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Structural breaks & volatility spillover: Effects on the Norwegian financial market

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Abstract

This paper investigates the structural breaks in and volatility spillover between Norwegian and several international indices with ties to Norway. Daily returns from 2000 to 2020 of the two Norwegian indices OSEBX and OSESX, as well as indices from the US, the UK, Germany, France, Sweden and Denmark, are analyzed through the use of the CCC-GARCH (Bollerslev, 1990), the DCC-GARCH (Engle, 2002) and the BEKK-GARCH (Engle & Kroner, 1995). By applying the Iterated Cumulative Sums of Squares (ICSS) algorithm (Inclan & Tiao, 1994) on the time series data, we detect multiple structural breaks in all aforementioned indices.

From the DCC-GARCH(1,1) we find evidence of a decline in correlation between the Norwegian index OSEBX and other indices during structural breaks. From our results of the BEKK-GARCH(1,1) model we find evidence of volatility spillover from several international indices to the Norwegian OSEBX; as well as structural breaks in other indices affecting the volatility in OSEBX. Most controversial, we find strong evidence that volatility spillover between OSEBX/FTSE and OSEBX/DAX have a unidirectional relationship from OSEBX to FTSE/DAX. The occurrence in OSEBX/FTSE can be explained by the oil price dependence on the global economy. The relationship between OSEBX/DAX, however, can not be explained by the oil price.

Acknowledgement

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Summary

Denne masteroppgaven utforsker strukturelle brudd i, og volatilitetsoverføring fra og mot det Norske finansmarkedet. Vi har brukt daglig avkastning fra 2000 til 2020 for de to norske indeksene OSEBX og OSESX, i tillegg til indekser fra USA, Storbritania, Tyskland, Frankrike, Sverige og Danmark. Tidsseriene er analysert gjennom bruk av CCC-GARCH (Bollerslev, 1990), DCC-GARCH (Engle, 2002), og BEKK-GARCH (Engle & Kroner, 1995). Ved å applikere Iterated Cumulative Sums of Squares (ICSS) algoritmen (Inclan & Tiao, 1994) har vi funnet flere strukturelle brudd i tidsseriene til de nevnte indeksene.

Fra DCC-GARCH modellen finner vi bevis for en nedgang i korrelasjon mellom OSEBX og andre indekser under strukturelle brudd. Fra resultatene gitt av BEKK-GARCH modellen finner vi bevis for volatilitetsoverføring fra flere internasjonale indekser til den norske OSEBX indeksen, i tillegg til at strukturelle brudd i andre indekser påvirker volatiliteten i OSEBX. Det mest kontroversielle funnet i denne avhandlingen, er et sterkt empirisk bevis for at volatilitets overføringen mellom OSEBX/FTSE og OSEBX/DAX er et enveis forhold fra OSEBX til FTSE/DAX. Begrunnelsen for funnet i OSEBX/FTSE kan forklares av oljepris-avhengigheten i den globale økonomien, noe som ikke gjelder for OSEBX/DAX.

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1 Introduction

1.1 Background

The dynamic relationships between international financial markets hold interest for investors, academics, and policymakers. This thesis seeks to investigate the effects of volatility spillover towards the Norwegian stock market. We consider some of the main indices from Norway's largest trading partners - Sweden, Denmark, the US, the UK, France and Germany. In addition we include a Norwegian index tracking companies with lower market capitalization to research the domestic spillover between high cap and low cap companies, as well as the Brent crude oil because of its effect on the Norwegian economy. We have adopted a BEKK-GARCH(1,1) model, a CCC-GARCH(1,1) model and a DCC-GARCH(1,1) model, to analyze the volatility spillover between the chosen markets.

The global financial markets essentially consist of complex financial networks, which become more interconnected during crises (Lai & Hu, 2021). Because major economies such as the United States and the United Kingdom hold a central position in the global financial markets, they may spread crisis or volatility to other parts of the network, especially in high volatility periods. Through the time span between February and March 2020 we saw a major decrease in the Norwegian stock market, as well as the international stock markets in general, resulting in a decrease of over 30% for the Oslo Børs Benchmark Index (OSEBX). Engelhardt et.al. (2020) found that the increase in COVID-19 announcements led to higher volatility in financial markets, and that the market´s reactions depend on the level of trust in the countries they are analyzed. They found that high trust is related to lower impact from COVID-19 on the volatility (Engelhardt, Krause, Neukirchen & Posch, 2020). As a result of the pandemic central banks across the globe were forced to take action, which for the Norwegian Central bank was to decrease interest rates. The action taken can imply a structural break if the change goes against the previous changes and trends in interest rate changes. Structural breaks or shocks in the market as well as the spurious volatility spillover effect is both statistically significant, and has substantial economic implications in terms of hedging (Caporin & Malik 2020).

Several studies have investigated volatility spillover in financial markets both within and between countries, and a significant number of these studies use univariate and multivariate GARCH-models for investigating the mentioned effect. This thesis will analyze the volatility and return spillover from Sweden, Denmark, the US, the UK, France and Germany, to Norway, as well as Norway's spillover to the aforementioned countries. Through a review of the existing literature on this topic we did not find any studies including these countries with a focus on spillover to the Norwegian financial markets, and our thesis will hopefully be part of filling this gap.

1.2 Problem statement

"How are the Norwegian stock market affected by structural breaks and volatility spillover?"

To answer our research question we intend to use multivariate GARCH-models to explore the possibility that structural breaks in volatility can cause volatility spillover. We intend to use CCC- and DCC-GARCH models to explore the significance of time variability in our time series, and use the BEKK-GARCH to explore the significance of structural breaks and volatility spillover.

