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The Explanatory Power of Traditional Multi-Factor Models in the Norwegian Mutual Fund Market

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Abstract

The purpose of the thesis is to investigate how well traditional Multi-Factor models explain the returns of a large selection of Norwegian mutual funds. We assess how much of the variance of the returns can be captured by using the aforementioned factor models, and in turn evaluate how the different funds are exposed to the individual risk factors. The main contribution of this thesis is to provide computations on the subject matter in an as recent time period as possible, in order to add to and reinforce the somewhat scarce pre-existing literature on the topic. We generate evidence as to how very few mutual funds generate statistically significant excess returns when adjusted for the included systematic risk factors. Furthermore, we provide evidence that there is a positive relationship between management fees and excess returns in Norwegian mutual funds.

Sammendrag

I studien vår gjennomfører vi vurderinger om hvorvidt tradisjonelle multifaktor modeller fortsatt er godt egnet til å forklare avkastningen et utvalg norske aksjefond genererer sammenliknet med benchmark indeksen OBX. Utvalget består av 34 norske aksjefond og innhentet data utspiller seg over en sammenhengende periode på 10 år fra og med 2010 til 2020. I databehandlingen benytter vi oss av regresjons verktøy som OLS for å bygge regresjonsmodeller som deretter lar oss evaluere hvorvidt hvert av fondene er eksponert mot de ulike risikofaktorene. Regresjonsmodellene vi implementerer er i hovedsak femfaktor-modellen til Fama & French(2015), men også deres trefaktor-modell(1993) samt Treynor's CAPM(1961). De inkluderte risikofaktorene består av markeds beta, Fama & French(1993) sin SMB samt HML faktor, og til sist Næs' (2009) iterasjon av momentum-faktoren og Næs' (2009) likviditets-faktor. Avslutningsvis deler vi opp fondene i tre ulike kategorier basert på forvaltningskostnad for å se om vi finner bevis på at økte forvaltningskostnader leder til økt meravkastning.

Vi finner bevis for at enkelte fond oppnår signifikant meravkastning etter justering for de fem faktorene. Dette forklares i hovedsak gjennom økt eksponering mot størrelsesfaktoren, samt negativ eksponering mot likvididetsfaktoren. Videre finner vi at de resterende faktorene momentum og verdi er mindre utslagsgivende. Totalt sett viser det seg at store deler av fondenes avkastning kan forklares gjennom eksponering mot de fem risikofaktorene. Videre i utredningen analyserer vi hvorvidt ulik forvaltningskostnad viser en tydelig forskjell i eksponering til de ulike faktorene, samt hvordan den totale forklaringsgraden til de ulike modellene endrer seg etterhvert som prisen øker og om det oppstår signifikant meravkastning, i et forsøk på å rettferdiggjøre høyere forvaltningskostnad i norske fond.

Funnene i denne studien indikerer at tradisjonelle multifaktor modeller og deres tilhørende faktor-premier er godt egnet til å forklare avkastningen for norske aksjefond. Videre finner vi at forvaltningskostnad spiller en stor rolle for modellenes forklaringskraft og estimerte meravkastning. Vi observerer at fond med høy forvaltningskostnad er vesentlig mindre indeksnære og genererer statistisk signifikant meravkastning sammenlignet med lavere prisede norske fond.

Preface

This thesis is written as a part of the master programme in finance at Nord Universitet. This independent piece of research constitutes 30 credits and has been constructed within the duration of one semester.

Our choice of topic was selected based on the shared interest we have for factor models and the concept of decomposing asset returns. Throughout our education, the theory of factor models has been elaborately discussed, and exploring their effectiveness in the Norwegian economic environment, in an as recent time frame as possible, appealed strongly to us.

Constructing this elaborate piece of research has proven to be immensely interesting as well as strongly challenging. It has given us the possibility to put theory to practice and provided us with the opportunity to utilize established theory in an exploratory fashion.

We would like to thank our Finance professor, Thomas Leirvik, for facilitating our education through thorough lectures, as well as arranging a Master seminar and several opportunities for students to present the progress of their master thesis. This allowed us to receive feedback from our peers, and made us aware of additional aspects of our thesis that could otherwise have gone unnoticed.

Lastly we would like to show our utmost gratitude to our supervisor, Frode Sættem, for his invaluable insight and guidance which has helped us stay on track during the span of our work on this thesis.

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1 Introduction

Throughout the years investors have employed the use of a broad range of models that take into account different kinds of factors in order to quantify and predict asset returns. While the factors used in these models have allowed famous investors such as Warren Buffet to achieve great success in the stock market, some of these appear to have reduced their effectiveness over time.

In modern history, many attempts have been made to develop forecasting-strategies and -models in order to capture future returns. One fundamental concept in theory that is often used as an example in terms of future return predictions is the CAPM-model. In retrospect, it is a relatively simple model that can be utilized when tasked with conducting a rough estimate of asset returns. The capital asset pricing model only considers an asset's sensitivity to the market beta-value. However as research matures and new research is conducted, more complex and sophisticated models emerge in an attempt to explain and forecast an even larger portion of asset returns with higher accuracy. Several elaborate models have been developed as extensions to the CAPM, such as the Fama & French 3-factor model, which has proven itself as a cornerstone in modern portfolio selection. The theoretical groundwork provided by the creators of the models has paved the way for countless new iterations of Multi-Factor models.

The aforementioned CAPM features a formula with a very simplistic expression that takes the systematic risk in its entirety and combines it into a single risk term referred to as Beta. This simplifies the calculations quite a bit, however it also leads to large parts of the variance related to the calculation being left unexplained. This is the point where the Multi-Factor models serve their purpose. The introduction of the Multi-Factor models aim to explain the individual components of the systematic risk related to computing the expected returns.

In this study we will be conducting regression analysis with the CAPM, Three-factor- and Five-factor models as basis, in order to assess Norwegian mutual fund returns. We will analyze how the explanatory power of the models change as we increase the model complexity, and its apparent relationship with fund management fees.

1.1 Problem Statement

To analyse the relevance and explanatory power of traditional Multi-Factor models while also examining the added value provided by fund managers, we came up with the following problem statement for this research:

"Are traditional factor models able to explain the returns in the Norwegian mutual fund market?"

In order to answer the problem statement, we will be using our knowledge in the subject of statistics & machine learning to properly conduct supervised training and testing of the datasets with the help of R software. By doing this we will be able to determine whether or not the preexisting factors explain a sufficient portion of future returns.

With basis in our problem statement we would also like to introduce our two resulting hypotheses, namely:

Hypothesis 1 "Traditional factors explain a large portion of the returns of Norwegian mutual funds"

Hypothesis 2 "Higher management fees are justified by higher excess returns and less explainability by traditional factors."

2 Theory

2.1 Mutual Funds

A mutual fund is defined as a financial vehicle that is derived from a pool of monetary value, consisting of deposits from several individuals which is then used to invest in securities such as stocks, bonds, money market instruments and other assets. Mutual funds are governed by professional money managers responsible for allocating the aforementioned assets, and generate returns on behalf of their customers. Fund managers are responsible for articulating the funds' investment objective in a formal document called the prospectus, which is a tool that customers utilize to orient themselves as to what investments and strategies the fund is concerned with. The fund managers in turn have an obligation to their customers to adhere to the prospectus, in order to maintain transparency and predictability in terms of strategy and risk. The immediate upside to mutual funds is that they enable individual investors to access professionally managed portfolios without having to build comprehensive competence and experience with financial instruments. Therefore each individual shareholder partakes proportionally towards the gains or potential losses of the fund. Mutual funds are typically invested in several different securities, thus the performance of a fund is expressed as the change in total market cap of the aggregated constituents adjusted for their part or weight in the total composition of the fund. Another notable upside related to investing through mutual funds is the investor's opportunity to conduct thorough diversification without executing numerous trades and aggregating transaction costs for said trades. Furthermore, the majority of mutual funds are readily accessible and require relatively low capital in order to become a shareholder. This enables individual investors to conduct elaborate and diversified investments that generate competitive returns, with little to no time and effort consumed.

2.2 Factor models

2.2.1 Capital Asset Pricing Model

The Capital Asset Pricing Model(CAPM) was constructed over time during the 1960s by the combined efforts of the known economists William Sharpe, Jack Treynor, John Lintner and Jan Mossin. (Perold, 2004)
The CAPM concept in itself was introduced in 1962 by american economist and board member of the Journal of Investment Management Jack Treynor. This economic model would go on to become one of the most cited concepts in finance. Treynors capital asset pricing model(CAPM) is a model that is used to calculate and determine a hypothetically appropriate required rate of return for a specific asset. With the introduction of the CAPM Treynor became the first individual to highlight and explain the interplay between expected returns of an asset and its covariance with regards to a market-portfolio. The development of the capital asset pricing model can be regarded as an extension of the fundamental theories provided by Markowitz (1952) and Tobin (1958) in which the diversification and modern portfolio theory is discussed. Treynor regarded his model as a sophisticated tool which would be used to make informed decisions about whether or not an investment manager should add a certain asset to a well-diversified portfolio.

As with most theories with quantifiable elements, the CAPM has a particular set of assumptions upon which the model is based:

1. All investors have the opportunity of borrowing money at the risk-free rate.
2. All investors are rational, risk-averse and thus share the same expectations.
3. Investing is a near cost-less process without considerable transaction costs, taxing or partial stock acquisitions.
4. All investors distinguish between investment alternatives through examining returns and variance.

In accordance with the model, an investor will be holding the market-portfolio, which entails that the only relevant type of risk the investor assumes is that which cannot be removed through diversification. Thus the risk is given through the covariance of each asset and its related market index. The capital asset pricing model describes the relationship between expected returns and the relevant risk. Considering this, the CAPM utilizes several specific elements, one of which is the assets sensitivity to non-diversifiable risk i.e. systematic risk, referred to as beta . Furthermore the model is concerned with the expected return of the market and the theoretical risk-free rate to be expected for a particular asset. Treynor formulated the following equation for the model:

$$ER_i = R_f + \beta_i(ER_m - R_f)$$

Explanation:

ER_i is the expected return of an asset.

R_f is the risk-free interest rate.

β_i is the sensitivity for expected return of an asset against the expected return of the market.

ER_m is the expected return in the market.

The model aims to explain that the expected return of an asset is dependent on the exposure to systematic risk factors. Thus the model explains that the expected return is equal to the risk-free rate added with the market premium of the return multiplied with the corresponding beta-value. It is implied that the only way for an investor to attain higher returns, is to take on a higher level of risk. In other words the model can be utilized as a way to describe the risk-premium$(ER_m - R_f)$ that an investor would require in order to invest in an asset with a higher corresponding risk.
As stated in the assumptions, the CAPM functions in a scenario where risk-averse investors are operating and/or where investors borrow funds at the risk-free rate. In theory, this enables the possibility for investors to secure parts of their portfolio at

a given return level, thus creating a minimum required rate of return for alternative investments. The alternative investments have to create additional returns if they are to be considered attractive investment alternatives, and to justify a higher corresponding risk. Therefore the risk-level of a particular asset is captured through the beta-value β, where a higher value is representative of higher returns, yet at the cost of the investor having to take on more risk relative to the market.

