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ORIGINAL PAPER

Market equilibriums for transport operators with several goals

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Abstract
Purpose The paper addresses a transport market consisting of two firms with goals extending beyond pure profit maximisation. Considering that transport companies often have public owners and that managers have different objective that the owners, it is argued the firms maximise a weighted sum of profits, revenues and total consumer surplus.
Methods The paper analyses equilibrium fares and quantities arising from collusion and competition on price (Bertrand) and quantity (Cournot), when the firms produce symmetrically differentiable services and have identical cost and goal functions.
Results Special focus is given to analyzing how the firms' costs, the degree of substitutability and complementarity between their services and their goal functions influence equilibrium prices in the three different competitive situations. The influence of parameters included in the model regarding the differences between the equilibrium prices is also addressed.
Conclusions The study provides relevant knowledge for transport authorities of how transport firms respond to changes in competitive regimes depending on their objectives and competitive situation.

Keywords Collusion · Equilibrium prices · Goal functions · Passenger transport · Price competition · Quantity competition

1 Introduction

It is well recognized that the design of optimal fares for transport firms depends on the goals that are to be maximised, e.g. [1]. Various recent studies have, for example, focused on how fares are set in firms maximising a weighted sum of profits and consumer surplus, see [2, 3] and [4]. However, firms and transport authorities may also maximize other goals than profit and consumer surplus (e.g. [5, 6]). [5] discusses mixed goal functions and deduces the conditions for optimal fares and vehicle-km supplied by bus companies wanting to maximise social surplus, passenger-km or vehicle-km subject to a budget constraint.

It has been argued that private transport operators may not purely maximise profit. This is, according to [3] and [7], mainly due to two reasons. Firstly, at least in Scandinavia, local businesses, local authorities and states own substantial equity interests in the transport companies.[1] All these groups of owners concern themselves with the standard of public transport. Local businesses in the license area of the company are interested in good public transport so that customers gain easy access to their facilities. Local authorities holding shares in transport companies operating locally are also concerned about the transport quality they offer, because good transport is important for the inhabitants' wellbeing, and therefore for the development of the community. These arguments are also relevant for the state as an owner and support the inclusion of consumer surplus in the goal function.

The second reason why firms could pursue other goals than profit is the separation of ownership and leadership. This

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[1] In Norway, for example, public bodies held in 2004 the majority of shares in 36 of the 95 bus companies [21] and the states of Norway, Sweden and Denmark held, respectively, 14 %, 21 % and 14 % of the shares in the dominant air carrier (SAS). There are also various degrees of public ownership in air and rail companies in many European countries [14].

implies that managers (agents) may have other goals than the owners (principals) and also some power to pursue them. Managers working and living in the transport firms' operating areas are naturally interested in keeping popular amongst the inhabitants by offering good quality public transport; i.e. they place weight on the consumers' wellbeing. Moreover, they may in particular be interested in running a big company since their salaries and status are often positively related to the firms' size; for example measured in terms of the firms' revenues. The relevance of the leaders' power and interests on the firms' goal functions and their subsequently influences on prices and quantities supplied have been addressed previously by [8–13]. These managerial discretion models are, thus, also relevant as far as transport firms are concerned [1].

Bearing the above arguments in mind, the aim of this paper is to expand the scope of the analysis provided by [4] by incorporating revenue in the transport firms' goal functions in addition to the profit and consumer surplus included in the original model. Transport operators maximise, thus, a weighted sum of profits, revenues and consumer surplus. In accordance with [4] we focus on the cases in which two firms compete simultaneously with regard to quantity (Cournot) and price (Bertrand) and when they collude. Even though the liberalization of the transport markets in industrialized countries has increased competition,[2] many transport routes are still served by one or two suppliers, at least when it comes to passenger transport (e.g. [14–17]). One or two suppliers are commonplace on many routes in both bus and air transport and in the UK most trains are served by one or two companies. Moreover, since passengers often make use of more the one company to complete a trip, the same companies may produce both substitutable and complementary services. Hence, the model takes into account a specific feature of the transport market that the infrastructure, e.g. airports and roads, are provided by the government and that transport firms provide scheduled routes on a commercial basis.

The aim of this paper is to derive equilibrium prices and quantities and discuss how the prices are influenced by 1) the weights firms place on profits, revenues and consumer surplus; 2) their competitive situation; 3) the degree of substitutability or complementarity between the services they offer and, finally; 4) their costs. Specific attention is also given to analysing the combinations of weight put on revenue and consumer surplus resulting in equal equilibrium fares. This knowledge can help regulators aiming to meet politically decided objectives through regulation.

The further organization of the paper is: in Section 2 the model with equilibrium prices and quantities are presented. Then, Section 3 takes the analysis further by conducting comparative analyses of the different equilibrium prices. Finally, implications specifically aimed at regulators are suggested in Section 4.

2 Market solutions

2.1 The model

2.1.1 Demand and cost conditions

Let us assume a transport market operated by two firms able to set fares and quantity freely. Following the model originally developed by [18] and later applied by [4] for the transport markets, it is assumed that a representative passenger has the following utility function, based on the use of the two services X_1 and X_2

$$U(X_1, X_2) = X_1 + X_2 - \frac{X_1^2 + X_2^2 + 2sX_1X_2}{2} \tag{1}$$

in which s denotes the degree of substitutability or complementarity between the two services. If $s = 1$ and $s = -1$ then the services are perfect substitutes and perfect complements, respectively. A value of $s = 0$ represents the case of independent markets. Consequently, this parameter indicates the degree of competition between the firms in any specific market situation. In the following analyses it is assumed that the parameter value for s is restricted to $-1 < s < 1$.