1.3 Hypothesis

The main objective of this thesis is to investigate the effect structural breaks and spillover effects have on the Norwegian stock index, OSEBX. Whether the structural breaks or volatility spillover affects the returns of OSEBX or another market in comparison. Norway's economy is severely influenced by the countries biggest export, oil (Statistisk sentralbyrå [SSB], 2018). We believe this influence can show itself when investigating the relationship between certain indices and OSEBX. If we encounter this problem, further analysis is required.

H_1: *Structural breaks in one index affect the returns in another index.*

H_2: *Spillover effects from one index affect the returns in another index.*

2 Literature review

Volatility in the financial markets have been researched quite extensively from the 1950´s until today. Markowitz (1952) addressed the issues of diversifying the assets included in a portfolio to obtain lower volatility overall. Since then, the factors driving volatility as well as the possibility of volatility transmission or spillover have increased its popularity both in practice and academically. Morgenstern (1959) was one of the first to research the modeling of volatility spillover, and in the last 20 years, over 10.000 scientific articles on this theme have been published in peer-reviewed journals. Naturally, as volatility transmission may be caused by a crisis, and the world being in a pandemic, the number of articles on this topic increased in 2020.

In terms of researching volatility spillover, Granger (1969), Granger (1980), was the first to provide a model by regressing the squared residuals of variables. Hong (2001) further improved Granger´s model with a class of new tests, by including a standardized version of a weighted sum of squared sample cross-correlations between two squared standardized residuals. Further, through the use of the extensive model, the researcher found that for causality in variance, there exists strong simultaneous interactions between the Japanese Yen and the Deutsche Mark, as the Deutsche Mark volatility caused a change in the Japanese Yen volatility.

Engle and Susmel (1993) investigated if international stock markets shared the same volatility process. Using an univariate Autoregressive Conditional Heteroskedasticity model (ARCH-model) they found that there are groupings of stock markets sharing the same time-varying volatility. Norway shared the same characteristics as Germany, Belgium and Sweden, resulting in the possibility of a volatility spillover from one of these countries to the Norwegian market.

Ewing and Malik (2005) explores the asymmetry in the predictability of the volatilities of large cap stocks vs small cap stocks and how it allows for sudden changes in variance. Taking advantage of the recent advances in time series econometrics (at the time), they used the ICSS (iterated calculated sums of squares) algorithm to detect the time periods of sudden change in volatility of large and small cap stocks. And further use to implement that information in a Bivariate Generalized Autoregressive Conditional Heteroskedasticity model (GARCH-model), which are different to the univariate model due to the inclusion of multiple variables. Their findings indicate that the volatility transmission and spillover effects are reduced if we account for volatility shifts.

van Dijk, Osborn and Sensier (2005) investigated whether structural breaks affect the appearance of volatility spillover effects. Demonstrated through the use of Monte Carlo simulations they found that if breaks are neglected, the causality-in-variance tests will suffer from severe size distortions. To conclude, the authors found that size problems arise particularly when their two time series exhibit volatility changes in close temporal proximity, resulting in an incorrect attribute to the occurrence of an underlying causality. They further recommend that the causality-in-variance tests should be applied only after pre-testing for breaks in volatility.

Ewing and Malik (2010) explores how shocks affect the volatility of oil prices over time.

By incorporating endogenously determined structural breaks into a GARCH model they accurately estimate the volatility persistence in oil prices under structural breaks.They show that oil shocks have a strong initial impact, but dissipate rather quickly. Their findings contradicted previous research on the topic. Their research is found useful and important for hedging decisions and derivative valuation.

Allen, Amram and McAleer (2013) investigates whether there is evidence of volatility spillover from the Chinese stock market to its trading partners. They use a number of variants of GARCH to test for constant conditional correlations and spillover in volatility. Their findings show evidence of volatility spillover across the markets before the financial crisis in 2008. However, after the finance crisis they find little evidence of the presence of spillover in volatility across the markets compared.

Caporin and Malik (2020) use extensive Monte Carlo simulations and bivariate GARCH models to test if the effects of spurious volatility transmissions actually are significant. Through their simulation they find that the spillover effect is statistically significant, and further that it has substantial economic implications in terms of hedging financial investments as the breaks in volatility change the average estimated hedge ratios. The researchers further state that other studies have ignored the frequent occurrence of volatility shifts, and conclude that their empirical findings may be deficient due to lack of including this.

3 Methodology

In the following chapter the economic theory that will be used in our thesis will be presented.

3.1 Volatility

To obtain a better understanding of the financial markets we have to include two main aspects. First, we need to understand expected return on the assets included in the market, and how these returns are dispersed around the mean. Dispersion is the variability around the central tendency which addresses the riskiness of an asset, also defined as volatility (DeFusco, McLeavey, Pinto & Runkle, 2015). Typically, the variance and standard deviation are the most used measurements on volatility by investors. Variance is defined as the average of the squared deviations around the mean, while standard deviation is the square root of variance (DeFusco et.al,2015). Second we need to know what factors drive volatility. Aggarwal, Inclan and Leal (1999) found that global and local events are causing shifts in emerging markets´ volatility. Through extensive research they found that factors such as political, social and economic events, were the main drivers of shifting volatility. For the sake of our thesis, these factors may also be a driver with regards to creating breaks or shocks in the market, and will be further discussed later on.

3.1.1 Structural breaks

It is often assumed that coefficients in a model are constant. More sophisticated models allow parameter estimates over time. Changes in how organizations, individuals or even governments interactions frequently occur and these interactions may change the correlation structure between the variables in the model. For example, when COVID-19 caused a lockdown in Norway, the central bank decided to lower its interest rates, creating repercussions for both individuals and organizations. We call these changes structural breaks (Bjørnland & Thorsund, 2015). As the structural breaks can cause shifts in estimation parameters, it can result in misleading estimation results.