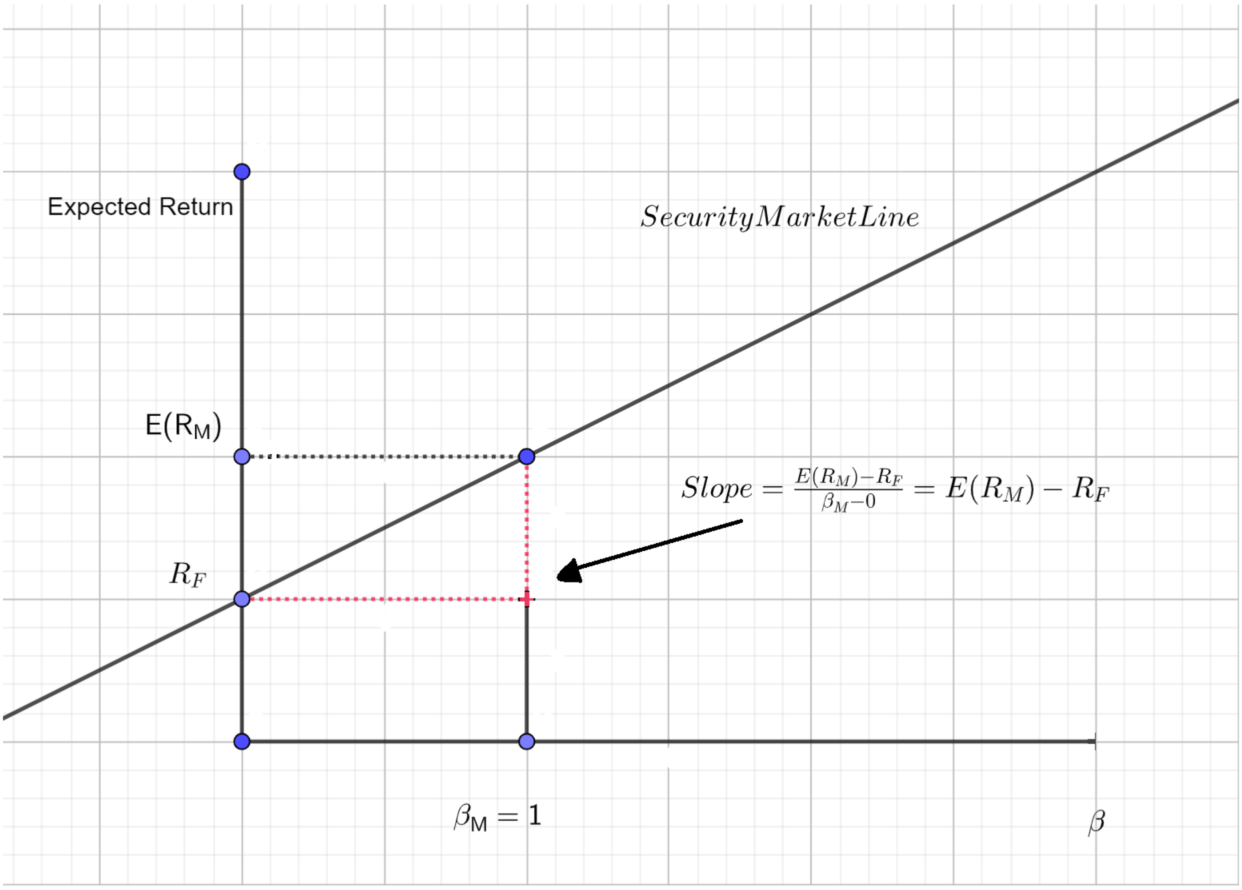

Figure 1: SML CAPM

The capital asset pricing model features a linear relationship between expected return and measure risk, that can be illustrated in a proportional function such as the one seen above.

The line observed is oftentimes referred to as the security market line(SML), and represents the expected return for any given value of risk β. In relation to the aforementioned assumptions of the CAPM, one can observe that the expected return for any asset has a linear relationship to the covariance of the asset with regards to that of the market. In the figure above, it is shown that a beta-value of 1 indicates an expected return that is in line with the expected return of the market. If an asset were to deviate from the SML, the result would be the emergence of an arbitrage opportunity.

As with all models, the CAPM is a simplification of reality and operates within the confines of its own fundamental assumptions. Naturally, some challenges arise when adopting the model and putting it to use in reality. For example, the underlying assumption which states that there exist risk-free assets is a gross simplification of reality, where the closest thing there is to risk-free assets are short-term government bonds. These bonds have an insignificant probability of being defaulted, however the inflation rate creates a problem when regarding them as risk-free rates. Another assumption that creates a problem in reality for utilizing the CAPM, is the fact that the measured beta-values for an asset changes over time, which makes historical beta-values insufficient as means for deciding the future risk of an asset.

2.2.2 Fama & French 3-factor model

In 1992 an extension of the capital asset pricing model was developed by Eugene Fama and his colleague Kenneth French. They set out with the motivation to solve the problem as to why the CAPM didn't hold up in a variety of scenarios for different assets. Furthermore the authors proved through rigorous research that the average returns of the assets in the New York Stock Exchange and NASDAQ did not correlate closely with the market-beta values of the included assets. Instead the authors found that the average returns correlated more closely with the size of corporations and also the value of the corporation. As a response to their new findings, the authors proceeded to articulate a more comprehensive and elaborate model for estimating the returns of an asset.

Today we refer to their model as the Fama French 3-factor model, and it has proven itself to be the foundation of what we regard as modern factor-investing. The model expands upon the CAPM by adding corporation size risk and also value risk factors to the already present market risk factor as seen in the capital asset pricing model. This model incorporates the observation that stocks that are considered as value-stocks and/or small-cap stocks, generally outperform the corresponding market relatively consistently. Thus by including the two factors size-risk and value-risk, the Fama & French 3-factor model adjusts calculations for the observed outperforming over the market. In theory this should make the model a more appropriate and accurate tool for evaluating investment performance. The authors articulated the following model for calculating the returns of an asset:

$$ER_i = R_f + \beta_1(R_m - R_f) + \beta_2 SMB + \beta_3 HML + \epsilon$$

Explanation:

- ER_i = Expected rate of return for an asset
- R_f = Risk-free rate
- β_n = Sensitivity to the corresponding factor
- $(R_m - R_f)$ = Market risk premium
- SMB = Excess returns of small companies over large ones based on historical data.
- HML = Excess returns of high-value companies over low-value ones based on historical data.

The influence on expected returns that the small minus low and high minus low factors introduced by Fama & French brings are not immediately evident. However, the authors argued that the two factors they included in their model, also incorporates the effects of more fundamental variables. Thus capturing a broader variety of contributing factors than one might think when initially reviewing the model. For example, Fama & French argued for their choice of the SMB factor included in their model, by stating that smaller companies are more prone to finding themselves in economic uncertainty and/or troubles.

In theory, the SMB-factor also captures the sensitivity related to risk factors in the macroeconomic environment, and is in theory able to incorporate the sensitivity towards business cycles/economic cycles and the fluctuations in gross-domestic product growth trends. In accordance with this way of thinking, a portion of the size and value premiums would explain the increased returns on the basis of assuming riskier investments.

In their research, Fama & French determined that approximately 71% of the excess returns were explainable through calculations using an assets beta-value towards the market-factor, i.e the returns explainable by deploying the capital asset pricing model. Thus by incorporating the two aforementioned factors of their own design, they argued that their model could explain an astonishing 95% of the excess returns.

In the wake of Fama & Frenchs work, several prominent individuals have theorized on and expanded upon their renowned three-factor model. Arguably the most notable addition to the three-factor model initially emerged as a result of the work done by N. Jegadeesh and S. Titman, who in 1993 created and introduced the momentum-factor. The momentum-factor emerged as an attempt to capture the observed effect where assets that perform well in a specific period, are statistically more likely to also perform well in the following period. However it wasn't until Mark Carhart explicitly added the momentum-factor to the Fama & French three-factor model, that it was deployed in conjunction with already existent factors. The result of Carharts work was a model that in theory was able to capture and explain even more of the excess returns of an asset, than the three-factor model.

Another extension of the Fama & French three-factor model that gained a lot of renown, was the model created and introduced in 2003 by Lubos Pastor and Robert Stambaugh. The authors added a liquidity-factor to the model, predicated on the assumption that private and institutional investors demand higher returns for investments in assets that are less liquid than other comparable assets. Pastor and Stambaugh stated that liquidity is the option or ability to sell a large portion of an asset relatively quickly, without this influencing the price.

The affinity of liquidity on excess asset returns was reinstated following the discoveries made by Bodie et al. (2018), where the authors proved that the average returns on assets with a high sensitivity to liquidity were notably higher, than for assets with low sensitivity to liquidity. In conclusion the authors found that assets with a high sensitivity to the liquidity factor performed approximately 7% better in terms of average returns, than the assets with low sensitivity. Thus the authors argued that by including their liquidity-factor in a Multi-Factor model it would yield an even higher accuracy in terms of calculating expected returns of an asset.

2.2.3 5-factor models

After the emergence of the Fama & French 3-factor model, several takes and additions to the model have been made. Most noteworthy of which is the contribution done by Jagadeesh and Titman, who in 1993 discussed the tendency that well performing stocks have to continue performing well in the short term. Likewise, the authors found that underperforming stocks were likely to continue to underperform in the short term. This effect was described by the authors as the momentum-effect, from which they derived a factorial addition to the Multi-Factor model aimed to contribute to the explanatory power that Multi-Factor models have for returns in the stock market. This factor however, has faced its fair share of controversy and there has emerged several conflicting articles with contrary observations of the momentum effect.

Carhart (1997) contributed to the academic literature on the momentum-factor through stating that the findings of Jagadeesh are true, but not as a result of an active strategy by equity managers, but as an incidental tendency that managers have of holding a larger part of last years "stock winners" rather than losers. Carhart continues by describing how a conscious momentum-strategy will end up losing returns after one adjusts for the costs-related to frequent trading from rebalancing the portfolio for emergent stock-winners. The author proceeds to argue that the momentum-factor has great relevance when utilized as an addition in conjunction with other factors, thus Carhart added the momentum-factor to the 3-factor model in order to explain a larger part of the variation in returns observed in the traditional model.

The Norwegian author Døskeland (2014) argued that the momentum-factors uniqueness and relevance is large due to it not being a typical risk-premium like the other factors, but instead the momentum-factor explains variations caused by systematic irrational behaviour in the stock market. Thus reinforcing the descriptions made by Carhart of the momentum-factor.

The momentum-factor utilized in this piece of research is constructed as Carhart described it in his 1997 article. Firstly the stocks in the representative selection are sorted into three separate portfolios, based on the last twelve months returns.

Secondly you simulate entering long-positions for the best performing portfolio, and you enter short-positions for the worst performing portfolio, then this new long-short portfolio is held onto the next month, at which point the process is performed anew. The measured factor-returns are therefore equal to the difference between the best and the worst performing portfolio (Næs & Ødegaard 2008). The factor-returns utilized for the momentum factor in this piece of research is based on the continuation of Ødegaard et al's work from 2008, in the form of free available factor data-sets.

Another notable addition to the five-factor model is the liquidity factor. The factor was first introduced by Pastor and Stambaugh in 2003, where they proved how the risk of holding illiquid stocks is a priced systematic risk factor. They analyzed the period 1966-1999 and concluded that the average returns on stocks that had a high sensitivity to liquidity outperformed stocks with considerably lower sensitivity to liquidity by approximately seven percent per year. Pastor and Staumbaugh defined liquidity as the ability to sell considerable amounts of shares rapidly at a low cost and without influencing the assets price considerably.

Ibbotson et al. (2013) reinforced the findings of Pastor and Stambaugh and argued that by holding stocks that are illiquid, the investor or fund manager is taking on risk in the sense that one can be stuck with large positions and thus reducing one's own flexibility in the market. Ibbotson et al. constructed several portfolios with long-positions consisting of both small-, value- and momentum-stocks with considerably low liquidity and concluded that these portfolios achieves a greater alpha-value observed in both the CAPM, three- and four-factor models. The authors proceeded to refute and disprove the common misconception that small stocks are illiquid and thus already adjusted for by the size factor. They provided evidence in their article that this is not the case and describes how the liquidity factor remains relevant when deployed in conjunction with other traditional factors.

How to articulate a liquidity factor remains a matter of definition, and Ibbotson et al. defined their liquidity-factor as asset-turnover. The alternatives to that particular way of defining liquidity would be to view liquidity as a dimension of cost, time or quantity which is the case in the article by Næs et al.

For this thesis we utilize the quantity definition of liquidity, based on Næs et al's conclusions as to how this particular definition holds the highest degree of relevance when deployed in the Norwegian economic environment. Thus the liquidity factor is computed by dividing the stock selection into three portfolios based on low, medium and high cost spread. Then you proceed to assume long-positions in the high spread portfolio, and assume short-positions in the low spread.