By maximisation of the passenger's consumer surplus $\left(S = U(X_1, X_2) - \sum_{i=1}^{2} P_i X_i\right)$ the symmetric inverse demand functions are given by:

$$P_1 = 1 - X_1 - sX_2 \text{ and } P_2 = 1 - X_2 - sX_1 \tag{2}$$

where P_i represents the price for firm $i = \{1,2\}$. Assume, for example, that the demands for transport services provided by firms 1 and 2 in equilibrium are represented by X_1^* and X_2^*, respectively. Using Eqs. (1) and (2) in combination with $\left(U(X_1, X_2) - \sum_{i=1}^{2} P_i X_i\right)$ gives the following expression for passengers' total consumer surplus (S)

$$S = \frac{X_1^{2*} + X_2^{2*} + 2sX_1^*X_2^*}{2} \tag{3}$$

[2] In air transport, this trend started with the Air Deregulation Act of 1978 in the US and with the three liberalization packages between 1988 and 1997 in Europe [22].

The firms are assumed to have the following identical cost functions[3]:

$$C_i(X_i) = cX_i \quad \text{where} \quad 0 < c < 1 \tag{4}$$

The above functions lead to the following profit expressions, π_i (pi), for the firms:

$$\pi_1 = (1-X_1-sX_2)X_1-cX_1 \quad \text{and} \quad \pi_2 = (1-X_2-sX_1)X_2-cX_2 \tag{5}$$

2.1.2 The firms' goal functions

Following the argument that goals are influenced by ownership structure and the power of leadership, firms' are assumed to maximise the following weighted sum, G_i, of profits, revenues and total passengers' surplus of which all are measured in pecuniary terms

$$G_i = \pi_i + \beta(P_iX_i) + \gamma S \quad \text{where} \quad 0 \le \beta, \gamma \le 1 \tag{6}$$

In (6) the revenue of firm i is given by (P_iX_i) and the parameters β (beta) and γ (gamma) represent the weight put on revenue and consumer surplus, respectively. This goal function implies that both operators have identical goals and that each operator is only concerned about the total surplus for the passengers (S).[4] It is reasonable to focus on the consumer surplus that both firms in total bring about since they serve the same population. The restrictions placed on β and γ imply that the firms cannot place lower weight on profits than on revenues and passenger surplus. If $\beta=\gamma=0$, then the firms are pure profit maximisers, and if $\beta=\gamma=1$ then they place equal weight on profits, revenues and passenger surplus. If ($\beta=0$, $\gamma>0$) and ($\beta>0, \gamma=0$) then the firms are only concerned about profits and passenger surplus and profits and revenues, respectively. In intermediate cases where $\beta,\gamma>0$, the firms put weight on all three separate factors and the values of β and γ depend on, as emphasized earlier, their ownership structure and the power of their leadership.

There are few studies of which values these parameters can take in practice. By combining fare schemes and information on costs [3] was able to calculate that bus and ferry operators in Norway weighted profit 38 % and 8 % higher than consumer surplus, respectively. This would correspond to $\gamma \approx 0.72$ for bus and $\gamma \approx 0.93$ for ferry. The value of s was assessed by [16] using the own and cross-price derivative. For example, a s-value of 0.5 (−0.5) implies that $\frac{\partial X_1}{\partial P_1} = \frac{1}{s^2-1} = -1.33$ and $\frac{\partial X_1}{\partial P_2} = -\frac{s}{s^2-1} = 0.67(-0.67)$ meaning that an increase in own price by one unit will decrease own demand by 1.33 units and increase (decrease) the rival's demand by 0.67 units. The valuation of these parameters in a specific context could be revealed by studying how prices deviate from profit maximization in combination with a manager interview and an assessment of the market conditions.

2.2 Stability and existence conditions

For later discussion it is useful to study whether the conditions for which the interior equilibriums exist impose more restrictions on the parameters in question. Conditions for stability and concavity for the cases of Cournot, Bertrand and collusion are presented in Table 1. The stability condition implies that the absolute value of the cross derivative of the response function is less than 1; that is $|\partial X_1/\partial X_2|$, $|\partial X_2/\partial X_1|<1$. The concavity condition implies that $\partial^2 G_i/\partial X_i^2<0$. See e.g. [19] and [20] for discussions of these conditions in oligopoly models.

From Table 1 it follows that the concavity conditions are always met when $0<\beta,\gamma<1$. The stability conditions for the case of Cournot and Bertrand when the firms place no weight on revenues ($\beta=0$) and when they value profits and revenues equally ($\beta=1$) are given as the areas under the lower envelopes of the unbroken and broken curves in Fig. 1, respectively. The curves show that the stability conditions are also met under the previous restrictions placed on β and γ. Hence, neither the concavity nor the stability conditions lead to more bindings on β and γ.

2.3 Equilibrium prices for different competitive situations

2.3.1 Simultaneous price competition (Bertrand)

When the transport firms maximise their goal functions in (6) by setting prices strategically, we obtain the following common equilibrium price (P^{B*}) and quantity (X^{B*})[5]:

$$P^{B*} = \frac{(1-s)(\gamma-\beta-1)-c}{\gamma(1-s)+(s-2)(1+\beta)} \quad \text{and} \quad X^{B*} = \frac{1+\beta-c}{(1+s)(\gamma(s-1)+(2-s)(1+\beta))} \tag{7}$$

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[3] This may be a reasonable assumption when operators using the same modes compete; for example when two bus operators, two airlines etc. compete. Their services can, however, still be different. These costs do not consider the cost indivisibility often found in the transport industry, but will be valid for the use of existing capacity or when capacity can be made available on a short notice e.g. if resources can be allocated from other parts of the firm.

[4] Whether social surplus is maximized when maximizing G_i depends on the values of β, γ and the shadow price of raising public funds. If the shadow price is 20 %, as suggested by [3], and $\beta=0$ then the transport operator must put 21 % higher weight on profit compared to consumer surplus when aiming to maximize social welfare.

[5] It can be seen from (2) that when finding direct demand and inserting for X_2 in X_1 then $X_1 = \frac{1}{s^2-1}(s+P_1-sP_2-1)$. Inserting for X_1 in $G_1 = (P_1-c)X_1 + \beta P_1 X_1 + \gamma\frac{X_1^2+X_2^2+2sX_1X_2}{2}$ and solving for P_1 yields the response function $P_1 = \frac{1}{2\beta-\gamma+2}(c-s+(1-s)(\beta-\gamma)+sP_2\,(1+\beta-\gamma)+1) = R_1(P_2)$. Price in Eq. (7) is derived by inserting the symmetric response function for firm 2 and the expression is equal for the two firms. Inserting for P_1 and P_2 in the expression for X_1 presented above produces the equilibrium quantity given in (7). The same relationships form the basis when deriving the equilibriums for Cournot in (13) and collusion in (19).