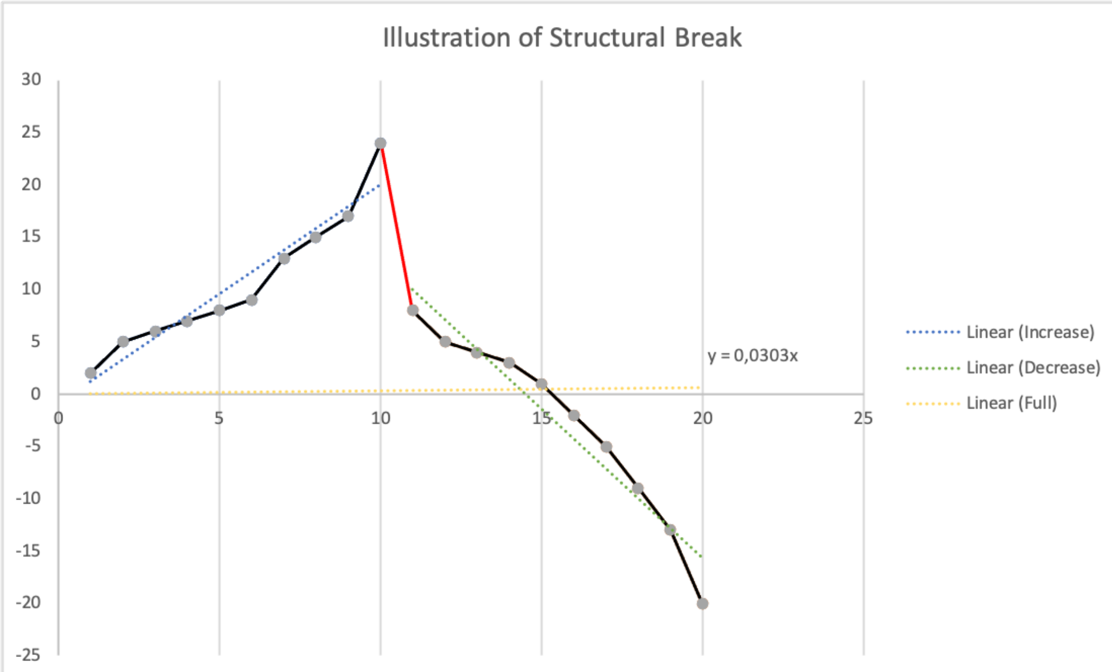

Figure 1: Illustration of a structural break

A possible structural break during visible trends is illustrated in Figure 1. We can see a positive trend in the first period, the structural break, and lastly a negative trend. A structural break in volatility would be fairly similar to our illustration, however preferably with a larger dataset or more observations and no negative values for the volatility itself. If we know exactly when the break occurred we can split the dataset into two time periods, one before and one after the break. In this simple example it would be to split the dataset into the first ten observations, and the last ten observations. The problem with this procedure is that we do not know exactly when the break happens, but it can be answered when a structural break occurred; we can use statistical testing as well as performing a split of the dataset.

3.1.2 Volatility transmission/spillover

Volatility is often related to the rate of information flow, and if this comes in clusters it may result in an exhibit in volatility in asset prices or returns, even if the market adjusts to the news perfectly (Ross, 1989). Therefore, the study on volatility spillover can help understand how information is transmitted across assets and markets. If there is an absence of volatility transmission, it implies that the sources of disturbances are changes in asset or market fundamentals, and the shock increases the volatility in one asset or market alone. As for the existence of volatility transmission, it implies that one shock causes an increase in the volatility for several markets or assets (Hong, 2001).

3.1.3 ICSS-Algorithm

The procedure of an Iterated Cumulative Sums of Squares (ICSS) is used to detect the number of significant sudden changes in variance in a time series, as well as estimating the time point and magnitude of each detected sudden change in the variance (Aggarwal et.al., 1999). The algorithm detects both increases and decreases in the variance, and the results provide output for which observation in the time series where the breakpoint is detected. The high volatility characteristics of emerging markets is recognized by frequent, sudden changes in variance, or breaks. These breaks are often associated with important events in each country rather than global events, and are therefore also possible to detect in developed financial markets. The algorithm was initially created by Inclan and Tiao (1994), as they found that there were series, particularly in the area of finance, that do not follow the usual assumption of constant variance underlying most models for time series.

3.1.3.1 Centered cumulative sums of squares

The main idea of the ICSS algorithm was to research the variance of a given sequence of observations retrospectively, so they could use all the information on the series to indicate the points of variance change. For indicating a single break, let $Ck = \sum \alpha_t^2$ be the cumulative sum of squares of a series of uncorrelated random variables α_T with a mean 0 and variances $\sigma^2, t = 1, 2, ...T$. Let

$$D_k = \frac{C_k}{C_t} - \frac{k}{T}, k = 1, ..., T., with : D_0 = D_T = 0 \tag{1}$$

be the centered and normalized cumulative sum of squares (Inclan & Tiao, 1994). The plot of D_k against k will oscillate around 0 for series with homogeneous variance. When there is a sudden change in variance, the plot of D_k will exhibit a pattern going out of some specified boundaries with high probability. These boundaries can be obtained from the asymptotic distribution of D_k assuming constant variance (Inclan & Tiao, 1994).

3.1.3.2 Multiple changes

Iterated Cumulative Sums of Squares Note that for indicating the possible existence of a single point of change, the D_k function would provide a satisfactory procedure. We however are interested in indicating several points, which makes the D_k function questionable due to the masking effect. Therefore we have to use the Iterated Cumulative Sums of Squares over the previous Centered Cumulative Sums of Squares. A solution, also provided by Inclan and Tiao (1994), is an iterative scheme based on successive application of D_k to pieces of the series, dividing consecutively after a possible break is found. This procedure proposes to look for breaks in order to isolate each point systematically. Inclan and Tiao (1994) provides the following steps for the application of the ICSS algorithm with regards to detecting multiple changes.

Step 0: Let $t_1 = 1$.