2.3 Factor Selection

When going through the process of factor selection, it is important to maintain a good balance between implementing too many factors and too few factors. The reasoning behind the importance of this balance is that one is effectively lowering the regressors efficiency by using an excessive amount of factors. Likewise one would be prone to miss the corresponding correlation relationship between the asset returns by utilizing too few factors in the model. In order to combat the problem of how many factors to include in any given model we can use the two solutions suggested by Serges Darolles et al.(2013) Their first suggested solution is to make use of a predefined set of observable factors that are already documented for their ability to explain the cross-section of asset returns. These predefined factors are then used to produce both long-term and short-term benchmarks with the help of style analysis through the following formula:

$$r_{p,t} = i = \sum_{i=1}^{N} a_i r_{i,t} = a' r_t,$$

Where $r_t = [r_{1,t} ... r_{N,t}]' and a = [a_1 ... a_N]'$ is the vector of fixed asset weights in the portfolio. This style analysis provides us with a distribution of $r_{p,t}$ which in turn can be interpreted as the included factors' explanation of the specified portfolios performance and thus we have a benchmark. Furthermore, these factors can also be used to reduce the size of the related covariation matrix in the instance it would be required to compute optimal portfolio allocation for a larger set of risky assets. If some risk factors were to be overlooked during the style analysis, both of the benchmarks would be unable to properly specify the portfolio performance and the corresponding added value of the portfolio manager wouldn't be properly calculated. The second solution

as stated by Serges Darolles et al.(2013) is to make use of statistical approaches to help filter the unobservable factors able to explain the asset returns' cross-section from the asset returns distribution itself. This method however has a few drawbacks that are also mentioned by Serge Darolles et al.(2013) The statistical factors discovered by the use of this method can be difficult to interpret in an economic setting, as well as this type of factor representation not being unique. The lack of uniqueness means that *"Any linear combination of a given set of factors defines an equivalent factor model"* ~ Serge Darolles et al.(2013)

2.4 Active vs. Passive investing

Active and Passive investing are the two main investment strategies when it comes to generating returns on a portfolio. Active portfolio management does conduct frequent trades as the name implies. Examples of such portfolio management could be that a portfolio has set weights for each asset included in the portfolio, and the fund manager will then actively redistribute the value of the portfolio in order to maintain these weightings over time.

In contrast we have passive portfolio management, which as the name implies, includes far fewer trades than its counterpart. An example of a passive form of portfolio management can be made through the act of a buy & hold strategy. This strategy mainly involves transactions at the beginning and at the end of the portfolio's lifetime, and contrary to the active management it conducts no redistributions to maintain the portfolio weights.

When choosing to implement an active management strategy an investor believes the market to not be entirely efficient, and is therefore trying to outperform the market benchmark in order to achieve excess returns. These excess returns are achieved through conducting analyses with the goal of determining if stocks are either correctly-, under-, or over-priced, thus allowing the investor to profit on the mispricing that exists in the market.

On the other hand if an investor chooses to implement a passive management strategy they are of the belief that the market is efficient, and are thus attempting to mimic the market index with their portfolio. By doing this they achieve the same risk and returns as the market without the need of continuous analysis as is needed with its active counterpart.

2.5 Factor investing

In terms of asset returns there are three major types of returns that we often divide into separate categories based on how they are estimated. Firstly we have returns that are estimated by the asset's exposure to market risk, which are commonly estimated with the help of models such as the CAPM model introduced earlier. Secondly we have returns that are estimated by the asset's exposure to predetermined known factors, this is commonly referred to as the Multi-Factor models and include the famous Fama & French models. Lastly we have returns that are estimated by alpha, or in other terms how well an investors strategy produces excess returns in comparison to the market index, which can not easily be attributed to exposure to systematic risk factors.

While all of these promise hypothetical returns, one has to wonder to what degree these returns are actually attainable in a real life setting. Huij and van Gelderen (2014) took a look at the long-term performance of mutual funds with significant exposure to established factor premiums in order to find an answer to this question. When they did this, they found strong evidence of the mutual funds' ability to capitalize on factor premiums with a probability amounting to 66% for funds that seek to benefit from the value premium. Meanwhile, funds that lacked positive correlation with one or more of these factors only had a probability of about 20%.

The large difference in probability between funds actively seeking to benefit from value premiums and the funds lacking positive correlation strongly suggests that factor investing indeed works, and could be employed in the process of fund management.

To further elaborate on this type of investing we will in the following subsections go further into the different factors previously introduced, in order to properly flesh out their respective definitions.

2.5.1 Size

This factor is commonly referred to as the firm size premium, and was first coined by Banz, 1981. This premium related to smaller companies has been one of the better known anomalies of the academic market. The premium itself was discovered in Banz's study and showed how smaller firms outperformed the stocks of larger firms due to the difference in size of their market capitalization. In 2014 Kalesnik & Beck fronted their skepticism in regards to whether or not investors would be able to earn higher returns simply by preferring small stocks rather than large stocks. However, a paper written by Cho (2019) documents how the size effect still exists, but how it's only visible while the economy is in periods of high volatility. In summary this implies that should this factor be used when constructing a modern Multi-Factor model, it would have to be done with caution as there exists evidence both for and against it.

2.5.2 Momentum

When talking about momentum we are referring to something with a direction and a given velocity in said direction. In financial terms this refers to the direction of a specific stock's share price. Share price often has a tendency to maintain its direction, something that can be used in so called momentum-strategies in order to make predictions of future returns based on historical data. Hong et al. (2000) attempted to demonstrate the profitability related to strategies such as these based upon the gradual-information-diffusion model of Hong and Stein (1999).

When testing this model they were able to establish three key results. Firstly they discovered that once they passed the smallest stocks the related profitability of the momentum strategies faced a sharp decline relative to increasing firm size. Furthermore, they discovered that if they maintained a fixed holding size their strategies were better at achieving profitability when applied to stocks with low analyst coverage. Lastly, they saw an increased effect of analyst coverage for stocks that had previously been losers, than for stocks previously being winners. Based on these findings they landed upon the conclusion that bad news indeed travels slowly, thus only diffusing gradually across the investing public. Because news in general only gradually diffuses across those investing it will take time for the market as a whole to react to said news, resulting in the momentum of the stock continuing its course for a while longer than it should.

2.5.3 Volatility

Volatility measures how the price of an asset varies over time, and thus it tells an investor how risky it would be to invest in said asset. When investing in assets with higher risk most investors demand compensation in terms of a risk-premium, which is somewhat of a reward for their willingness to take the extra risk. However, if we look at the study conducted by Jordan Riley (2015) they showcase their findings that suggest otherwise. As a result of their study on the connection between highly volatile assets and a corresponding high risk-adjusted return, they found that higher volatility doesn't necessarily lead to the highest risk-adjusted returns. On the contrary, their study indicates that funds with historically low volatility are able to generate a positive alpha within 1-2% per year, while the highly volatile funds had a negative alpha of about 3% in the same time frame. Seeing as the historical volatility of the funds examined by Jordan & Riley (2015) were able to produce such results, it would be reasonable to make use of Volatility in future estimations to predict future expected returns based on historical data.

2.5.4 Value

Using value as a factor to predict expected future returns is primarily done by estimating a company's actual value and determining whether or not the corresponding stocks are correctly valued, overpriced or underpriced. In order to take advantage of this info an investor would buy any stocks that were undervalued in comparison to the corresponding company's actual estimated value. By doing this the investor would effectively be able to achieve excess future returns as the value of the stock appreciates and catches up to the estimated company value.

As Daniel & Titmaan (1996) suggests, the reason behind the achievable excess return is related to the market efficiency not being perfect. This leads to the markets taking time to realize the higher than expected EPS-growth thus allowing investors to acquire the stocks in an undervalued state. This effectively makes the value evaluation of stocks suitable for producing a factor which can be used in Multi-Factor models to help predict expected excess returns.

2.5.5 Liquidity

When talking about liquidity one is often referring to how easily or quickly an asset can be bought or sold for a price that reflects its intrinsic value. Assets with a high liquidity are naturally efficient means of conducting trade, and it is the major reason why we've implemented cash as the main means of payment in society. For instance if you were to purchase a brand new TV you wouldn't't be able to pay for it with apples, signifying that apples have a lower liquidity than that of cash, and would generally be an unaccepted form of payment. If taken further this example can also showcase the topic of Market Liquidity, a term which refers to what degree any market allows assets to be traded at transparent and stable prices. Seeing as no one would sell a brand new TV in exchange for a basket of apples we could for all purposes with certainty say that such a market is non existent. In terms of being a factor, Liquidity can be measured over time and benchmarked against an assets performance in the same period in order to derive the correlation between the two. The correlation can be used further as a factor that shows the sensitivity of the asset, and how its value moves together with liquidity and vice versa. This allows us to use liquidity as a factor in order to predict the value asset by implementing it into a Multi-Factor model.

2.6 Measuring risk-adjusted returns

2.6.1 Inflation rate

In short terms, Inflation is what occurs when the supply of a currency outpaces its demand. This causes every unit of said currency to be worth less which consequently leads to reduced purchasing power, thus driving prices up. In theory there is support that due to inflation increasing the general price level combined with stocks being capital goods, we should observe a positive correlation between the general price level and the stock prices. According to Fisher's basic hypothesis it is expected that the expected nominal rate of return is equal to expected inflation plus the expected real rate of return, where the expected real rate of return is either independent of the expected inflation or constant. With this in mind it would be possible to make use of the inflation rate in the process of making an estimator for a factor that could be implemented into a Multi-Factor model.

2.6.2 Sharpe ratio

The Sharpe ratio is used to evaluate an asset's risk-adjusted return. In other words it adjusts a portfolio's expected future performance for the excess risk taken by the investor. In terms of the Sharpe ratio we are mostly interested in using it to make decisions about future investments, and as such we are primarily interested in the ex-ante version that produces a forward looking estimate contrary to the ex-post version that mainly takes a look at past performance. The formula for computing the Sharpe Ratio is as follows:

$$SharpeRatio = \frac{R_p - R_f}{\sigma_p}$$

In this formula we have three components, namely the portfolio return(R_p), the risk-free rate(R_f), and the standard deviation of the portfolio's excess return(σ_p). By looking at the relationship between the variables included in this formula it is clear that if we are to reduce the standard deviation or risk-free rate we will end up with a higher Sharpe Ratio. Likewise we will also end up with a higher Sharpe Ratio if we are to increase the size of the portfolio returns. While there is little we can do to reduce the risk-free rate, it is possible to effectively lower the related standard deviation through greater diversification of the portfolio. By doing this we are effectively able to lower the risk while still maintaining the same R_p, thus achieving a greater Sharpe Ratio. Due to the very nature of the Sharpe Ratio, it is possible to somewhat falsify the reality of the current situation by choosing a specific analysis period in order to achieve the best possible Sharpe Ratio instead of conducting the analysis on a neutral historical period. This leads to the possibility of the Sharpe Ratio being manipulated, thus often making another benchmark, the Information Ratio more suitable.

2.6.3 Information Ratio

The Information Ratio is a value that reflects an investor/portfolio manager's skill by looking at their ability to produce excess returns relative to the returns of an index in comparison to the volatility of the same returns. The following formula is what we use when calculating the Information Ratio:

$$InformationRatio = \frac{PortfolioReturn - BenchmarkReturn}{TrackingError}$$

As seen in the formula the Information Ratio is effectively a ratio between the portfolio's excess returns and the standard deviation of the difference between portfolio and benchmark returns ($TrackingError$).

By looking at the formula we can conclude that a higher portfolio return compared to benchmark return as well as a decreasing size in the tracking error would produce an increase in size of the Information Ratio. Thus, a higher Information Ratio would indicate that the investor/portfolio manager has a higher skill than that of an individual with a lower Information Ratio. Compared to the previously mentioned Sharpe Ratio, the Information Ratio calculates the risk-adjusted return against a benchmark instead of a risk-free asset like the Sharpe Ratio does. This combined with the fact that the Information Ratio also measures an investors consistent performance, makes it a much more appealing metric for investors.