Table 1 Conditions for symmetric interior equilibrium

	Cournot	Bertrand	Collusion
Stability	$\gamma < \min\left\{\frac{(\beta+1)(s+2)}{1+s}, \frac{(\beta+1)(s-2)}{s-1}\right\}$	$\gamma < \min\left\{\frac{(\beta+1)(s+2)}{1+s}, \frac{(\beta+1)(s-2)}{s-1}\right\}$	None
Concavity	$\gamma<2(\beta+1)$	$\gamma<2(\beta+1)$	$\gamma<2(\beta+1)$

The bindings on β and γ leading to the stability conditions in Fig. 1 being met ensure that both P^{B*} and X^{B*} are positive. If the firms do not receive any subsidies, the conditions that guarantee non-negative profits ($\pi \geq 0$) imply that:

$$P^{B*} \geq c \rightarrow \gamma \leq \frac{(c-1)(s-1) + \beta(1-s-2c+cs)}{(c-1)(s-1)} \quad (8)$$

When the firms do not place any weight on revenues ($\beta=0$) the condition under (8) implies that $\gamma \leq 1$. Hence, the firms must value profits equal to or higher than consumer surplus. Increasing β leads to tighter (looser) restrictions on γ in (8) when $c>(<)(1-s)/(2-s)$. A further inspection of the equilibrium prices gives:

$$\frac{\partial P^{B*}}{\partial c} = \frac{1}{\gamma(s-1) + (2-s)(1+\beta)} > 0 \quad (9)$$

$$\frac{\partial P^{B*}}{\partial s} = (c-\beta-1)\frac{1+\beta-\gamma}{(2+s\gamma+2\beta-s-\gamma-s\beta)^2} < 0 \quad (10)$$

$$\frac{\partial P^{B*}}{\partial \beta} = \frac{\gamma(1-s) + c(s-2)}{(2+s\gamma+2\beta-s-\gamma-s\beta)^2} \geq (<) 0 \text{ when } \frac{\gamma}{c} \geq (<) \frac{2-s}{1-s} \quad (11)$$

$$\frac{\partial P^{B*}}{\partial \gamma} = (s-1)\frac{1+\beta-c}{(2+s\gamma+2\beta-s-\gamma-s\beta)^2} < 0 \quad (12)$$

The denominator in (9) is positive with the previous restrictions placed on the parameters. Then higher costs lead to higher prices. Moreover, increasing s leading to less complementary services when $s<0$ and more intense competition when $s>0$ gives lower fares. The same happens when the firms put more weight on consumer surplus (γ increases). However, for all these unambiguous relationships the magnitudes depend on the values of γ, β and s. When the firms do not care about consumer surplus ($\gamma=0$), it follows from (11) that more weight placed on revenues (β increases) will reduce prices. In intermediate cases ($\gamma>0$) it is ambiguous whether more weight placed on revenues will increase prices with the present restrictions imposed on the parameters. It is easily seen from (11) that the lower the value of γ and the higher the values of s and c, the more likely it is that higher β leads to decreasing prices.

2.3.2 Simultaneous quantity competition (Cournot)

Under Cournot competition the transport firms maximise their goal functions by choosing quantities. This gives the following common equilibrium price (P^{C*}) and quantity (X^{C*}):

$$P^{C*} = \frac{1+\beta+(1+s)(c-\gamma)}{(2+s)(1+\beta)-\gamma(1+s)} \text{ and } X^{C*} = \frac{1+\beta-c}{(2+s)(1+\beta)-\gamma(1+s)} \quad (13)$$

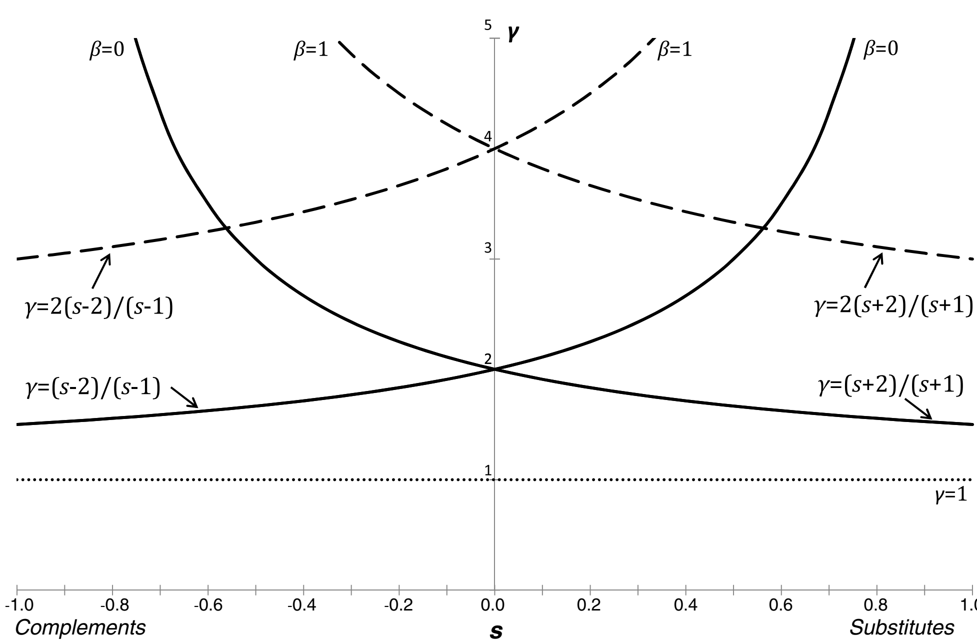

Fig. 1 Stability conditions for Cournot and Bertrand competition. The unbroken line indicates $\beta=0$ and the broken line indicates $\beta=1$. The dotted line indicates the parameter restriction $\gamma=1$

Also, for this competitive situation, the stability conditions in Table 1 and Fig. 1 ensure that both P^{C*} and X^{C*} are positive. The condition for non-negative profits ($\pi \geq 0$) when not receiving subsidies is now:

$$P^{C*} \geq c \rightarrow \gamma \leq \frac{(c-1) + \beta(2c + cs-1))}{(c-1)(s+1)} \tag{14}$$

When the firms do not place any weight on revenue ($\beta=0$), the condition under (14) implies that $\gamma \leq 1/(s+1)$, meaning that the firms will always make positive profits if they produce complementary services ($s<0$). The more intensely the firms compete ($s>0$ and increasing) the lower weight the firms can put on consumer surplus if they want positive profits. When $\beta>0$ and increases, the threshold weight the firms can put on consumer surplus resulting in positive profits increases (decreases) when $c<(>)1/(2+s)$.