Step 1: Calculate $D_k(a[t_1 : T])$ and let $k*(a[t_1 : T])$ be the points at which $MAX_k|D_k(a[t_1 : T])|$ is obtained, and let

$$M(t_1 : T) = max\sqrt{\frac{T - t_1 + 1}{2}}|D_k(a[t_1 : T])| \tag{2}$$

If $M(t_1 : T) > D^*$ consider that there is a changepoint at $k*(a[t_1 : T])$ and proceed to Step 2a. The value of D^* is $D_1^* - p$ for the desired value of p, usually p = 0.95. If $M(t1 : T) < D^*$, there is no evidence of variance changes in the series, and the algorithm stops.

Step 2a: Let $t_2 = k*(a[t_1 : T])$. Evaluate $D_k(a[t1 : t2])$; that is, the centered cumulative sum of squares applied only to the beginning of the series up to t_2. If $M(t_1 : t_2) > D^*$, then we have a new point of change and should repeat Step 2a until $M(t_1 : t_2) < D^*$. When this occurs we can say that there is no evidence of breaks in variance in $t = t_1, ..., t_2$ and, therefore, the first point of change is $k_{first} = t2$.

Step 2b: Now do a similar search starting from the first changepoint found in Step 1, toward the end of the series. Define a new value for t_1. let

$$t_1 = k*(a[t_1 : T]) + 1 \tag{3}$$

Evaluate $D_k(a[1 : T])$ and repeat Step 2b until $M(t_1 : T) < D^*$. Let $k_{last} = t_1 - 1$.

Step 2c: If $k_{first} = k_{last}$, there is just one changepoint. The algorithm stops there. If $k_{first} < k_{last}$, keep both values as possible changepoints and repeat Step 1 and Step 2 on the middle part of the series, that is, $t_1 = k_{first} + 1$ and $T = k_{last}$. Each time that Steps 2a and 2b are repeated, the result can be one or two more points. Call N_T the number of changepoints found so far.

Step 3: If there are two or more possible changepoints, make sure they are in increasing order. Let cp be the vector of all the possible changepoints found so far. Define the two extreme values $cp_0 = 0$ and $cp_{nt+1} = T$. Check each possible changepoints by calculating

$$D_k(a[cp_{j-1} + 1 : cp_j + 1]), \qquad j = 1, ..., N_T \tag{4}$$

If

$$M(cp_{j-1} + 1 : cp_{j+1}) > D^* \tag{5}$$

then keep the point; otherwise, eliminate it. Repeat Step 3 until the number of changepoints does not change and the points found in each new pass are "close" to those on the previous pass (Inclan & Tiao, 1994).

3.2 Time Series Models

Time series is a sequence of numerical data points in a timely order, that can be used to explain the past or predict the future (DeFusco et.al 2015). These series of data points can be used in a variety of fields. For example in economics we are exposed to daily stock market quotations or unemployment figures. Social scientists follow population series such as birth rates or school enrollments. An epidemiologist in today's climate is interested in the number of COVID-19 cases over a time period, and so on (Shumway and Stoffer, 2017). As an illustration we have included a time series plot of the Oslo Børs Benchmark Index (OSEBX).

Figure 2: Time series of OSEBX prices

Figure 2 shows an annual visualization of the OSEBX prices from 2000 until 2020.

The primary objective of time series analysis is to develop mathematical models that provide plausible descriptions for sample data (Shumway and Stoffer, 2017). For the case of OSEBX in Figure 2, a mathematical model could hypothetically describe why the price movement has had an overall increase.

However, as many results in econometrics and statistics depend on having many observations, we should not think too much of the sample itself, but instead consider the number of observations as more important. When analyzing stocks it is preferred to use daily data, or data with high frequency, to obtain a broader basis of data and conduct a more thorough analysis (Bjørnland & Thorsund, 2015).

3.2.1 ARCH/GARCH Models

Working with financial data, such as indices, there are several precautions the scientist must take into consideration. The Autoregressive Conditional Heteroskedasticity Model takes issues like volatility clustering/volatility pooling into consideration, that being described as the phenomenon of large(or small) changes in asset prices to follow large(or small) changes. The current level of volatility tends to follow the trend of previous periods (Brooks, 2008). Historically, a simple method of estimating volatility have been widely used. Historical volatility is computed with the sample standard deviation over a short period of time. This raises several issues such as the length of period the standard deviation should be sampled from; too short and it is too noisy, too long and it may not be relevant (Engle, 2004).

Volatility itself is risk over a future time period, therefore a forecast is a prediction of future volatility and a measurement of volatility today. The initial assumption that volatility was constant (homoskedastic). Engle's (1982) ARCH and its extensions however was under the assumption that volatility behaved dynamically (heteroskedastic). It is logically inconsistent to assume that volatility is constant for any given time (Engle, 2004).

For economic applications, the ARCH model is useful where the underlying forecast variance may differ over time and is predicted by earlier forecast errors (Engle, 1982). Where portfolios of financial assets are based on the variance and expected means of the return, any shifts in asset demand must be connected to changes in the variance and expected means of the return. Here the use of an exogenous variable to understand the changes in variance is not appropriate (Engle, 1982). The ARCH is for instance used in Engle (1982) and Engle (1983) to construct models for the inflation rate in the U.K. and the US, as the inflation tends to differ over time (Bollerslev, 1986).

Under ARCH, the equation for conditional mean, y_t, describes the changes in the dependent variable over time. The conditional mean equation could take any form the scientist wishes. One example of a full ARCH model would be (Brooks, 2008):

$$y_t = \beta_1 + \beta_2 x_{2,t} + \beta_3 x_{3,t} + \beta_4 x_{4,t} + u_t \qquad u_t \sim N(0, \sigma_t^2) \tag{6}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \tag{7}$$

Where y_t is the equation for the conditional mean and σ_t is the equation for the conditional variance.