2.7 Utilizing the model with Norwegian data

This piece of research is largely predicated on the findings of Næs et al. from 2008. The authors found through their studies that the book-to-price factor and the momentum factor have little to no relevance in the Norwegian securities market. Their conclusion described how a factor-model consisting of marked-returns, size and liquidity explains the returns on Oslo Børs to a greater degree. Through our piece of research we aim to assess whether our results contradicts or reinforce this conclusion, and investigate whether there are other less traditional factors that are beneficial in explaining returns on Oslo Børs. With this in mind, adapting traditional models and utilizing traditional scientific literature which is primarily focused on the US market, will be a major challenge.

Particularly relevant is the size of the Norwegian market as compared to, for example, the US'. The Norwegian market is relatively small in comparison, which can ultimately result in an effect where large funds influence asset prices through large trades. This effect in turn puts a limit on the mutual funds freedom of action.

The Norwegian market has another particular trait, in which an abundance of large companies have concentrated ownership. More often than not for these large companies, the government or other large investors own a considerable number of shares in the companies. As a result there is a considerable amount of shares which are not available to the market. This in turn results in lower liquidity, seeing as there are fewer shares readily available for trade. Næs et al.(2008) makes mention of this effect and thus comes to the conclusion that in the particular case of the Norwegian market, size and liquidity are more or less the same. The authors base this conclusion on their own research in which they find that the factor-related returns of the liquidity-portfolio is positively correlated with that of the size-portfolio.

2.8 Literature review

2.8.1 "What factors affect the Oslo Stock Exchange?"

Working paper by Randi Næs, Johannes A. Skjeltorp and Bernt Arne Ødegaard. 2009.

Traditional Multi-Factor models are rigorously researched within the discipline of finance. Thus, the empirical and theoretical literature on asset pricing is very internationally extensive. The existing research on the topic is however mostly conducted in the economic environment of the USA. It is established that there are unique differences when adapting the traditional Multi-Factor models for deployment in different countries. There are possible specific considerations and limitations to address when attempting to apply these models in different countries, thus the degree of model explainability can vary across countries. These particularities would have to be unveiled through extensive empirical and statistical analyses.

In relation, there are very few thorough analyses in which traditional Multi-Factor models are researched in the Norwegian economic environment. The most notable study of this topic in the Norwegian market is "What factors affect the Oslo Stock Exchange?" by Næs, Skjeltorp Ødegaard. In their paper, the authors conduct thorough analyses of traditional factor models and factors on the Oslo Stock Exchange in order to assess their relevance and explainability in Norway. The article is often considered to be the cornerstone of factor-model research on Norwegian assets. The authors report results from thorough empirical analysis of the Norwegian stock market, with the purpose of evaluating whether traditional factors and factor models are well suited to explain the returns of these assets.

The authors based their research on a dataset which consisted of all the listed stocks on the Oslo Stock Exchange / OSE in a time period that stretched from 1980 to 2006. They utilized this data to analyze and identify what systemic factors actually demand risk compensation in the Norwegian market. The authors proceeded to argue that the research is of great importance and contribution, because systematic risk factors can be used to set required returns for investments and also to evaluate a single stock's individual contribution within a portfolio.

In their analysis the authors aimed to test the traditional international factors, including the local stock market, the Fama & French factors regarding firm size, book value and momentum. In addition they also tested typical macroeconomic factors such as consumption, inflation, investments and money stock without discovering any statistically significant relationships. The authors argue that these results are typical for macroeconomic factors, which implies that the stock market in a specific country is a leading indicator for the economy, rather than being the other way around.

In conclusion the authors find that the returns of the assets on the OSE can be explained by traditional factors and factor models reasonably well. Their results highlight how market, firm size and also liquidity stand out as statistically significant and priced risk factors in the Norwegian economic environment. On the other hand, the two factors book value and momentum stand out as being near insignificant and irrelevant for use in a Norwegian setting. Furthermore, the authors also provide solid evidence with which they concluded that in contrast to public opinion, the oil price is not a priced risk-factor in the Norwegian asset market. (Næs et al, 2009)

Seeing as Næs et al's paper is the most extensive piece of research on the topic of Multi-Factor models, we aim to reinforce or dispute their conclusions with the results of our thesis.

2.8.2 "On persistence in mutual fund performance"

Article by Mark M. Carhart. 1997.

In his 1997 article, Carhart presents his work on mutual fund performance, and argues that common systematic risk factors in stock returns and differences in mutual fund expenses and transaction costs explain nearly all of the mutual fund returns predictability. The author utilizes a sample of mutual funds, free of survivorship bias for his computations. Carhart utilizes two models for performance measurement, namely the CAPM, the Fama & French 3-factor model and lastly his own articulation of the 4-factor model introduced in 1995. The author finds that systematic risk factors account for a substantial part of the mutual funds performance, and concludes by specifying three rules-of-thumb for investors seeking to invest in mutual funds: Firstly Carhart urges investors to avoid funds with consistently poor performance, secondly he concludes that the momentum effect is evident, meaning that funds that over-performed this year are more than likely to over-perform the next year, but not in the years thereafter. Lastly the author argues that transaction costs and expenses have a direct negative impact on performance.

2.8.3 "Mutual fund's R^2 as predictor of performance"

Article by Yakov Amihud, Ruslan Goyenko featured in The review of Financial studies. 2013.

The authors of this article wanted to investigate further upon the common conception that fund performance is positively affected by active management. The authors thus proposed that fund performance can be predicted by its inherent R2-value, obtained through a regression of the funds returns with respect to a Multi-Factor benchmarking model, which in this case were the Fama-French(1993) and Carhart(1997) four-factor models. In their results the authors state that a lower R2-value indicates greater selectivity for the fund, and thus significantly predicts better performance. They elaborate on their conclusion by stating that the lower R2-value corresponds with greater activity within the funds investments, thus increasing the fund-manager's opportunity to achieve higher risk adjusted returns when compared to the benchmark.

2.8.4 "European Mutual Fund Performance"

Article by Roger Otten & Dennis Bams, featured in European Financial Management. 2008.

In their 2009 article, the authors Roger Otten and Dennis Bams present an overview and evaluation of the European mutual fund industry, and more importantly conduct assessments based on the Carhart(1997) 4-factor pricing model. The authors utilize a bias controlled sample of over five-hundred funds derived from the five most prominent mutual fund countries in Europe. In their results, the authors present evidence that small cap funds are better able to add value as indicated by their positive and significant alpha values. Furthermore they found that four-out-of-five countries in the selection presents positive statistically significant performance over the market level. Thus the authors highlight that their results deviate from the majority of similar studies conducted on the US mutual fund market, in which the conclusions often are that mutual funds underperform the market by an amount equal to their charged management fees.

2.8.5 "Faktoreksponering i det norske fondsmarkedet"

Master thesis by Mads Haug Johansen & Fredrik Strømberg. 2015.

In this master thesis, the authors conducted several multiple regression computations with traditional factors for Norwegian mutual funds, based upon data spanning from 2002-2014. The authors utilize the factors; market, firm size, book value, momentum and liquidity, and their conclusions are largely in line with what Næs et al. found in their 2009 article. Thus the authors found that market, firm size and liquidity were of statistical significance for the majority of the funds in their selection, which indicated that those are priced systematic risk factors in the Norwegian mutual fund market. In addition, they investigated whether major differences in managed capital yielded any significantly different results. The authors concluded that the amount of management capital is of little to no relevance with regards to the funds factor sensitivities and corresponding statistical significance.

2.8.6 "Flaks versus dyktighet i det norske aksjefondsmarkedet"

Master thesis by Michael Kjelstrup & Ragnar Vagle Urving. 2020.

As a part of this piece of research, the authors conducted computations with traditional Multi-Factor models to assess their explanatory power when utilized on Norwegian mutual funds. The authors utilize forty Norwegian mutual funds in their analyses where they conducted several computations involving Information Ratio, traditional factor models and the CAPM. Their results are somewhat in line with the conclusion of Næs et al.(2009), in that they find market and size to be statistically significant. However, they do not find the liquidity factor to be of any significance, which could be attributed to using a different definition of the liquidity premium, and/or using a different data source for the liquidity factor premium. It is worth mentioning that their computations regarding traditional multi factor models are conducted on a selection of the four "best" funds from their total of forty funds, which reduces the generalizability of their results.

3 Data

Conducting computations on funds operating in the Norwegian economic environment is a key part of this research, therefore the funds included has to be defined as Norwegian funds by the official fund agency VFF(Verdipapirfondenes forening). The requirement that needs to be met in order to be considered a Norwegian fund entails that eighty percent of the funds' value needs to be invested in Norway. In addition, we have chosen to exclude funds that have had or currently have a short existence, in order to create the most representative selection of the funds factor exposure. The data frame utilized in this research stretches from 2010 to 2020, therefore all the included funds have a coherent return-history of 10 years or more. The reasoning behind the particular selection of funds is that this time frame allows for the highest number of coherent observations, whilst maximizing the number of included funds.

There exists no universal rule as to how long of a time period one should utilize, however the aforementioned time frame yields us more than 100 observations over 10 years which we regard as more than satisfactory. The selection includes actively managed funds as well as passively managed funds. We expect the passively managed funds to be similarly exposed to the factors as the index in comparison. Whilst the actively managed funds may produce more interesting and varying results.

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3.1 Fund information

Table 1: Fund Information

	Funds	Purchase fee	Sales fee	Management fee	capital	type	Stock size
1	Alfred Berg Aktiv	0.00	0.00	1.50	2991.63	Mixed	Medium
2	Alfred Berg Gambak	0.00	0.00	2.00	9623.74	Mixed	Medium
3	Alfred Berg Humanfond	0.00	0.00	1.20	171.64	Mixed	Medium
4	Alfred Berg Indeks I	0.00	0.00	0.08	2564.45	Growth	Medium
5	Alfred Berg Norge C	0.00	0.00	1.20	2303.22	Mixed	Medium
6	C WorldWide Norge	3.00	1.00	1.30	405.28	Growth	Medium
7	Danske Invest Norge Aksj. Inst 2	0.20	0.20	0.90	10202.39	Mixed	Medium
8	Danske Invest Norge I	2.00	0.30	1.50	598.87	Mixed	Medium
9	Danske Invest Norge II	1.50	0.30	1.25	1561.66	Mixed	Medium
10	Danske Invest Norge Vekst	2.00	0.30	1.75	2886.13	Growth	Medium
11	Delphi Norge	0.20	0.20	2.00	1624.59	Mixed	Medium
12	DNB Norge D	0.00	0.00	0.73	185.59	Growth	Medium
13	DNB Norge Selektiv E	0.00	0.00	0.80	3873.30	Growth	Medium
14	DNB SMB A	0.00	0.00	1.21	2489.42	Growth	Small
15	Eika Norge	2.00	0.50	1.50	2367.01	Mixed	Medium
16	Fondsfinans Norge	3.00	0.30	1.00	1223.30	Mixed	Medium
17	Handelsbanken Norge	0.00	0.00	1.50	1892.42	Mixed	Medium
18	Holberg Norge A	0.00	0.00	1.50	1512.48	Mixed	Small
19	KLP AksjeNorge Indeks	0.00	0.00	0.10	15663.50	Growth	Medium
20	KLP AksjeNorge Indeks II	0.00	0.00	0.18	3883.79	Mixed	Medium
21	Nordea Avkastning	0.00	0.00	1.50	4346.71	Mixed	Medium
22	Nordea Kapital	0.05	0.05	1.00	5217.36	Mixed	Medium
23	Nordea Norge Verdi	0.00	0.00	1.50	4773.44	Value	Small
24	Pareto Aksje Norge A	1.00	0.50	1.50	990.70	Value	Medium
25	Pareto Aksje Norge B	1.00	0.50	2.00	646.23	Value	Medium
26	Pareto Aksje Norge I	1.00	0.50	0.50	3546.60	Value	Medium
27	PLUSS Aksje	0.50	0.50	1.00	150.76	Mixed	Medium
28	PLUSS Markedsverdi	0.50	0.50	0.90	155.99	Mixed	Medium
29	Storebrand Aksje Innland	0.30	0.30	0.60	1968.25	Mixed	Medium
30	Storebrand Norge	0.20	0.20	1.50	583.42	Mixed	Medium
31	Storebrand Norge I	0.20	0.20	0.28	10784.78	Mixed	Medium
32	Storebrand Vekst	0.20	0.20	2.00	890.44	Growth	Medium
33	Storebrand Verdi A	0.00	0.00	2.00	679.44	Mixed	Medium
34	XACT OBX	0.00	0.00	0.30	831.56	Growth	Large

Source : Morningstar.no(26.04.21)

In order to make sense of the table above we will elaborate on some of the values so that they will be easily understandable even if some of the values are fairly self-explanatory in nature.