The derivatives of the common equilibrium price with respect to c, s, β and γ are now:

$$\frac{\partial P^{C*}}{\partial c} = \frac{1+s}{(2+s)(1+\beta)-\gamma(1+s)} > 0 \tag{15}$$

$$\frac{\partial P^{C*}}{\partial s} = -(1+\beta)\frac{1+\beta-c}{(2+s+\beta(2+s)-\gamma(1+s))^2} < 0 \tag{16}$$

$$\frac{\partial P^{C*}}{\partial \beta} = (1+s)\frac{\gamma(1+s)-c(s+2)}{(2+s+\beta(2+s)-\gamma(1+s))^2} \geq (<) 0 \text{ when } \frac{\gamma}{c} \geq (<) \frac{2+s}{1+s} \tag{17}$$

$$\frac{\partial P^{C*}}{\partial \gamma} = -(1+s)^2\frac{1+\beta-c}{(2+s+\beta(2+s)-\gamma(1+s))^2} < 0 \tag{18}$$

Also under Cournot competition increasing s and more weight put on consumer surplus (γ) lead to lower prices whilst increasing costs (c) implies higher prices. Another similar result compared to Bertrand competition is that higher weight placed on revenues (β increases) leads to lower prices when firms are not concerned about consumer surplus ($\gamma=0$). Contrary to the Bertrand case, more intense competition between the firms (s increases) makes it less likely that prices decrease when the firms put more weight on revenue, given that $\gamma>0$.

2.3.3 Collusion

When the firms collude, they maximise the total goal function $G=G_1+G_2$. Equilibrium price (P^{COLL*}) and quantity (X^{COLL*}) are now:

$$P^{COLL*} = \frac{1+c+\beta-\gamma}{2(1+\beta)-\gamma} \text{ and } X^{COLL*} = \frac{1+\beta-c}{(1+s)(2+2\beta-\gamma)} \tag{19}$$

The bindings previously imposed on the parameters in (19) ensure that P^{COLL*} and X^{COLL*} are positive. The condition for non-negative profit when not receiving subsidies ($\pi \geq 0$) is now:

$$P^{COLL*} \geq c \rightarrow \gamma \leq \frac{(1-c) + \beta(1-2c))}{(1-c)} \tag{20}$$

When the firms do not place any weight on revenues ($\beta=0$), the condition under (20) is similar to the one under Bertrand competition; i.e. the firms must value profits equally or greater than consumer surplus ($\gamma \leq 1$) in order to obtain non-negative profits. Increasing β leads to tighter (looser) restrictions on γ when c is greater (lower) than 0.5.

The derivatives of the common equilibrium price with respect to c, s, β and γ are now:

$$\frac{\partial P^{COLL*}}{\partial c} = \frac{1}{2+2\beta-\gamma} > 0 \tag{21}$$

$$\frac{\partial P^{COLL*}}{\partial s} = 0 \tag{22}$$

$$\frac{\partial P^{COLL*}}{\partial \beta} = \frac{\gamma-2c}{(2+2\beta-\gamma)^2} \geq (<) 0 \text{ when } \gamma \geq (<) 2c \tag{23}$$

$$\frac{\partial P^{COLL*}}{\partial \gamma} = -\frac{1+\beta-c}{(2+2\beta-\gamma)^2} < 0 \tag{24}$$

From Eqs. (21) through (24) it follows that the collusive price increases in costs and decreases in the weight put on consumer surplus. The more weight the firms put on consumer surplus (γ increases) and the lower the costs (c decreases) the more likely it is that the collusive price increases when the firms put more weight on revenues. Note that the equilibrium price under collusion is independent of the degree of substitutability or complementarity between the services (the s value).

3 Comparisons of equilibrium prices

3.1 The collusion and the Bertrand cases

Using Eqs. (7) and (19) leads to the following difference, denoted by Δ (delta), arising between the collusive and the Bertrand cases:

$$\Delta = P^{COLL*} - P^{B*} = -\frac{s(1+\beta-c)}{2+2\beta-\gamma}\frac{1+\beta-\gamma}{s(1+\beta-\gamma)-2(1+\beta)} \tag{25}$$

With the bindings we have imposed on the parameters s, β and γ it can be verified that the first denominator in (25) is positive and the second is negative. Consequently, $\Delta \geq (<) 0$ when $s \geq (<) 0$. No matter how the firms value profit versus revenues and consumer surplus, the collusive price will always be higher (lower) than Bertrand prices when the firms produce substitutable (complementary) services. Differentiating Δ in (25) with respect to c, s, β and γ, we can deduce the following based on some mathematical computations:

$\frac{\partial \Delta}{\partial c} \geq (<) 0$ when $s \leq (>) 0$, $\frac{\partial \Delta}{\partial s} > 0$, $\frac{\partial \Delta}{\partial \beta} \geq (<) 0$ when $s \geq (<) 0$, $\frac{\partial \Delta}{\partial \gamma} < 0$ when $s > 0$ and also when $s < 0$ provided that $\gamma < q(\beta, s) = \frac{1+\beta}{1-s}\left(1 - s + \sqrt{1-s}\right)$, $\frac{\partial \Delta}{\partial \gamma} > 0$ otherwise

All above differentiations can be proved exactly, except the condition for the sign of $\partial \Delta / \partial \beta$ when $\gamma > 0$ which according to simulation is the same as when $\gamma = 0$. Increasing cost will always decrease the difference between the collusive price and the Bertrand prices irrespective of whether the firms produce complementary or substitutable services. Increasing s decreases (increases) the difference between collusive price and Bertrand prices when the firms produce complementary (substitute) services. It can be noted that for substitutes a larger value of s, in the meaning of more positive, will increase the positive difference in prices. Oppositely, for complementary services a larger value of s means less negative and must be understood as reducing the negative difference since the Bertrand price in this case is higher than the price in collusion.