An extension of the ARCH model, the GARCH (Generalized ARCH) model, was developed by Bollerslev (1986). Where the ARCH allows the conditional variance to differ over time as a function of past errors leaving the unconditional variance constant (Bollerslev, 1986); the GARCH allows the conditional variance to be dependent upon previous own lags (Brooks, 2008). The equation for the conditional variance is then:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{8}$$

For the univariate GARCH, there is a broad academic understanding that given a correct model specification and a large enough sample, the GARCH is enabling researchers to conduct statistical interference with a reasonable amount of confidence (Comte & Liebermann, 2003).

3.2.2 CCC model Bollerslev

Bollerslev (1990) proposed a model that had time varying conditional variances and covariances, with constant conditional correlation. The Constant Conditional Correlation-GARCH model is a generalization of the constant conditional correlation-ARCH model by Cecchetti, Cumby, and Figlewski of 1988 (He & Teräsvirta, 2004). The CCC-GARCH is a model in the class of "Models of conditional variances and correlations." In this class the covariance matrix, H_t, can be broken down into D_t and R_t, which is the conditional standard deviations and a correlation matrix, respectively. The conditional correlation matrix in the CCC-GARCH is time invariant, i.e. $R_t = R$.

CCC-GARCH can therefore be expressed as:

$$H_t = D_t R D_t \tag{9}$$

where D_t is an N x N stochastic diagonal matrix which contains the elements $\sigma_{i,t}$. D_t follows a univariate GARCH process. R is the N x N conditional correlation matrix. It follows that H_t will be positive definite for all t if R is positive definite and conditional variances are well defined (Bollerslev, 1990).

In our use of the CCC-GARCH model we will use a modified DCC Copula GARCH where we remove the time-varying properties in the conditional correlation and make it constant. A copula is a multivariate distribution function whose one-dimensional margins are uniform on an interval from 0 to 1 (Nelsen, 2007). Due to limitations in selection of packages and lack of modifiability in said packages in R, we ended up using a modified copula version of the DCC-GARCH with the time varying aspect set to null.

3.2.3 DCC-GARCH

The Dynamic Conditional Correlation GARCH Model (DCC-GARCH) was introduced by Engle and Shepard (2001) as an extension to Bollerslev's (1990) CCC-GARCH model. The DCC-model is used to capture the degree of volatility correlation changes or spillover between two or more variables. The DCC- is, as the CCC-model, in the class of "Models of conditional variances and correlations." Since the conditional correlation is designed to be dynamic in the DCC-model, both D_t and R_t are time varying.

DCC-GARCH is expressed as:

$$H_t = D_t R_t D_t \tag{10}$$

where D_t is the N x N diagonal matrix of the conditional standard deviation and R_t is the N x N conditional correlation matrix. Engle (2002) states that two requirements have to be fulfilled when specifying the form of R_t:

1. H_t has to be positive definite. To ensure that H_t is positive definite, R_t also has to be positive definite.
2. The elements of the correlation matrix R_t need to be equal or less than one.

To ensure that both requirements in the DCC-model are met, R_t can be decomposed to:

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{11}$$

Following:

$$Q_t = (1 - \alpha - \beta)\bar{Q} + \alpha \varepsilon_{t-1} \varepsilon_{t-1}^T + \beta Q_{t-1} \tag{12}$$

where α and β non-negative scalars which ensures the positive definiteness of Q_0, and in turn H_t positive definiteness. $\overline{Q}$ is the unconditional covariance matrix of the standardized errors that can be depicted as $Cov[\epsilon_t \epsilon_t^T]$.

Q_t^{*-1} is the inverted N x N diagonal matrix that includes the square root of the diagonal elements of matrix Q_t.

This results in the correlation structure of the DCC-GARCH model (Engle, 2002):

$$Q_t = (1 - \sum_{i=1}^{P} \alpha_i - \sum_{j=1}^{Q} \beta_j)\bar{Q} + \sum_{i=1}^{P} \alpha_i \varepsilon_{t-1}\varepsilon_{t-1}^T + \sum_{j=1}^{Q} \beta_j Q_{t-j} \tag{13}$$

3.2.4 BEKK model

The BEKK model is an extension to Bollerslev's (1986) GARCH model by Engle and Kroner (1995). The BEKK model is under the class of multivariate GARCH models, which means it involves more than two variables. The BEKK aims to parameterize the multivariate process to ensure that positive definiteness is happening as well as allowing complicated interactions among the variables (Engle & Kroner, 1995). For instance, shocks could provide a spillover effect on another market. Positive definiteness in this case means that the variance-covariance matrix will have positive digits on the leading diagonal (Brooks, 2008). The variance can never be negative, and the covariance will always be the same regardless to which of the series is taken first. This is what positive definiteness ensures from a mathematical standpoint. Other multivariate GARCH models, such as the VECH, struggle with ensuring that the covariance matrix always is positive definite (Brooks, 2008). This is why in some cases, such as from a risk management point of view, the BEKK is superior. One example of a BEKK model would be:

$$H_t = C_0 C_0^* + \sum_{k=1}^{k}\sum_{i=1}^{i} A_{ik}^* \varepsilon_{t-1}\varepsilon_{t-1}^* A_{ik} + \sum_{k=1}^{k}\sum_{i=1}^{i} B_{ik}^* H_{t-1} B_{ik} \tag{14}$$