The purchase fee shown represents the percentage amount of the investment which can be charged by the funds when investors buy shares. Such a fee is used to alleviate some of the costs associated with selling the shares. Commonly, this type of fee is mostly seen within funds that have inherently high transaction costs, and there are several funds with little to no purchase fee as can be seen in the table above. The implementation of a fee like this ensures that the investment of other fund investors are not diminished by the high transaction costs that occur on every purchase.(SEC, 2021) Next up comes the sales fee. As the name implies it is a fee that is paid as an investor seeks to exit his/her investment in the fund. This fee is dependent on the returns yielded by having invested in the fund and is also given as a percentage value.(SEC, 2021)

One of the primary aspects that an investor is concerned with when selecting a mutual fund is the annual management fee. The management fee is presented as a percentage, and details how much is paid to the fund managers for managing-, operating- and administrative expenses. Depending on the magnitude of the management fee, it can have significant consequences for an investors final return on his/hers investment.(SEC, 2021)

In the above table we included the popular Morningstar investment style box categories, in which a mutual fund is classified in a simple matrix based on the type and size of its underlying assets. .Management capital refers to the sum total assets handled by a specific mutual fund. Whilst 'stock type' is a classification attributed to a mutual fund in order to categorize the assets in which it is invested in. 'Value' indicates that the fund is heavily invested in value stocks, whilst 'Growth' indicates that the fund is concerned with assets that are considered to be growth stocks. The classification 'Mixed' naturally indicates a mix of the two aforementioned classifications (Morningstar, 2021).

3.2 Returns data

In order to gather the data needed to calculate the monthly returns for the funds included in our selection, we utilized historical price data derived from TITLON, a widely recognized financial data tool for use by academic institutions in Norway. The database registers daily price data entries for all mutual funds in Norway, which we downloaded and cleaned through excel. We proceeded to compute monthly returns from the daily data in order to eliminate high daily fluctuations, and to be more precisely in line with the data for factor premiums, which are also computed on a monthly basis.

The historical prices utilized as the fundament for calculating returns are adjusted, which implies that any dividends are calculated into the price, which in turn does a better job at evaluating fund performance over the long term.

3.3 Descriptive statistics

Table 2: Descriptive Statistics

Funds	10yr-return	ST.Dev	Min	Max	Sharpe	IR
Alfred Berg Aktiv	0.910	4.079	-12.739	11.762	0.212	0.264
Alfred Berg Gambak	1.043	4.028	-12.680	10.763	0.247	0.421
Alfred Berg Humanfond	0.851	3.883	-11.354	10.573	0.206	0.167
Alfred Berg Indeks I	0.793	3.841	-9.979	10.899	0.192	0.011
Alfred Berg Norge C	0.892	3.900	-10.977	11.297	0.215	0.295
C WorldWide Norge	0.756	4.157	-10.955	11.605	0.172	-0.114
Danske Invest Norge Aksj. Inst 2	0.852	3.943	-11.499	11.525	0.203	0.187
Danske Invest Norge I	0.767	3.956	-11.653	11.576	0.181	-0.077
Danske Invest Norge II	0.826	3.949	-11.520	11.581	0.197	0.106
Danske Invest Norge Vekst	1.008	4.155	-9.947	14.380	0.232	0.427
Delphi Norge	0.873	4.344	-11.836	11.548	0.193	0.144
DNB Norge D	0.696	4.023	-10.042	11.195	0.161	-0.323
DNB Norge Selektiv E	0.812	4.338	-10.622	13.875	0.179	0.056
DNB SMB A	0.677	5.102	-13.207	15.532	0.133	-0.121
Eika Norge	0.597	4.139	-11.904	10.226	0.134	-0.433
Fondsfinans Norge	0.835	4.654	-11.723	12.738	0.175	0.075
Handelsbanken Norge	0.890	4.229	-11.688	12.035	0.201	0.183
Holberg Norge A	0.625	4.139	-11.789	10.550	0.141	-0.271
KLP AksjeNorge Indeks	0.787	3.856	-10.086	10.934	0.190	-0.043
KLP AksjeNorge Indeks II	0.782	3.844	-9.990	10.849	0.189	-0.091
Nordea Avkastning	0.852	4.168	-10.832	11.581	0.195	0.176
Nordea Kapital	0.850	4.087	-10.358	11.616	0.197	0.185
Nordea Norge Verdi	0.899	3.548	-9.270	11.257	0.197	0.185
Pareto Aksje Norge A	0.563	3.855	-11.425	14.008	0.235	0.204
Pareto Aksje Norge B	0.565	4.021	-11.592	13.993	0.129	-0.383
Pareto Aksje Norge I	0.688	4.028	-11.465	14.160	0.159	-0.176
PLUSS Aksje	0.750	3.900	-10.461	11.203	0.179	-0.125
PLUSS Markedsverdi	0.778	3.988	-10.642	12.523	0.183	-0.050
Storebrand Aksje Innland	0.819	3.774	-10.059	11.149	0.202	0.177
Storebrand Norge	0.829	4.039	-11.389	11.370	0.194	0.097
Storebrand Norge I	0.834	3.821	-10.045	11.693	0.204	0.195
Storebrand Vekst	1.088	4.611	-10.648	12.937	0.231	0.327
Storebrand Verdi A	0.678	3.798	-10.598	11.187	0.164	-0.382
XACT OBX	0.761	3.979	-10.078	10.703	0.179	-1.029

Prior to commencing our empirical analysis we wanted to create an overview of the descriptive statistics related to each fund included in our research. All data presented in the table is based on monthly data, while the Sharpe is based on Ødegaards(2021) risk-free rate as mentioned in subsection 3.4, and the IR is based on the OBX as a benchmark. The data includes monthly mean-returns, the standard deviation of said monthly means, as well as the minimum and maximum returns of that same period of 10-years. By analyzing 10-years of data we ensure that all funds have an equal amount of observations, at the same time as we leave out market anomalies such as the 2008-recession. This leaves us with data representative of a relatively consistent growth period which allows us to focus on how the funds perform relative to each other in a modern environment without being influenced by different starting points for the funds and major market anomalies.

To give a brief introduction to how the numbers in the table above are to be interpreted we will provide some examples of the extremes related to each type of value:

The fund with the highest related mean return is Storebrand Vekst with a value of 1.08%, meanwhile we find Pareto Aksje Norge A at the other side of the spectrum with a related mean return of 0.563%.

When examining the column displaying the Standard deviation of the monthly means, we find that the largest value belongs to DNB SMB A which has a standard deviation of 5.1%. Meanwhile, the lowest standard deviation belongs to Nordea Norge Verdi which has a more modest value of 3.54%.

Seeing as the minimum and maximum columns are two sides of the same coin it only makes sense to take a look at both in relation to each other. The highest maximum return belongs to DNB SMB A. This would ordinarily be considered good, but as we can see from the table above it is also the fund with both the highest standard deviation as well as the lowest minimum value. This context taken into consideration along with the funds' related low Sharpe ratio, indicates that the fund is exposed to high risk, while not necessarily yielding a corresponding level of returns.

Lastly the remaining two columns display estimations of the Sharpe ratio and Information Ratio which are elaborated upon in subsection 2.7.2 and 2.7.3 respectively.

3.4 Risk-free rate

There were several approaches available to us in terms of collecting the risk-free rate for use in our analysis. However, seeing as the systematic-risk factors used in our analysis were produced by Ødegaard(2021) specifically with the Norwegian stock market in mind, we deemed it appropriate to employ his iteration of the risk-free rate as well. His iteration specifically calculates the risk-free rate with the help of government bonds and NIBOR.

In order to properly match the periodicity of our collected data, we of course use the monthly risk-free rate in all computations shown previously, as well as all computations going forward.

3.5 Factor data

We have fetched the monthly returns computed for the factor-portfolios regarding size(SMB), book/price (HML) from AQR, a global investment firm who produces and distributes the aforementioned data for several countries in Scandinavia. In addition, we have fetched data regarding the momentum (PR1YR) and liquidity (LIQ) factors from the website of professor Bernt Arne Ødegaard. The factor data is computed as described in the theory part of this thesis.

3.6 Market-data

Seeing as the funds included in the selection are composed of at least 80% Norwegian stocks, the main Norwegian index OBX is utilized as the market-portfolio. OBX is an index constructed of the 25 most actively traded stocks at Oslo børs, the stocks included in the index are revised semiannually.

4 Methodology

The methodology chapter serves the purpose of fleshing out the methodological approach and the research methods that are utilized in this thesis in order to properly answer the problem statement.

4.1 Research design

This piece of research consists of a qualitative analysis of the Norwegian mutual-fund market. The thesis can be classified as positivistic in terms of methodological approach, seeing as the research is dependent on quantifiable observations that is later used in statistical analysis. Furthermore the study aims to test and evaluate existing theory in light of as recent data as possible. The study utilizes exclusively quantifiable data made available to the general public through different databases. By definition the study is based on secondary data, which contributes to the objectivity of the research by minimizing the possibility for human influence and thus reinforcing the positivistic approach of the research.

4.2 Data collection

In this thesis, publicly available data on asset prices is utilized, as well as publicly available data for the different factor premiums. This enables a higher degree of reproducibility of the results and also allows the researcher to analyze a considerably larger pool of data without having to generate the entirety of the data first-hand.

4.3 Regression analysis

Regression analysis is a popular quantitative research method which is used to study and evaluate the relationship and interplay of one or more variables. Thus the primary statistical tool for analysis in this piece of research is linear regression. Linear regression models are used to test and evaluate the relationship between a set of variables. The coefficients derived from linear regression models for a particular variable determines how much the dependent variable increases or decreases as the independent variable is increased by one absolute unit. However this research will mostly feature multiple linear regression models, in which the previous remark is only true if all other independent variables remain unchanged.

4.3.1 Simple linear regression

The simple linear regression model explains the relationship between the dependent variable and an independent variable through a mathematical expression for a straight line. In this primitive regression model the slope of the line illustrates how much the dependent variable y is changed when the researcher increases the independent variable x by one unit. This form of regression can be illustrated as such:

$$y_i = \alpha + \beta x_i + \epsilon_i$$

Where:

- y_i = Dependent variable
- x_i = Independent variable
- β = Coefficient of the independent variable
- α = Intercept
- ϵ_i = Error term / residuals

The parameters and are initially unknown and are thus estimated values in the regression model. In order to find the regression expression that explains the covariation best, the researcher can utilize the method called Ordinary Least Squares(OLS). In OLS the unknown parameters are estimated by the principle of least squares, where the sum of the squares of the differences between the dependent variable and those computed by the linear function of the independent variables are minimized. In essence, minimizing the difference makes the regression model fit the dataset better. OLS can be illustrated as such:

$$OLS = \sum_{i=1}^{N} (y_i - \hat{\alpha} - \hat{\beta} x_i)^2$$

The method of OLS results in selecting the model which minimises the sum of squared residuals.

4.3.2 Multiple linear regression

Computations with Multi-Factor-models implies the inclusion of more than one independent variable. Thus multiple linear regression is required in this piece of research in order to assess and evaluate the different models and their included factors' influence on asset returns. Multiple linear regression can be expressed as such:

$$y_t = \alpha + \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 x_{3t} + ... + \beta_n x_m + \epsilon_t$$

In which the variables $x_{1t}, x2t, x3t\ldots, x_{nt}$ are the different independent variables or factors that are computed to influence the dependent variable or asset returns as is the case with this piece of research. For the multiple linear regression models, the -coefficients constitute the independent variables' multiplicators. Thus the basic principle for the multiple linear regression model is similar to that of the simple linear regression, however one must alter the interpretation slightly. Therefore the computed coefficients only imply the change yielded by a one unit increase given that all other independent variables remain unchanged.