The difference between the collusive price and Bertrand prices will increase the more weight they place on revenues (β increases). Provided that the firms produce substitute services ($s > 0$), the price rise when Bertrand competitors start to collude will be lower the more weight the firms place on consumer surplus. When the operators produce complementary services and at the same time place low emphasis on travellers' wellbeing and great weight on revenues such that $\gamma < q(\beta, s)$ an increase in γ can, however, increase the price fall when the rivals start to collude.

3.2 The collusion and the Cournot cases

The difference between the collusive price and the Cournot prices, denoted by θ (theta), is found by using Eqs. (13) and (19). This leads to:

$$\theta = P^{COLL*} - P^{C*} = s \frac{(1+\beta)}{2 + 2\beta - \gamma} \frac{1 + \beta - c}{(s+2)(1+\beta) - \gamma(1+s)} \tag{26}$$

Both denominators in (26) are always positive implying that $\theta \geq (<) 0$ when $s \geq (<) 0$. Irrespective of the firms' cost conditions and their valuation of profits versus revenues and consumer surplus, prices will increase (decrease) when the firms start to collude rather than compete and produce substitute (complementary) services. After some mathematical calculation we can derive the following using Eq. (26):

$\frac{\partial \theta}{\partial c} \geq (<) 0$ when $s \leq (>) 0$, $\frac{\partial \theta}{\partial s} > 0$, $\frac{\partial \theta}{\partial \beta} \geq (<) 0$ when $s \geq (<) 0$ and $\gamma = 0$, $\frac{\partial \theta}{\partial \beta} \gtreqless 0$ when $\gamma > 0$, $\frac{\partial \theta}{\partial \gamma} \geq (<) 0$ when $s \geq (<) 0$

All above differentiations can be proved exactly, except the condition for the sign of $\partial \theta / \partial \beta$ when $\gamma > 0$ which is found by simulation. The difference in equilibrium prices for collusion and Cournot will decrease with costs. Increasing s will reduce (increase) the price difference when the firms start to collude rather than compete and produce complementary (substitute) services. Simulation indicates that the sign of $\partial \theta / \partial \beta$ is ambiguous when $\gamma > 0$. Hence, we cannot conclude in which direction more weight put on revenues (β increases) will influence the difference between collusive and Cournot prices. For the special case when the firms disregard consumer surplus ($\gamma = 0$) an increase in β will always lead to larger price differences when they start to collude rather than compete. Similarly, the more weight the firms place on consumer surplus (γ increases), the greater is the price difference between collusion and Cournot equilibriums, both when they produce complementary and substitutable services.

3.3 The Bertrand and the Cournot cases

Using Eqs. (7) and (13) the difference between the Cournot prices and Bertrand prices, denoted by Ψ (psi), can be calculated as follows:

$$\Psi = P^{C*} - P^{B*} = s(c - 1 - \beta) \frac{s(1+\beta) - \gamma(1+s)}{s^2\left(1 + \beta^2 + \gamma^2 + 2\beta - 2\gamma - 2\beta\gamma\right) - 4\beta(\beta - \gamma + 2) - \gamma^2 + 4(\gamma - 1)} \tag{27}$$

A closer inspection of the denominator in (27) shows that it is negative. It thereby follows that $\Psi > 0$ when $s < 0$ and when $s > 0$ provided that $s(1+\beta) - \gamma(1+s) > 0 \Rightarrow \gamma < s(1+\beta)/(1+s)$, otherwise $\Psi < 0$. This means that Cournot prices are higher than Bertrand prices irrespective of the weight the firms put on revenues and consumer surplus when the firms produce complementary services ($s < 0$). The sign of Ψ is ambiguous when firms produce substitutable services ($s > 0$). The conditions for prices in Cournot being higher than in Bertrand when the firms place no weight on revenues ($\beta=0$) and when they weight revenues and profits equally ($\beta=1$) are illustrated in Fig. 2 by the lower envelopes of the unbroken and broken lines, respectively. The lower weight the firms put on revenues (decreasing β), the greater weight they put on consumer surplus (increasing γ) and the less fiercely they compete (decreasing s), the more likely it is that Cournot prices are lower than Bertrand prices. For example, if $s=0.5$ and $\beta=0.6$ then $\Psi \geq (<) 0$ when $\gamma \leq (>) 0.53$.

When differentiating Ψ in (27) with respect to c it follows that $Sign \frac{\partial \Psi}{\partial c} = sign(-\Psi)$, meaning that an increase in costs will always decrease the difference between the Cournot and the Bertrand prices. The conditions for the signs of the derivatives of Ψ with respect to s, β and γ are all, however, very complicated when $\beta, \gamma > 0$. The indicated signs below of these derivatives when $\beta, \gamma > 0$ are, therefore, based on simulations. This gives the following results:

$\frac{\partial \Psi}{\partial s} < 0$ when $s < 0$, $\frac{\partial \Psi}{\partial s} \gtrless 0$ when $s > 0$, $\frac{\partial \Psi}{\partial \beta} > 0$ when $\gamma=0$, $\frac{\partial \Psi}{\partial \beta} \gtrless 0$ when $\gamma > 0$, $\frac{\partial \Psi}{\partial \gamma} \geq (<) 0$ when $s \leq (>) 0$

When the firms produce complementary services, simulations indicate that the difference between Cournot and Bertrand prices is reduced when the degree of complementarity between the services decreases. The relationship is inconclusive when the firms produce substitutes. In the special case when the firms disregard consumer surplus ($\gamma=0$), increasing β leads to a greater difference in prices between Cournot and Bertrand, both when the firms produce complementary services ($s<0$) and substitutes ($s>0$). When $\gamma>0$, the influence on Ψ when β increases is ambiguous. Finally, the more weight the firms put on consumer surplus (γ increases) the greater is the difference between Cournot prices and Bertrand prices when the firms produce complementary services. When they produce substitutes it is, however, ambiguous how more weight placed on consumer surplus will influence Ψ.