Where H_t is the conditional covariance matrix and C_0 is the N x N upper triangular matrix. Positive definiteness is guaranteed in the covariance matrix as long as $C_0C_0^*$ is positive definite. Element A_{ik} of the N x N matrix A reflects the ARCH effect on volatility as well as indicating the impact of market i volatility on market k. Element B_{ik} of the N x N matrix B reflects on the GARCH effect of volatility as well as indicating the persistence of volatility transmission between market i and k (Kumar, 2013). As for our thesis, we plan to run a bi-variate BEKK-GARCH (1,1). The model can be written as:

$$\begin{aligned}\begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} &= \begin{bmatrix} c_{11,t} & c_{12,t} \\ c_{21,t} & c_{22,t} \end{bmatrix}' \begin{bmatrix} c_{11,t} & c_{12,t} \\ c_{21,t} & c_{22,t} \end{bmatrix} \\ &+ \begin{bmatrix} a_{11,t} & a_{12,t} \\ a_{21,t} & a_{22,t} \end{bmatrix}' \begin{bmatrix} \epsilon_{1,t-1}^2 & \epsilon_{1,t-1}, \epsilon_{2,t-1} \\ \epsilon_{2,t-1}, \epsilon_{1,t-1} & \epsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11,t} & a_{12,t} \\ a_{21,t} & a_{22,t} \end{bmatrix} \\ &+ \begin{bmatrix} b_{11,t} & b_{12,t} \\ b_{21,t} & b_{22,t} \end{bmatrix}' \begin{bmatrix} b_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11,t} & b_{12,t} \\ b_{21,t} & b_{22,t} \end{bmatrix}\end{aligned} \tag{15}$$

where $h_{11,t}$ and $h_{22,t}$ represents the variance of change rate of the stock index returns, $h_{12,t}$ represents the covariance of the change rate of two stock index returns. When testing for the volatility spillover effects from one index to another, the coefficients a_{12}, a_{21}, b_{12} and b_{21} are tested to be statistically different from zero. If the non-diagonal elements of matrices A and B are not significantly different from zero, there is no evidence of volatility spillover effects between the indices (Kumar, 2013).

3.3 Portfolio theory

According to Markowitz (1952) the process of selecting a portfolio can be divided into two stages. The first stage starts with observation and research of assets and ends with impressions about future performances. The second stage starts with those impressions about future performances and ends with the choice of portfolio. Asset allocation accounts for the variability in return of a portfolio. That is why the optimal asset allocation is perhaps the single, most important factor when constructing a portfolio or diversifying wealth (Sharpe, 1992). The trade-off between risk vs return is an important factor when allocating assets. The ability to adjust underlying weights in a portfolio with the goal of minimizing volatility and maximizing return.

Markowitz's (1952) Modern Portfolio Theory (MPT) involves how the risk-averse investor can create portfolios that optimizes or maximizes expected return based on a set level of market risk. The MPT argues that investment risk and return should not be interpreted separately, but rather how the investment affects the total level of portfolio risk and return. The expected portfolio return is given by:

$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i) \tag{16}$$

Where $E(r_p)$ is expected portfolio return, w_i is the weight of each security in the portfolio, and the $E(r_i)$ is the expected return on the security.

The expected portfolio variance is given by:

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i \sigma_i * w_i \sigma_i * Cor_{r_i, r_j} \tag{17}$$

Where σ_p^2 is the variance of a portfolio's expected return for the period, w_i and w_j is the weight of assets i and j, σ_i and σ_j is the standard deviation of assets i and j, and the last part is the correlation between the assets. The equation for portfolio variance and expected portfolio return does not contain any time varying properties, and therefore no opportunity of structural breaks.

4 Data description

The basis of our analysis are time-series data of main indices from two Norwegian indices, as well as indices from Norway's largest trading partners. We also included Brent Crude oil as it is a major part of the Norwegian economy. The OSEBX index is the main representative for the Norwegian Stock market, representing the largest companies in the Norwegian economy, and the OSESX index represents the low market capitalized companies in the same economy. Both the OSEBX and the OSESX time-series data were gathered from the TITLON database, with a time span from January 2000 until December 2020. The Standard & Poor 500 (S&P500) index as an indication of the US Stock Market. As an indication for the UK Stock markets, we included the Financial Times Stock Exchange 100 (FTSE100) index. Further we included the DAX and CAC index representing Germany and France respectively, as well as OMX Copenhagen and OMX Stockholm for Denmark and Sweden. The data from the US, UK, Germany and France were gathered from Yahoo Finance, and the Swedish and Danish data were gathered from Nasdaq. All these indices share the same time span as the Norwegian data, and the common denominator is that the data is in daily observations. We obtained the data for the Brent Crude oil from Refinitiv.

In the analysis, all price data is converted into daily returns by calculating the difference of the logarithmic daily closing prices.

$$R_t = log(Y_t/Y_{t-1}) \tag{18}$$

Where R_t denotes the return at time t, and Y_t denotes the weekly closing price at time t. Descriptive statistics for our dataset is presented in Table 1. All computations in our thesis are done using R.

	OSEBX	OSESX	S&P500	FTSE	DAX	CAC	OMX.S	OMX.D	Brent Oil
Mean	0.03	0.01	0.02	-0.00	0.01	0.00	0.01	0.02	0.00
Std.Dev	1.44	1.12	1.26	1.21	1.50	1.46	1.46	1.27	2.32
Min	-10.48	-11.43	-12.77	-11.51	-13.05	-13.10	-11.17	-11.72	-27.98
Q1	-0.62	-0.47	-0.46	-0.54	-0.67	-0.66	-0.70	-0.61	-1.12
Median	0.11	0.11	0.07	0.03	0.08	0.03	0.05	0.07	0.08
Q3	0.77	0.63	0.58	0.59	0.74	0.72	0.75	0.70	1.15
Max	10.14	5.72	10.96	9.38	10.80	10.59	9.87	9.50	19.08
MAD	1.03	0.81	0.77	0.84	1.04	1.03	1.07	0.97	1.68
IQR	1.40	1.10	1.04	1.13	1.41	1.38	1.45	1.32	2.27
CV	49.15	79.68	65.57	-508.70	100.21	508.51	178.78	58.43	5127.85
Skewness	-0.69	-1.35	-0.45	-0.29	-0.09	-0.10	-0.05	-0.35	-0.70
SE.Skewness	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
Kurtosis	7.18	9.10	11.78	8.34	5.82	6.43	4.60	5.58	12.92
N.Valid	4668.00	4668.00	4668.00	4668.00	4668.00	4668.00	4668.00	4668.00	4668.00
Pct.Valid	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Table 1: Descriptive statistics

As shown in Table 1, we see that the Norwegian index OSEBX has had the largest mean return during the last 20 years with 0.03%, closely followed by the American S&P500 with

0.02%. We also see that Brent Crude Oil had the largest negative daily return with -27.98%, in which can be explained by the oil crash in 2014. For all indices, we have a total of 2668 daily observations through 2000 until 2020. Consistent with earlier literature, all return series show excess kurtosis which indicates that a GARCH model is appropriate to model volatility. In Figure 3 we illustrate the prices from all indices used in this thesis, normalized to start at the same value to give a better view of growth.