4.3.3 T-test

An important tool in interpreting and analysing regression results is the t-test. It allows the researcher to evaluate the statistical significance of the coefficients in a regression model. For any number of observations of the independent variable, one can articulate the t-test as such:

$$t = \frac{\frac{\bar{x}-\mu}{S}}{\sqrt{N}}$$

In the above expression, is equal to the mean of the population, $\bar{x}$ is equal to the mean of the selection and S is equal to the estimated standard deviation of the population. For this piece of research we will utilize t-tests for two primary objectives:

1. *Test how different the factor-coefficients are from zero.*

2. *Test how different the market-beta is from one.*

By conducting our regressions in R-studio we output the t-statistic for every coefficient, with predetermined significance levels and illustrative symbols: 0,1% (***), 1% (**), 5% (*), 10% (‘)

4.3.4 The models' explanatory power

A key metric in regression models is the adjusted r-squared value. It enables the researcher to determine how much of the datasets variation the model is able to explain. In other words it is a metric that gauges how well the model fits the dataset, and it is preferable that the R^2-value corresponding to the regression model is as high as possible.
The explanatory power of a regression model can be explained mathematically as such:

$$R^2 = 1 - \frac{SS_{RES}}{SS_{TOT}}$$

Where:

- SS_{RES} = The residual sum of squares / the sum of squared residuals
- SS_{TOT} = The total sum of squares

Given this expression it is clear that the better the regression fits the data, the closer the R^2 - value will be to one.

4.3.5 Breusch-Godfrey test

When performing regression analyses on data over a given period of time there exists a reasonable concern that the data used is dependent on its previous values. Should this be the case, it would indicate that the data observed is in fact not random and we would have a serial correlation approaching a value of one. Since serial correlated data is dependent on past data, this would indicate that a previous overestimate of returns would have an effect on future data. Thus, serial correlation would lead to inaccuracies and could in the worst case scenario leave the resulting data rendered useless. In order to test our data for serial correlation we decided to make use of the Breusch-Godfrey test.

One of the benefits of using this test in comparison to other tests also aimed at looking for serial correlation is its ability to look for serial correlation of a higher order. Since our data is based on monthly returns and as such is to be classified as monthly data, we want to test for serial correlation of the 12th order.

If we use the linear model introduced in (4.3.2) as an example: We would assume that the associated correlated residuals could be expressed as such:

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + ... + \rho_p u_{t-p} + \epsilon_t$$

With this in mind we can begin testing for serial correlation by performing a new regression of the model by fitting this auxiliary regression model:

$$\hat{u}_t = \alpha_0 + \alpha_1 X_{t,1} + \alpha_2 X_{t,2} + \rho_1 \hat{u}_{t-1} + \rho_2 \hat{u}_{t-2} + ... + \rho_p \hat{u}_{t-p} + \epsilon_t$$

This model is based upon the null hypothesis:

$$H_0 : \{\rho_i = 0 \; for \; all \; i\}$$

The p would as previously stated be 12 in our case due to our data being of monthly periodicity. This also influences our test statistic, which in this case would be defined as being as follows:

$$LM = (T - p) * R^2 \sim \chi^2$$

As you can see from the test statistics' expression it is indeed following a chi-squared distribution. As such, we are able to determine whether to discard or keep our null hypothesis based on if the test statistic exceeds the critical limits according to the chi-squared distribution. If we discard our null hypothesis we then assume our data to be affected by serial correlation.

4.3.6 Variance Inflation Factor

Since we are using several explanatory variables in our analyses we might be exposed to what is known as multicollinearity. Multicollinearity is a phenomenon which occurs when some of the explanatory variables used in a regression model can be explained by the variation of the other variables used in the same model. As is stated by Salméron et al.(2018), a high collinearity in multiple linear regression models would imply that the resulting analysis conclusions would lack accuracy due to high variance among the estimators. In order to make sure that our models are not subject to multicollinearity we chose to include a Variance Inflation Factor test (VIF-test) as was used by Salméron et. al(2018).

The VIF is given by the following expression:

$$VIF\left(i\right) = \frac{var(\hat{\beta}_i)}{var(\hat{\beta}_i^o)} = \frac{1}{1 - R_i^2}$$

In this expression the R^2 is used to describe how well an explanatory variable is being explained by the remaining explanatory variables. As such, the better an explanatory variable is explained by the other explanatory variables, the higher the resulting R^2 will be. Seeing as the R^2 is in the denominator we see that an increase in R^2 would result in a higher VIF, meaning that the higher VIF we get, the more evidence of multicollinearity there exists within our model.

Salméron et. al(2018) states that *"commonly a VIF of 10 or even one as low as 4 have been used as rules of thumbs to indicate excessive or serious collinearity"* based on this we will determine that we have too high collinearity should our VIF exceed that of 4.

4.4 Evaluating the study

4.4.1 Reliability

A key part in any piece of research is reliability. It is a broad term that involves assessing to what degree the data is reliable and also how dependable the handling of data has been. As previously stated this piece of research is largely based on secondary data derived from or generated by recognized and acclaimed authors and databases. We have utilized historical data on mutual funds derived from TITLON, a widely recognized financial data tool for use by academic institutions. It is a database often utilized as the basis for historical data on financials in Norway. Furthermore we derive market and factor data from professor Ødegaards own web page, which is the continuation of his work from 2008, which is widely regarded as the cornerstone of research on the subject of factor-investing in Norway.

Reliability is generally considered to be high with quantitative studies based on secondary data, however there is still the matter of how the data is handled and utilized by the researcher. Therefore it is important for the researcher to articulate and describe how the data is handled and how it is being utilized for computations, in order to maximize the reliability of the study. Providing thorough description of details and procedure also contributes greatly to the reproducibility of results. The data in our thesis is generally unaltered and computations with the data are specified, in an attempt to maximize reliability and reproducibility. Anyone with access to the aforementioned databases should be able to replicate the results of this piece of thesis.

4.4.2 Validity

Validity is another relevant concept in evaluating quantitative studies. The concept involves the extent to which a measurement or conclusion corresponds accurately with the real world. In our thesis we attempt to provide a holistic assessment of factor investing on Norwegian mutual funds, and our selection aims to redeem this as far as possible. It is important to note that the relevance and applicability of factor-premiums differ from country to country. We recognize that the selection represents a wide extent of Norwegian mutual funds, thus representing an adequate share of the compounded capital for this type of investment in Norway. With this as the basis, we would argue that the results of this study are sufficiently representative and relatively generalizable within the confines of the Norwegian economic environment.

5 Data analysis

In this chapter we will be conducting analyses of the data resulting from our regression models. We explicitly used R-software to aid us in performing our regression models on the collected data.

We conducted regressions based on three different models in our search for answers to the pre-established problem statement:

"Are factor models able to properly capture and explain asset returns in the Norwegian mutual fund market?"

The first model utilized was the well-known CAPM-model, followed by the Fama & French Three-factor model and Five-factor model. We will go a bit more into detail in terms of the factors involved in each of these models in their respective sub-chapters of the data analysis.

5.1 CAPM

This model is the most simplified of the models used in this study. As is evident from the table below, the CAPM-model relies solely on the market-beta as its explanatory variable. This model operates with the expectation of complete market efficiency, and thus expects a linear relationship between the associated market-beta and the resulting historical returns. This expectation implies that should the model return an Alpha significantly different from zero, then it is unable to properly explain the returns through implementation of the associated market-beta. When the regression

model returns an Alpha that is significantly different from zero we can interpret it in two ways. Firstly given the assumption that all markets are highly efficient we would interpret these Alphas as the model being poorly specified at the factors responsible for the excess returns. The other interpretation would be that the fund managers based on experience and skill have been able to add value to the fund portfolio in the form of excess returns, and is of course based on the notion that markets are not entirely efficient.

Seeing as our selection of funds consists exclusively of Norwegian equity funds we expect most if not all of them to have a significant exposure to the inherent market-risk of the benchmark market portfolio OBX. The OBX can be viewed as a single buy & hold portfolio with a market-beta that equals 1. As the goal of this study is to examine whether or not factor models can serve as a competitive investment strategy in the current modern market it would be of high relevance to also examine if the included funds have a market-risk exposure that differs from that of the index benchmark.

Having stated this, we ended up with the following null-hypotheses:

$\alpha = 0, \qquad \beta_M = 1$

Table 3: CAPM-Model

Funds	α	β_M	R^2
Alfred Berg Aktiv	0.00180	0.934$^{\cdot}$	0.83
Alfred Berg Gambak	0.00386$^{\cdot}$	0.848**	0.70
Alfred Berg Humanfond	0.00124	0.921**	0.89
Alfred Berg Indeks I	0.00030	0.960***	0.98
Alfred Berg Norge C	0.00159	0.928*	0.89
C WorldWide Norge	-0.00030	1.002	0.92
Danske Invest Norge Aksj. Inst 2	0.00107	0.944*	0.90
Danske Invest Norge I	0.00020	0.947$^{\cdot}$	0.90
Danske Invest Norge II	0.000801	0.946$^{\cdot}$	0.90
Danske Invest Norge Vekst	0.00286	0.926	0.78
Delphi Norge	0.00138	0.952	0.76
DNB Norge D	-0.00069	0.971	0.92
DNB Norge Selektiv E	0.00002	1.036	0.90
DNB SMB A	0.00007	0.917	0.51

Funds	α	β_M	R^2
Eika Norge	-0.00145	0.952	0.83
Fondsfinans Norge	0.00038	1.038	0.78
Handelsbanken Norge	0.00164	0.935	0.77
Holberg Norge A	-0.00049	0.873*	0.70
KLP AksjeNorge Indeks	0.00022	0.964***	0.98
KLP AksjeNorge Indeks II	0.00018	0.961***	0.98
Nordea Avkastning	0.00072	0.995	0.90
Nordea Kapital	0.00077	0.983	0.91
Nordea Norge Verdi	0.00288·	0.772***	0.75
Pareto Aksje Norge A	-0.00075	0.817***	0.71
Pareto Aksje Norge B	-0.00100	0.856**	0.71
Pareto Aksje Norge I	0.00022	0.857**	0.71
PLUSS Aksje	0.00015	0.930*	0.90
PLUSS Markedsverdi	0.00014	0.967	0.93
Storebrand Aksje Innland	0.00071	0.939***	0.98
Storebrand Norge	0.00084	0.949	0.87
Storebrand Norge I	0.00087	0.940**	0.95
Storebrand Vekst	0.00506	0.789**	0.46
Storebrand Verdi A	-0.00047	0.916**	0.92
XACT OBX	-0.00030**	0.999	0.99

As is evident from the table above we are left with only a single fund that has an Alpha significantly different from 0. This Alpha belongs to XACT OBX which is an ETF made to passively follow the OBX index.This ETF has an annual management fee of 0,30% while the Alpha indicates that the monthly return of this fund is an estimated -0,03% at a 1% significance level. The negative related Alpha is in line with expectations if one takes into consideration that all returns are adjusted for any dividends and management fees. Since this fund is the only fund that has achieved any significant Alpha through this regression we cannot determine whether or not the remaining 33 funds have achieved excess returns or lower returns than the benchmark, despite having relatively high positive Alphas.

The second value we display in this table is the market-beta, also known as the market-risk. As previously stated in the null hypotheses we started with the expectation that the funds would align with the risk amount of the index. As such, we were interested to see whether some of the funds had lower market exposure. According to the significance classification from our regression analysis we have a total of 6 funds that are significantly different from 1 at a 0,1% significance level as well as 7 funds at a 1% level and 4 funds at a 5% level. We have employed the use of very strict significance levels and as a result of this we can be close to 100% certain that we can reject the null in the cases mentioned. One extra characteristic worth mentioning is that not a single one of the funds with a significant market-beta are higher than 1. This tells us that none of the 34 funds have higher exposure to market-risk than the index itself, which leads to the conclusion that all funds are of either neutral exposure, or lower exposure than the index.

