

In 2015, the market shares of the largest textbook series had undergone a radical shift. That of *De Wereld in Getallen* has increased by 25% points to 50%, making the oldest realistic textbook series by Huitema and his collaborators one of the most successful and influential textbook series in the history of Dutch mathematics education. The most recent edition (Huitema et al., 2010) has been revised for both organisational and didactical structure as content. Students receive a week task for independent working which they can do in the second part of each lesson, after the whole-class instruction. These tasks are available at three difficulty levels, minimum, basic and advantaged, which makes it possible that the students can work on a level that is in tune with their ability. Further revisions in the textbook series are that some of the teaching sequences for measurement have been adapted; less time is spent on digit-based algorithmic calculation; and addition, subtraction and multiplication are more geared towards whole-number-based written calculation.

The market share of *Pluspunt* has fallen by 25% points to 20%. Compared to the 2001 edition, more time is spent on algorithmic calculation, much more than in *De Wereld in Getallen*. The organisational structure of the semi-whole-class system with three lessons a week for students to work independently and two lessons for whole-class instruction has not changed. New is that, as in *De Wereld in Getallen*, for differentiation three difficulty levels are distinguished.

Also, most of the other textbook series published a revised version, but there were only a few changes in their already not large market share. *Alles Telt* became a bit larger and the market share of *Rekenrijk* has decreased. For *Wis en Rekenen* no new, revised version was published. The new textbook series *Wizwijs* (Van Groenestijn et al., 2009) did not acquire a substantial market share. The same is true for the textbook series *Reken Zeker* (Terpstra & De Vries, 2009), explicitly published to implement (again) a procedural approach, which has not obtained a market share of any significance.

Taking into account that replacing a textbook series in a school takes about ten years, the current situation means that for the time being, realistic textbook series are used most frequently. So, we may conclude that two centuries of working on mathematics education have been decided in favour of the conceptual approach. This choice for a conceptual approach rather than a procedural one, is exactly what Freudenthal argued for around forty years ago (Freudenthal & Oort, 1977, p. 337).

> When a child finishes primary school, it has processed between ten and twenty thousand arithmetic problems – the degree to which it succeeded with them will determine its further education and its road in life, following a type of lower vocational education or […] general secondary education. But foremost this fact of learning to calculate (and the achieved or not achieved success in this learning) will determine the mathematical (or rather anti-mathematical) attitude of the student – and, what is even worse – of the teacher who has to teach mathematics […]. [This learning to calculate reflects][7] a view of a human being as an efficiently to be programmed computer, while the performance typical of a computer will never be approached. The education that we develop has been determined by another image of a human being, and by another view of mathematics – not as subject matter, but as a human activity.
>
> I have previously given this the triple characterisation of
>
> – Linked to reality
> – Near to the children
> – And socially relevant.
>
> And I will now sum up these characteristics in one that encompasses all: human worthy, the human being as a learner, as a teacher, as a counsellor and as a creator of education. (translated from Dutch by the authors)

These words that were spoken by Freudenthal on accepting an honorary doctorate at the University of Amsterdam, where he stated his 'realistic' vision of mathematics education against the sharply contrasting background of procedural mathematics that dominated education at the time, have lost none of their relevance today.

[7] Added by the authors.

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