3.4 Equal prices for different goal functions

Having the possibility of putting weight on several goals, it is interesting to study further how they interact with equilibrium prices. Hence, this section elaborates more thoroughly on the relationships between the weight put on consumer surplus (γ) and revenues (β) producing equal equilibrium prices for each of the three studied competitive situations. Having in mind that $P^{j*}=P^{j*}(\gamma(\beta),\beta)$, implicit differentiation assuming P^{j*} being constant gives the expression in (28).

$$\left(\frac{\partial \gamma}{\partial \beta}\right)_j = -\frac{\frac{\partial P^{j*}}{\partial \beta}}{\frac{\partial P^{j*}}{\partial \gamma}} \quad \text{where } j = \{B, C, COLL\} \qquad (28)$$

From earlier analyses we have found that $\partial P^{j*}/\partial \gamma < 0$. It then follows from (28) that $Sign\ (\partial \gamma/\partial \beta)_j = Sign\, \partial P^{j*}/\partial \beta$. If

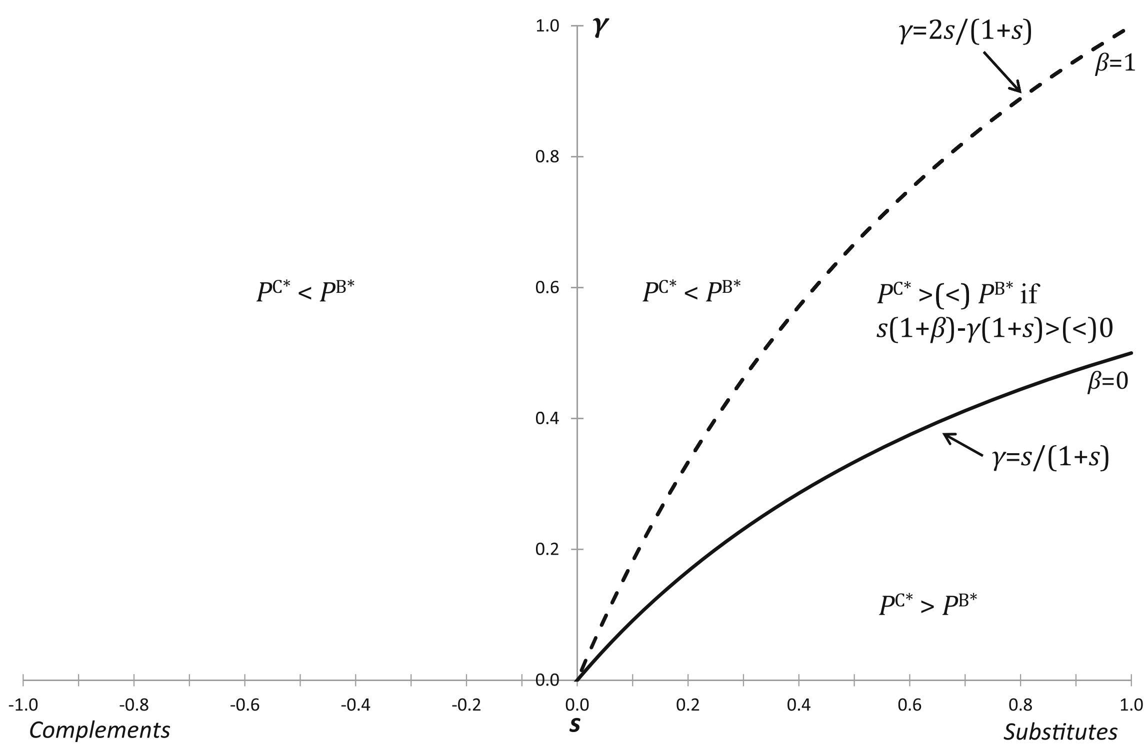

Fig. 2 Conditions for Cournot prices being higher than Bertrand prices for different levels of competition

equilibrium prices increase (decrease) when the firms put more weight on revenues, then the weight the firms put on consumer surplus must also increase (decrease) if the prices are to remain constant.

Inserting (11) and (12) in (28) gives for the Bertrand case:

$$\left(\frac{\partial\gamma}{\partial\beta}\right)_B = \frac{\gamma + cs - 2c - s\gamma}{(1-s)(1+\beta-c)} \geq (<) 0 \text{ when } \frac{\gamma}{c} \geq (<) \frac{2-s}{1-s}, \quad \left(\frac{\partial^2\gamma}{\partial\beta^2}\right)_B = 0 \tag{29}$$

Similarly, using Eqs. (17) and (18) in combination with (28) gives for the Cournot case:

$$\left(\frac{\partial\gamma}{\partial\beta}\right)_C = \frac{\gamma - 2c + s\gamma - cs}{(1+s)(1+\beta-c)} \geq (<) 0 \text{ when } \frac{\gamma}{c} \geq (<) \frac{2+s}{1+s}, \quad \left(\frac{\partial^2\gamma}{\partial\beta^2}\right)_C = 0 \tag{30}$$

Finally, we can deduce for the collusion case, using Eqs. (23), (24) and (28):

$$\left(\frac{\partial\gamma}{\partial\beta}\right)_{COLL} = \frac{\gamma - 2c}{1+\beta-c} \geq (<) 0 \text{ when } \gamma \geq (<) 2c, \quad \left(\frac{\partial^2\gamma}{\partial\beta^2}\right)_{COLL} = 0 \tag{31}$$

It follows from (29), (30) and (31) that the relationships between γ and β giving constant prices are linear for all competitive situations.The conditions for the signs of $(\partial\gamma/\partial\beta)_B$, $(\partial\gamma/\partial\beta)_C$ and $(\partial\gamma/\partial\beta)_{COLL}$ in (29), (30) and (31) are exactly the same as the conditions for the signs of $\partial P^{B*}/\partial\beta$, $\partial P^{C*}/\partial\beta$ and $\partial P^{COLL*}/\partial\beta$ in Eqs. (11), (17) and (23), respectively. The signs of $(\partial\gamma/\partial\beta)_j$ are independent of the weight the firms put on revenues (β), but depend on s, γ and c as illustrated in Fig. 3. The areas above the unbroken, broken and dotted curves indicate that the weight put on consumer surplus must be increased to maintain the same equilibrium price under Bertrand, Cournot and Collusion, respectively, when greater weight is placed on revenues.