5.2 Three-Factor Model

The CAPM is by many considered a primitive model, and existing empirical studies show that some money managers generate excess returns that are not attributable to market risk. Therefore we proceed to add more explanatory variables to the regression in order to increase the degree of explained variance and also to assess the significance of the computed alpha-values. We computed a multiple regression in accordance with the Fama & French three-factor model, where the systematic risk factors SMB and HML are utilized in conjunction with the market-beta. In this piece of research we are mainly concerned with the five-factor model, however it is interesting to assess how the incremental increase in model complexity affects the regression results. The null-hypotheses for the three factor model are slightly different, seeing as both the SMB and HML-factor premiums are calculated through long-short portfolios. Therefore the t-test is conducted to determine whether the factor coefficients are significantly different from zero rather than one, which is the case with the market-factor.

$Nullhypotheses: \ \alpha = 0 \ \ \beta_M = 1 \ \ \beta_{SMB} = 0 \ \ \beta_{HML} = 0$

Table 4: Three Factor Model

Funds	α	β_M	β_{SMB}	β_{HML}	R^2
Alfred Berg Aktiv	0.00219	1.031	0.324***	-0.057	0.86
Alfred Berg Gambak	0.00453*	0.954	0.376***	-0.042862	0.74
Alfred Berg Humanfond	0.00136	0.993	0.225***	-0.055$^{\cdot}$	0.90
Alfred Berg Indeks I	0.00026	0.985	0.070***	-0.026**	0.99
Alfred Berg Norge C	0.00172	0.997	0.217***	-0.052$^{\cdot}$	0.91
C WorldWide Norge	-0.00051	1.051$^{\cdot}$	0.125*	-0.061*	0.92
Danske Invest Norge Aksj. Inst 2	0.00104	1.008	0.185***	-0.059*	0.92
Danske Invest Norge I	0.00020	1.011	0.188***	-0.057*	0.92
Danske Invest Norge II	0.00079	1.009	0.186***	-0.057*	0.92
Danske Invest Norge Vekst	0.00357$^{\cdot}$	0.990	0.255**	-0.001	0.80
Delphi Norge	0.00278	1.051	0.424***	0.020	0.80
DNB Norge D	-0.00048	1.016	0.153**	-0.024	0.93
DNB Norge Selektiv E	0.00009	1.060$^{\cdot}$	0.076	-0.016	0.90
DNB SMB A	0.00290	1.072	0.726***	0.081	0.60

Funds	α	β_M	β_{SMB}	β_{HML}	R^2
Eika Norge	0.00010	1.030	0.378***	0.051	0.87
Fondsfinans Norge	0.00319·	1.094·	0.428***	0.170***	0.84
Handelsbanken Norge	0.00187	1.042	0.338***	-0.079·	0.81
Holberg Norge A	0.00103	0.962	0.408***	0.039	0.74
KLP AksjeNorge Indeks	0.00023	0.987	0.069***	-0.019*	0.99
KLP AksjeNorge Indeks II	0.00020	0.983·	0.069***	-0.019*	0.99
Nordea Avkastning	0.00120	1.066*	0.256***	-0.026	0.92
Nordea Kapital	0.00118	1.051·	0.242***	-0.029	0.93
Nordea Norge Verdi	0.00412*	0.844***	0.329***	0.032	0.78
Pareto Aksje Norge A	0.00137	0.853**	0.304***	0.135**	0.75
Pareto Aksje Norge B	0.00122	0.896*	0.326***	0.140**	0.76
Pareto Aksje Norge I	0.00245	0.897·	0.325***	0.140**	0.76
PLUSS Aksje	-0.00014	0.970	0.090·	-0.060*	0.90
PLUSS Markedsverdi	-0.000009	1.011	0.113**	-0.051*	0.93
Storebrand Aksje Innland	0.00061	0.966**	0.070**	-0.031**	0.98
Storebrand Norge	0.00051	1.034	0.222***	-0.103**	0.89
Storebrand Norge I	0.00102	0.973	0.112***	-0.018	0.96
Storebrand Vekst	0.00608*	0.963	0.612***	-0.077	0.55
Storebrand Verdi A	0.00013	0.942*	0.134**	0.024	0.92
XACT OBX	-0.00034**	0.998	-0.006	-0.001	0.99

In comparison to the CAPM-Model the Three-Factor model sees an additional three funds achieve significant alphas, while the negative alpha pertaining to XACT OBX remains at the same level of significance. Keeping in mind our strict significance levels, a single shooting-star still indicates a significance of a 5% level. According to the above table, the statistical significance of the coefficients for the market factor is drastically reduced. Whereas seventeen mutual funds showed statistical significance for the market factor with the CAPM regression, now only five of them return the same level of significance. This implies that the addition of two extra systematic risk factors account for the majority of difference from a market beta of one.

It is particularly noteworthy that statistical significance for the market coefficient only occurs in mutual funds where it is less than one. It is apparent that these observations are paired with the lowest sensitivity to the SMB factor for our selection. It is apparent that the coefficients for the HML factor are positive for these particular funds, which is somewhat rare for our selection of mutual funds. Furthermore it is observed that the coefficient for market-beta is increased with the addition of the

SMB and HML factors, which is suggestive of a negative correlation between the market-factor and SMB. Thus, by computing a model based purely on the CAPM would result in an underestimation of the market-beta. We reinforce this notion with our results and would argue for the three-factor model as a better tool for dissecting mutual fund returns.

As expected by adding more explanatory variables to the regression, one can observe that the degree of variance-explainability increases across the board. This suggests that the three-factor model is better suited at accounting for the observed variance and does a better job at explaining the returns of mutual funds.

5.3 Five-Factor Model

The main emphasis of this piece of research revolves around the five-factor model. It is composed of the SMB & HML factors introduced in the Fama & French three-factor model, used in conjunction with the momentum-factor and the liquidity factor. In this paragraph we will be assessing and evaluating how the addition of these two factors affect the regression results.

$$Nullhypotheses: \quad \alpha = 0 \ \ \beta_M = 1 \ \ \beta_{SMB} = 0 \ \ \beta_{HML} = 0 \ \ \beta_{PR1YR} = 0 \ \ \beta_{LIQ} = 0$$

Table 5: Five Factor Model

Funds	α	β_M	β_{SMB}	β_{HML}	β_{PR1YR}	β_{LIQ}	R^2
Alfred Berg Aktiv	0.002158	0.964	0.348***	-0.030	0.056	-0.109·	0.87
Alfred Berg Gambak	0.003701·	0.879	0.414***	0.004	0.132*	-0.133·	0.76
Alfred Berg Humanfond	0.001438	0.916·	0.251***	-0.026	0.054·	-0.123**	0.91
Alfred Berg Indeks I	0.000305	0.978	0.072***	-0.024**	0.001	-0.010	0.99
Alfred Berg Norge C	0.001821	0.935	0.237***	-0.029	0.040	-0.098*	0.91
C WorldWide Norge	-0.000025	0.949	0.153**	-0.031	0.037	-0.156***	0.93
Danske Invest Norge Aksj. Inst 2	0.001965·	0.931	0.198***	-0.048·	-0.021	-0.108*	0.92
Danske Invest Norge I	0.001148	0.934	0.201***	-0.047·	-0.023	-0.108*	0.92
Danske Invest Norge II	0.001742	0.931	0.199**	-0.046·	-0.022	-0.110*	0.92
Danske Invest Norge Vekst	0.005410**	0.967	0.236**	-0.032	-0.144**	-0.008	0.81
Delphi Norge	0.001978	1.020	0.447***	0.049	0.095·	-0.062	0.81
DNB Norge D	0.001156	0.941	0.155***	-0.029	-0.085**	-0.096*	0.93
DNB Norge Selektiv E	0.001482	1.022	0.069	-0.031	-0.093*	-0.039	0.90
DNB SMB A	0.005362	1.036	0.703***	0.042	-0.189*	-0.021	0.61
Eika Norge	0.001505	0.937	0.389***	0.058	-0.050	-0.128*	0.88
Fondsfinans Norge	0.005332**	0.989	0.433***	0.165***	-0.106*	-0.136·	0.85
Handelsbanken Norge	0.000504	1.018	0.366***	-0.039	0.140**	-0.060	0.82
Holberg Norge A	0.000962	0.978	0.404***	0.035	-0.005	0.024	0.74
KLP AksjeNorge Indeks	0.000286	0.981	0.070***	-0.018*	-0.0005	-0.007	0.99
KLP AksjeNorge Indeks II	0.000271	0.978	0.070***	-0.018·	-0.001	-0.008	0.99
Nordea Avkastning	0.001844	1.026	0.260***	-0.024	-0.024	-0.055	0.92
Nordea Kapital	0.001910·	1.012	0.245***	-0.029	-0.033	-0.052	0.93
Nordea Norge Verdi	0.004782**	0.810**	0.331***	0.031	-0.031	-0.043	0.79
Pareto Aksje Norge A	0.002683	0.787**	0.308***	0.133**	-0.063	-0.086	0.75
Pareto Aksje Norge B	0.002804	0.818*	0.330***	0.136**	-0.078	-0.100	0.76
Pareto Aksje Norge I	0.004046·	0.818*	0.329***	0.136**	-0.079	-0.101	0.76
PLUSS Aksje	0.000881	0.848***	0.117*	-0.034	0.005	-0.182***	0.91

Funds	α	β_M	β_{SMB}	β_{HML}	β_{PR1YR}	β_{LIQ}	R^2
PLUSS Markedsverdi	0.000854	0.920^{*}	0.132^{**}	-0.034	-0.004	-0.133^{***}	0.94
Storebrand Aksje Innland	0.000711	0.957^{*}	0.072^{***}	-0.030^{*}	-0.0008	-0.013	0.98
Storebrand Norge	0.000610	0.981	0.238^{***}	-0.084^{*}	0.032	-0.083	0.90
Storebrand Norge I	0.001108	0.959	0.115^{***}	-0.014	0.003	-0.021	0.96
Storebrand Vekst	0.007058^{*}	0.968	0.596^{***}	-0.100	-0.090	0.023	0.55
Storebrand Verdi A	0.000163	$0.929^{\cdot}$	0.139^{**}	0.029	0.008	-0.020	0.92
XACT OBX	-0.000335^{**}	0.995	-0.004	-0.0005	0.002	-0.005	0.99

When advancing to the implementation of the Momentum & Liquidity factors we see some interesting results. The five-factor model leaves us with a reduction in significance for the alpha of Alfred Berg Gambak and Danske Invest Norge I, while Storebrand Vekst retains its level of significance. Furthermore, we are also faced with an increased significance for alphas related to Nordea Norge Verdi, Fondsfinans Norge & Danske Invest Norge Vekst.

While we expected the size and significance of the alphas to decrease as the model developed to become more complex by containing more explanatory variables, we observe the contrary for some of the funds mentioned above. As we observe almost no alphas of significance in the CAPM, a few more in the three-factor model and the most in the five-factor model.

5.4 Sorting by management fee

In addition to assessing the explanatory power of Multi-Factor models in Norwegian mutual funds, we find it intriguing to evaluate the results in light of management fees. One would expect the mutual funds with the highest management fees to bring some added value to their funds in the form of extra information capable of returning excess returns compared to cheaper alternative investments. To better justify the wide differences in mutual fund pricing, it would be ideal to find that the more expensive funds return a higher yet also a more significant alpha value than the cheaper ones, whilst decreasing the variance explained by the model. In order to test this common conception, we classified all the mutual funds in our selection by their adherent management fees. We constructed three separate price brackets titled; ‘Low’, ‘Medium’ and ‘High’. With the corresponding management fee intervals: ‘0-0,99%’, ‘1-1,49%’ and ‘1,5%+’ respectively.

Proceedingly, regressions were performed for each of the three models, for each price bracket.

5.4.1 CAPM cost divided bracket

Table 6: CAPM Bracket

	α	β_{SMB}	R^2
Low	0.000418	0.952^{**}	0.97
Medium	0.000398	$0.951^{\cdot}$	0.92
High	0.001290	0.895^{**}	0.85

When estimating the alpha related to each price bracket using the CAPM-model our expectations are somewhat met. We can see that according to our expectations the high-cost bracket has the highest alpha of the three which indeed aligns with our hypothesis of more expensive fund managers adding value relative to the increased management costs. However, we can also observe that the alpha related to the low bracket outperforms the alpha of the medium bracket, suggesting that the increased management cost does nothing to improve excess returns. All that being said, none of the alphas returned by the CAPM-model prove to be significant. Thus meaning that we cannot confidently conclude that any of the brackets actually perform as shown. We can only observe tendencies based on this data, and will have to increase the model complexity in order to better prove the managerial skill provided by the related fund managers.