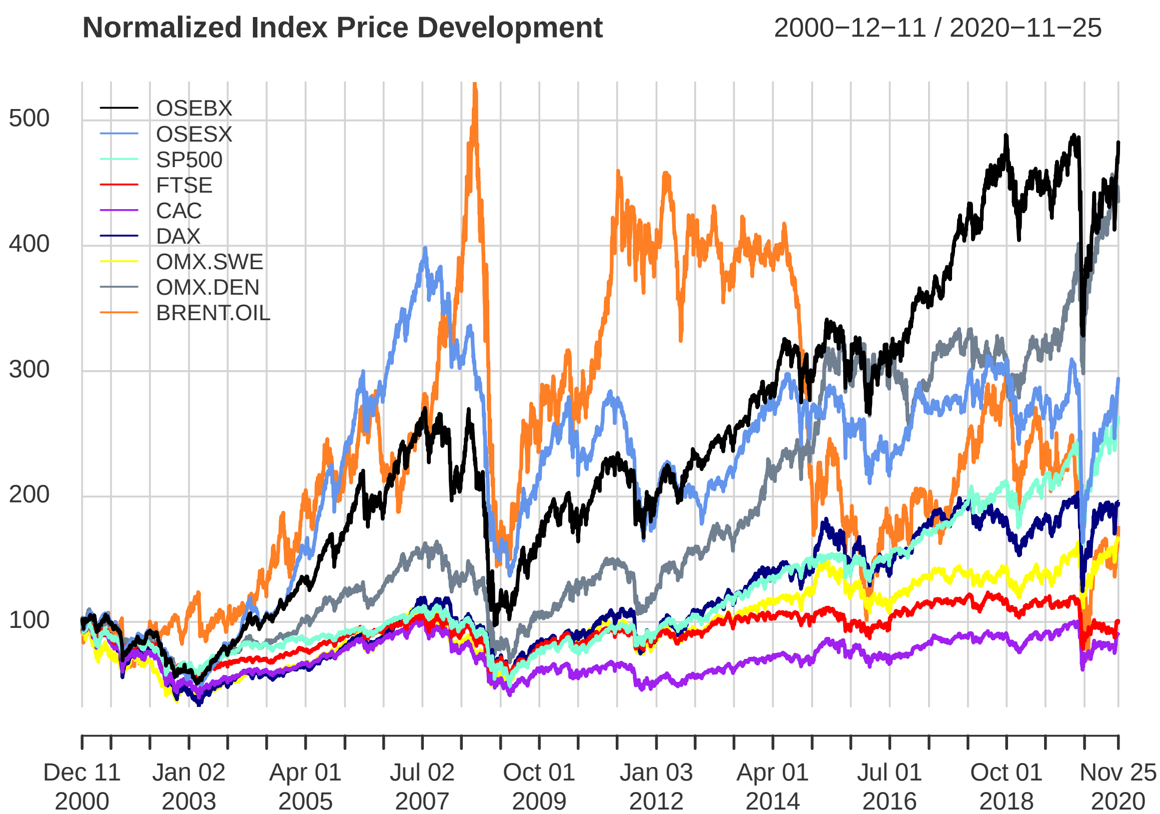

Figure 3: Normalized price development

Figure 3 illustrates the normalized price development of the indices included in this thesis. By dividing all observations on the first observation in the time series for each index, we see the growth instead of price movement. The reason for this is that when only using price movements, some indices are illustrated in the bottom of the plot, making it difficult to see. We see from Table 1 that OSEBX had the largest mean return during the time period of 2000 to 2020, however the Brent Crude oil had the largest overall price in early 2008. Illustrated in Figure 4 is the returns of all indices and the Brent Crude oil given.