Since $(2-s)/(1-s), (2+s)/(2-s) > 1$ it follows that $\gamma > c$ if $(\partial\gamma/\partial\beta)_j > 0$. For all competitive situations Fig. 3 also shows that it is more likely that γ will increase with β, the more weight the firms initially place on consumer surplus and the lower their costs (c) are. Moreover, it is more (less) likely that $(\partial\gamma/\partial\beta)_B > 0$ than $(\partial\gamma/\partial\beta)_C > 0$ when the firms produce complementary (substitutable) services. As expected, the degree of complementarity or substitutability between the services (the value of s) does not influence the relationship between γ and β when the firms collude.

Suppose, for example, that $\gamma = \beta = c = s = 0.5$ meaning that the firms produce substitutable services and initially put twice as much weight on profits than on revenues and consumer surplus. It then follows from (29), (30) and (31) above that $(\partial\gamma/\partial\beta)_B = -1.0$, $(\partial\gamma/\partial\beta)_C = -0.33$ and $(\partial\gamma/\partial\beta)_{COLL} = -0.5$. If equilibrium prices are to remain constant when the firms put more weight on revenues, the weight they put on consumer surplus must decrease most when they compete in prices and least when they compete in quantities.

3.5 Model results – Discussion

Regardless of the weight the firms put on profits, revenues and consumer surplus decreasing cost (c decreases) and greater weight put on consumer surplus (γ increases) will reduce equilibrium prices, P^*, under all types of competition between the operators. Moreover, when the degree of complementarity

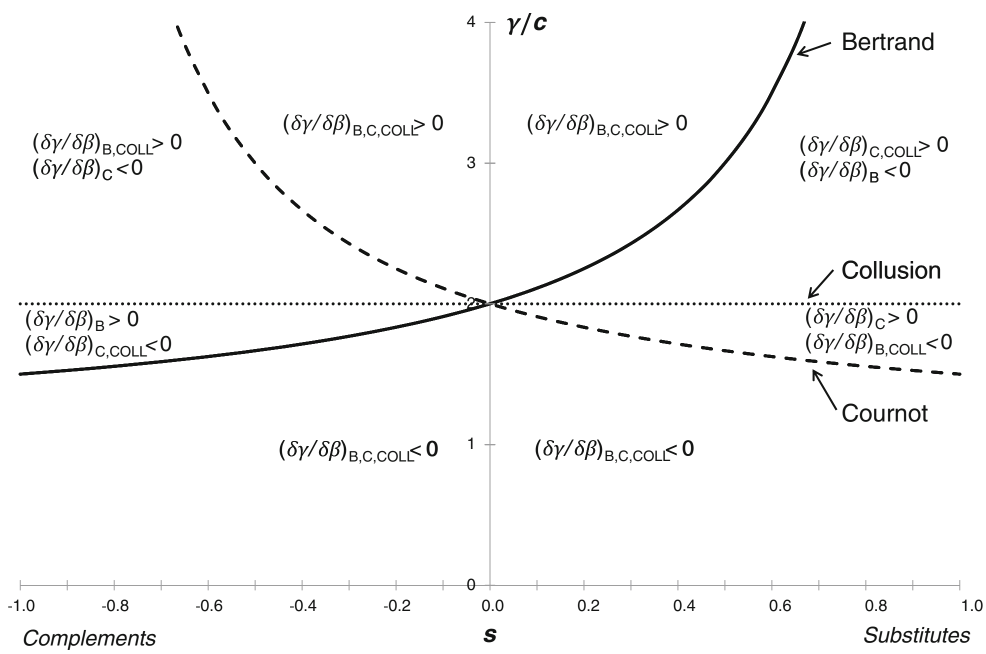

Fig. 3 Conditions for equal price when changing the weight put on consumer surplus and revenues

between the services decreases or the degree of substitutability between them increases (increasing absolute value of s), equilibrium prices under Bertrand and Cournot competition will decrease whilst the collusive price is independent of s.

The influence on prices of more weight put on revenue (increasing β) is, however, not so clear-cut. For all types of competition the sign of the derivative of price with respect to the weight put on revenue critically depends on the firms' costs (c), the weight they put on consumer surplus (γ) and how fiercely they compete (s). When the firms place no weight on consumer surplus ($\gamma=0$), an increase in β always leads to lower prices. In intermediate cases we can, in general, conclude that a necessary condition for increasing equilibrium prices with β is that $\gamma>c$. The more weight the firms place on consumer surplus (γ) and the lower their costs (c) are, the more likely it is that equilibrium prices increase with β. Lower degree of complementarity or higher degree of substitutability between the services (increasing s) makes it less (more) likely that prices increase with β under Bertrand (Cournot) competition. A closer inspection of the relevant derivatives also shows that it is less likely that an increase in β leads to higher equilibrium prices when the firms compete in prices than when they compete in quantities and produce substitutable (complementary) services. Finally, when the firms collude, more weight placed on revenues will increase prices when $\gamma>2c$.