5.4.2 Three-Factor Model cost divided bracket

Table 7: Three-Factor Bracket

	α	β_M	β_{SMB}	β_{HML}	R^2
Low	0.000658	0.986	0.121^{***}	-0.011	0.97
Medium	0.001007	1.013	0.239^{***}	-0.007	0.94
High	$0.002301^{\cdot}$	0.984	0.356^{***}	-0.0002	0.89

When expanding the model with both the SMB & HML factors we see that generally speaking most of the funds are significantly exposed to the SMB-factor. This offloads the significance from the market-beta, which suggests that a portion of the factor exposure in the CAPM-model was contained in the error-term, and has now been successfully attributed to the SMB-factor. As we have increased the complexity of our model we have now achieved an alpha significant at the 10% level for the high cost bracket. Contrary to the CAPM-model the excess returns of each bracket is now also in line with our expectations. Seeing as the lowest alpha belongs to the low cost bracket and the highest alpha belongs to the high cost bracket.

5.4.3 Five-Factor Model cost divided bracket

Table 8: Five-Factor Bracket

	α	β_M	β_{SMB}	β_{HML}	β_{PR1YR}	β_{LIQ}	R^2
Low	0.001225$^{\cdot}$	0.949*	0.125***	-0.009	-0.021	-0.049*	0.97
Medium	0.001578	0.950	0.253***	0.004	-0.001	-0.092*	0.94
High	0.003044*	0.933	0.363***	0.003	-0.025	-0.070	0.89

For the last bracket regression we added the remaining PR1YR-factor & LIQ-factor. The addition of these factors show an increase in significance of both the high cost alpha and the low cost alpha, while the medium cost alpha remains insignificant. The increased significance of the high cost alpha further solidifies the proof that the high management cost of some funds do indeed translate to higher excess returns not explained by exposure to systematic risk-factors.

5.4.4 Bracket correlation

Table 9: Bracket Correlation

	Low	Medium	High	OBX
Low	1	0.988	0.967	0.992
Medium	0.988	1	0.984	0.974
High	0.967	0.984	1	0.944
OBX	0.992	0.974	0.944	1

In order to further solidify the justifications for higher fund management costs, we expect to find less correlation with the index as we look at the individual brackets ranging from low price to high price. We also expect the low cost bracket to be highly correlated with the OBX, seeing as the low cost bracket is composed of several index- and near index-funds. As is evident from our computations in the table above, the index correlation decreases as the management cost increases, which contributes to justifying the higher management costs for the mutual funds included in our selection. Further studies could be conducted to reinforce this conclusion by specifying even more price brackets, in order to contribute with further depth and nuance.

5.5 Statistical evaluation

In our analysis we have utilized cohesive return history for every mutual fund. We have consciously selected a time period in which all the included funds have existed, thus eliminating any skewness that could arise due to different start and end dates for the funds. In addition to this, the time period has not seen any economic crises of a high magnitude which should contribute to enhancing the applicability of the models and underlying theory.

We have conducted several statistical evaluations and tests in order to solidify our results, and to uncover areas that are of particular risk or influence to our results.

5.5.1 Breusch-Godfrey test

Table 10: Breusch-Godfrey-test

Funds	X^2	p-value
Alfred Berg Aktiv	9.196	0.6860
Alfred Berg Gambak	4.228	0.9789
Alfred Berg Humanfond	14.998	0.2415
Alfred Berg Indeks I	14.129	0.2925
Alfred Berg Norge C	11.119	0.5187
C WorldWide Norge	21.022	0.0500*
Danske Invest Norge Aksj. Inst 2	10.756	0.5498
Danske Invest Norge I	10.956	0.5326
Danske Invest Norge II	11.830	0.4593
Danske Invest Norge Vekst	11.257	0.5070
Delphi Norge	19.641	0.0741*
DNB Norge D	11.120	0.5186
DNB Norge Selektiv E	19.692	0.0731*
DNB SMB A	11.063	0.5234
Eika Norge	10.156	0.6021
Fondsfinans Norge	4.804	0.9642
Handelsbanken Norge	16.179	0.1831
Holberg Norge A	14.874	0.2483
KLP AksjeNorge Indeks	15.960	0.1930
KLP AksjeNorge Indeks II	16.170	0.1835
Nordea Avkastning	9.371	0.6709
Nordea Kapital	12.275	0.4238
Nordea Norge Verdi	11.217	0.5104
Pareto Aksje Norge A	12.314	0.4207
Pareto Aksje Norge B	11.318	0.5018
Pareto Aksje Norge I	11.387	0.4960
PLUSS Aksje	11.753	0.4656
PLUSS Markedsverdi	20.034	0.0664*
Storebrand Aksje Innland	7.570	0.8176
Storebrand Norge	11.246	0.5079
Storebrand Norge I	14.616	0.2630
Storebrand Vekst	11.404	0.4946
Storebrand Verdi A	5.953	0.9184
XACT OBX	40.167	0.0000***

The table shown above displays the results of our Breusch-Godfrey test. The goal of this test is to check whether or not there exists any form of serial correlation within our data. The test was performed by testing for 12 periods for each and every fund. The value X^2 is distributed according to the chi-square distribution. This means that if we want to make sure there is no serial correlation at a 5% significance level, we have to make sure the $X^2 > 21,026$ and that the $p - value < 0,05$. While keeping these values in mind we can see that only a single fund contains serial correlation. This suggests that our OLS regression is efficient enough for this use case

5.5.2 VIF test for multicollinearity

Table 11: VIF-test

Factors	VIF
OBX	2.6822
SMB	1.2524
HML	1.2147
PR1YR	1.1309
LIQ	2.3946

In subsection 4.3.6 we elaborated on the specifics of the VIF, as well as what thresholds we would use to determine if our factors had too high multicollinearity or not. As previously stated we set this threshold to a VIF value of 4 as suggested by Salmerón et al.(2018). When performing our VIF test we observed our highest VIF to be a value of 2.6 which is way below that of our threshold, signifying that we are indeed without significant amounts of multicollinearity. To further emphasize this we can use the VIF equation introduced in subsection 4.3.6 with respect to R^2. By using the VIF results in the table above we are then able to calculate the corresponding R^2 value which further measures how well the variation of the variables are explained by the other variables. In this case the VIF of the OBX factor corresponds to an R^2 of 62.7%, which means that the majority of the factor variation remains unexplained by the other factors. Thus, leaving us without any problems related to the phenomenon of multicollinearity.

5.5.3 Factor correlation matrix

Table 12: Factor correlation(VIF)

	OBX	SMB	HML	PR1YR	LIQ
OBX	1	-0.420	0.291	-0.175	-0.740
SMB	-0.420	1	-0.204	0.017	0.376
HML	0.291	-0.204	1	-0.282	-0.085
PR1YR	-0.175	0.017	-0.282	1	-0.002
LIQ	-0.740	0.376	-0.085	-0.002	1

When having computed the related VIF for each of the factors implemented in our models, we also made a correlation matrix as seen above. The reason behind constructing this correlation matrix was rooted in the fact that a VIF test merely shows to what degree a factor is being explained by its cofactors, without any explanation as to which factors are closely linked. While it is possible to draw assumptions based on the fact that only two of the factors were above a VIF of 2, it would be of interest to verify this by other means as well.

As a result we can see that the OBX factor and LIQ factor are closely inversely correlated to each other as shown in our correlation matrix. This combined with the previous results showcased in the VIF-table we can with reasonable certainty conclude that these two factors are explaining each other to some degree. As was mentioned in section 2.8 Næs et al. (2009) suggested that an abundance of companies on the Norwegian index have concentrated ownership by the Norwegian state, effectively leaving the majority of shares out of normal circulation. This would indeed mean that most of the companies on the index are of low liquidity, something which our results further support.

6 Conclusion

There exists widespread consensus that exposure to systematic risk factors should yield a premium in terms of asset returns. Several renowned models have emerged over the years that attempt to price systematic risk exposure accurately. In this thesis we utilize three models as basis for conducting regressions, these are the CAPM, the Fama & French Three-factor model and lastly the Five factor-model initially based on Carharts momentum-factor and Ibbotsons liquidity factor. We regard all returns that are not captured by the models to be excess returns. Excess returns are often viewed as a fund manager's level of skill or competence, and thus describes their ability to achieve greater returns in relation to assumed risk.

We evaluate to what degree traditional factors can explain the returns of mutual funds in the Norwegian market. In order to evaluate this further we have formulated two hypotheses to generate conclusive results. Proceedingly, we evaluate our findings in light of the articles included in the literature review section, in order to reinforce or dispute their conclusions.

6.1 Hypothesis 1

"Traditional factors explain a large portion of the returns of Norwegian mutual funds."

We observe that as we increase the model complexity and add more systematic risk factors, the significance of the market betas observed with the CAPM regressions are steadily reduced. This indicates that a substantial part of the mutual funds returns can be attributed to exposure to systematic risk factors that are priced in the market, rather than being attributable to great individual stock picks. Furthermore we see that the degree of explained variance increases with the model complexity, implying that the more complex factor models are well suited for capturing the variance of the returns. Most notably, we find that with the five-factor model, two significant alpha-values emerge that were not evident with the CAPM. This discovery suggests that there exists a possibility to achieve risk-adjusted excess returns, or at least that there exists undiscovered adjustments and/or additions to the traditional models which can further improve the explanatory power and reduce the emergent significant alphas.

In conclusion, traditional factor models explain a substantial portion of the returns of the Norwegian mutual funds included in our selection. Furthermore we find that the vast majority of the funds exhibit an alpha value not significantly different from zero. With regards to the individual factors included in our regressions, we find that the size- and liquidity-premiums explain a substantial portion of the returns, implying that funds are largely invested in small companies and liquid companies. In line with Næs et al.(2009), we find through our research that the value- and momentum-factors are less relevant in the Norwegian market, and the returns of the funds in our selection are less attributable to these factors.

6.2 Hypothesis 2

"Higher management fees are justified by higher excess returns and less explainability by traditional factors."

We observe that mutual funds with higher management fees have a lower degree of explainability, when analyzed with traditional factor models. This is suggestive of the fund managers being able to apply something unique, that is not attributable to priced systematic risk factors. Furthermore we find that an alpha value with statistical significance emerges for the high-cost bracket which is suggestive of the fund managers ability of creating risk-adjusted excess returns. Both of these discoveries help to justify higher priced Norwegian mutual funds, however adjusted for all costs and compounded returns it is unlikely that higher-priced fund alternatives prove to be significantly better than lower-priced mutual funds.

The results are in line with the conclusions of Goyenko et al. (2013), with regards to a lower R^2 - value corresponding with a higher and more statistically significant alpha-value. This finding suggests that a lower R^2 - value could indicate more room for generating excess returns in mutual funds. Thus, it would imply that there could exist an inverse relationship between R^2 and alpha. It would be interesting to test this observation further on a broader and larger selection of mutual funds.

6.3 Suggestions for further research

While we've determined the funds performance after subtracting any expenses related to the expense ratio, there is still some uncertainty as to what portion of the resulting alpha can be attributed to the individual fund manager's actual skill. Seeing as the NOK has depreciated since 2015, some of the alpha could in fact be attributed to a weakening of the NOK value given that a fund holds assets in either USD or EUR. By not incorporating this effect into the study we run the risk of some of the resulting currency return being incorporated into the fund alpha. With this in mind, we would suggest future studies to incorporate the currency return into their work in order to control this effect as well as it would be interesting to investigate how this effect influences the fund returns.

A common conception is that Norwegian equities are subject to a change in oil prices, therefore it would also be interesting to incorporate the oil price as a priced risk-factor. Næs et al.(2009) touched on this subject in their working paper, however that was in the previous decade and did not assess its effect on mutual funds.

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