When the firms produce substitutable (complementary) services, the collusive price is always higher (lower) than prices in the competitive situations. Moreover, when the firms produce complementary services ($s<0$), Cournot prices are higher than Bertrand prices irrespective of the weights the firms put on revenues and consumer surplus. Also when the firms produce substitutes ($s>0$) and are not concerned about consumer surplus ($\gamma=0$), the usual result arises that Bertrand prices are lowest regardless of the weight the firms place on revenues. However, when the firms place sufficiently great weight on consumer surplus (γ is large), rather low weight on revenues (β is low) and compete to a moderate degree (s is positive but low), it is demonstrated that Bertrand prices can reach a higher level than Cournot prices.

The magnitudes of the differences between all equilibrium prices will decrease when the firms' costs (c) increase. The difference between the collusive price on the one hand and Bertrand and Cournot prices on the other hand is reduced with a lower degree of complementarity and increased with a higher degree of substitutability between the services (s increases). Also, an increase in s makes the Cournot prices lower than Bertrand prices when the firms produce complementary services. It is, however, ambiguous how a higher degree of substitutability between the services will influence the difference between Bertrand and Cournot prices.

If firms are not concerned about consumer surplus ($\gamma=0$), more weight placed on revenues (β increases) will always increase the difference between all equilibrium prices. When the firms also place weight on consumer surplus ($\gamma>0$) increasing β reduces (increases) the difference between the prices in collusion and Cournot when the firms produce complements (substitutes). The influence on the other differences when the firms put more weight on revenues is ambiguous. The difference between the collusive price and Cournot prices will always increase when the firms put more weight on consumer surplus (γ increases). As far as the difference between the collusive price and Bertrand prices is concerned, it decreases when the firms put more weight on consumer surplus and produce substitutable services ($s>0$). When $s<0$, the influence of a greater γ is ambiguous. The difference between Cournot prices and Bertrand prices also decrease when the firms produce complementary services and become more concerned about consumer surplus. Otherwise; that is when $s>0$, the influence of higher γ depends on the magnitudes of s, β and γ.

The relationships between the weights the firms place on consumer surplus (γ) and revenues (β) giving the same equilibrium prices are linear for all competitive situations and increase (decrease) when prices increase (decrease) with β. A necessary but not sufficient condition for $\partial\gamma/\partial\beta>0$ is that $\gamma>c$. Hence the more weight the firms initially place on consumer surplus and the lower their costs are, the more likely it is that increasing β must result in a greater γ, providing that equilibrium prices remain constant.

4 Conclusions and implications

The paper addresses a transport market with two firms that have identical cost functions, produce symmetrically differentiable transport services and compete either simultaneously in prices (Bertrand), in quantities (Cournot) or collude. The degree of competition between the firms is indicated by a variable (s) measuring the degree of complementarity. The firms have equal goal functions that extend beyond profit maximisation; we assume they maximise a weighted sum of profits (π), revenues (PX) and consumer surplus (S). Such a goal function is relevant for firms operating in the passenger transport industry due to the presence of state ownership, local public stakeholders such as municipalities and counties, and the separate interests of owners and management. The analysis is based on a theoretical model from existing literature [4] and now extended by including revenues of the firm in the goal function.

Generally, most of the well known results arising in a market where firms maximize profit are also valid when considering that firms have extended, and possibly more realistic, goal functions. This information has value in itself; it demonstrates how weights put on different goals and the competitive situation influence the interrelationships between the variables. The findings presented in this paper provide regulators of the market for public passenger transport with an understanding of how companies act under different regulatory regimes - depending on their goals and costs. This knowledge can help regulators aiming to meet politically decided objectives through regulation.

Firstly, if regulators are purely concerned about travellers' welfare, they should encourage the use of publicly owned transport companies, as they normally weigh consumer surplus more greatly (higher γ) than privately owned firms. This would give lower prices and will benefit travellers, but it is uncertain to which degree it benefits the society. It depends on the cost of rising public funds (see footnote 4). Also greater weight placed on revenues (increasing β) will probably lead to lower prices when the firms place low weight on consumer surplus (γ is low). Hence, the travellers' will benefit from powerful management with goals extending beyond pure profit maximization ($\beta>0$) in privately owned firms when γ is low.

Secondly, the decision-makers should endorse (oppose) that the firms start to collude if they produce complementary (substitutable) services. The difference between the collusive price on the one hand and Bertrand and Cournot prices on the other hand is reduced with a lower degree of complementarity and increased with a higher degree of substitutability between the services. In intermediate cases, i.e. when the firms produce complementary services for some customers and substitutable services for others, the regulators' view regarding collusion should depend on the weight they put on welfare provided for the different groups of customers. Economies (diseconomies) of scale also point in the direction that collusion becomes more (less) favourable.

Thirdly, since decreasing costs (c) make greater difference between the collusive price on the one hand and Cournot and Bertrand prices on the other hand, decision-makers should be more concerned about the firms' competitive situation when they become more productive. These effects are strengthened the greater the degree of substitutability or complementarity between the services.

Finally, it is discussed how the equilibrium price depends on the goal functions in the collusive case. How the weight the firms put on profits, revenues and consumer surplus influences the magnitude of price changes is not so clear-cut. One unambiguous result is, however, that when the firms are purely concerned about profits and revenues ($\gamma=0$), greater weight put on revenues leads to more significant price changes when the firms start to collude and when they start to compete in quantities rather than prices and vice-versa. This points in the direction that regulators should give special attention to the competitive situation for privately owned firms (where $\gamma=0$) with rather powerful management (high β).

From a travellers' point of view the regulators should in general stimulate the firms to compete with regard to price rather than quantity since Bertrand prices are lower than Cournot prices in most cases. Only when the firms produce substitutable services to a moderate degree, place great weight on consumer surplus and low weight on revenues, do we get the unusual result that the travellers may be better off when the firms compete in quantity rather than in prices. Since the difference between prices in Cournot and Bertrand increases the more productive the firms are, the regulators should pay greater attention to how the firms compete when they think the firms' productivity will increase. Also, a higher degree of complementarity between the firms' services should lead to the regulators being more concerned about whether the firms compete in quantity or prices.

Finally, it should be emphasised that more powerful management leading to greater weight being placed on revenue may result in higher prices in transport firms with substantial public ownership. A further development of this model could incorporate how asymmetry in costs and goal functions would influence the properties of equilibrium prices.

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