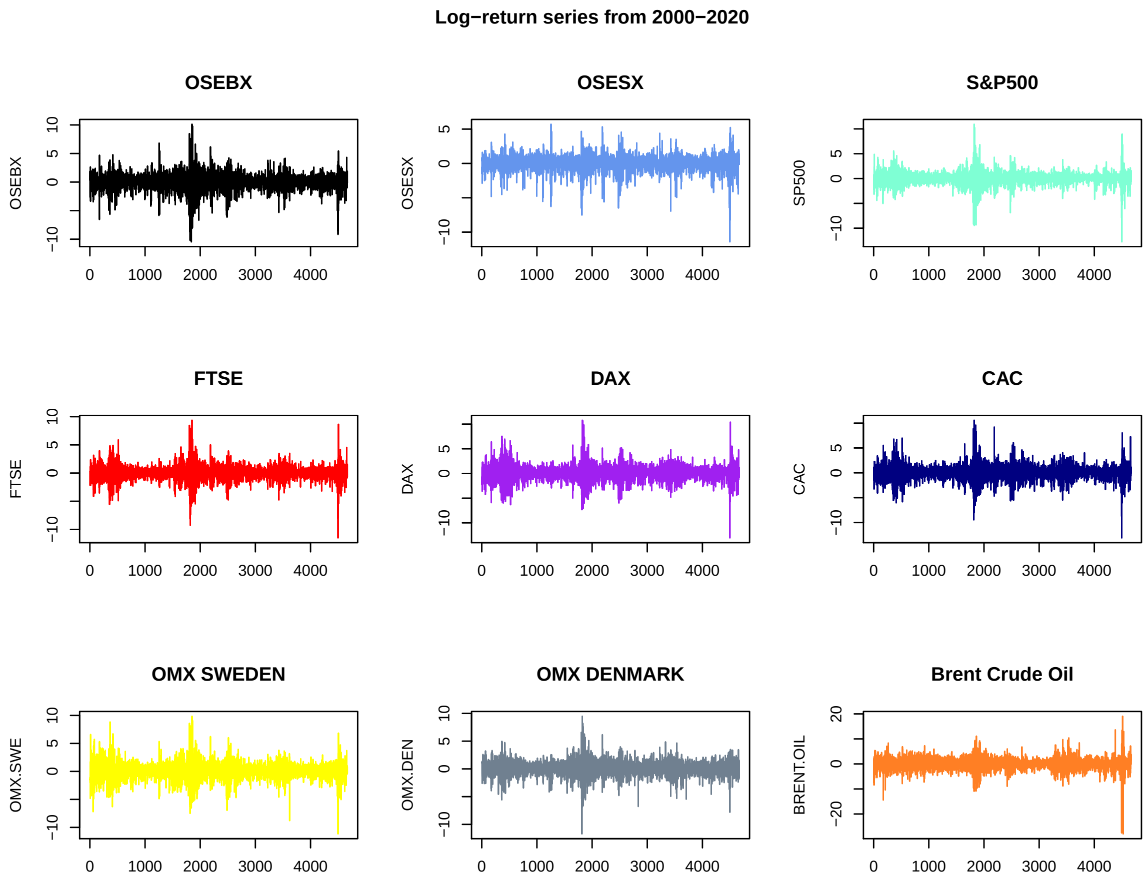

Figure 4: Return series

Figure 4 illustrates the return series from OSEBX, OSESX, S&P500, FTSE, DAX, CAC, OMX Stockholm, OMX Copenhagen and Brent Crude oil. We see from the figure that there is similarities with regards to highs and lows in terms of return.

4.1 Data specific details

The time period is from 2000/12/12 to 2020/11/27. This period is chosen to include both the financial crisis in 2007 as well as the more recent crash in oil prices in 2014. From acquiring time series from different dates in 2000, we removed some of the observations, getting a complete dataset with the same amount of daily data for all indices ranging from december 12th 2000 until november 27th 2020. For some time series, models converged only partially, resulting in testing with weekly series in an attempt to implement these models further. The latter part will be discussed further in the respective chapter of the BEKK-GARCH model.

To partially explain the volatility spillover and structural breaks, we included the Brent Crude oil as a time series of its own due to Norway being an oil-dependent country.

5 Analysis and results

This chapter will present all statistical analysis done in our thesis. As the previous chapter shows, we have applied a set of GARCH-models suitable for volatility spillover modeling. The analysis will follow a natural build-up towards the concluding results of our CCC- and DCC-GARCH, as well as the BEKK-GARCH model.

5.1 CCC-GARCH and DCC-GARCH

Before running the CCC-GARCH(1,1) on our dataset, several precautions have to be accounted for. A test for dynamic correlation in our data was made in order to see if the DCC-GARCH(1,1) was preferred over the CCC-GARCH(1,1). We used the function DCCtest in the package rmgarch (Ghalanos, 2019) in R version 4.0.3 to test the dataset for dynamic correlation between the indices. A p-value in the test below 5% is received, indicating an absence of constant correlation and the dataset is thus more suitable for use in the DCC-GARCH model compared to the CCC-GARCH model. The dataset was implemented in the CCC-GARCH as a precaution, but the model fails to provide any significant results. The main focus of this part of the analysis will thus be on the DCC-GARCH model, since our dataset consisted of dynamic correlation.

5.1.1 DCC-GARCH

The DCC-GARCH (1,1) model is used to explore the cooperative movements of the correlation of two sets of data. In our case, OSEBX and several of Norways trading partners. The optimal parameters for the first set of indices is shown in Table 2.

		OSEBX/OSESX		OSEBX/SP500		OSEBX/FTSE	
1		Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
2	[A]mu	0.000780	0.000000	0.000742	0.000000	0.000137	0.000000
3	[A]omega	0.000003	0.009328	0.000003	0.009913	0.000003	0.010154
4	[A]alpha1	0.116635	0.000000	0.117449	0.000000	0.118766	0.000000
5	[A]beta1	0.866693	0.000000	0.865070	0.000000	0.863846	0.000000
6	[B]mu	0.000918	0.000000	0.000589	0.000000	0.000290	0.014206
7	[B]omega	0.000007	0.000000	0.000002	0.073455	0.000002	0.158261
8	[B]alpha1	0.179307	0.000000	0.125953	0.000000	0.117474	0.000003
9	[B]beta1	0.770319	0.000000	0.859879	0.000000	0.868973	0.000000
10	[Joint]dcca1	0.046343	0.000000	0.008584	0.000628	0.035208	0.000000
11	[Joint]dccb1	0.904684	0.000000	0.989790	0.000000	0.952357	0.000000

Table 2: DCC results (1)

As mentioned earlier, when deciding whether a DCC is a good fit we first have to look towards the coefficients of $alpha1$ and $beta1$ for each index to check the assumption of GARCH(1,1) joint significance. Note that the output from the models gives the greek letters $\mu, \omega, \alpha, \beta$. The p-values of each indices α_1 and β_1 estimate is significant below 5%. This indicates that the GARCH(1,1) joint significance of α_1 and β_1 makes sense in each scenario of Table 2. We also see that $\alpha_1 + \beta_1 < 1$ which means that the time series data is considered to be stationary. While α_1 measures the short term impact volatility, the beta1 coefficient measures the lingering effect of structural breaks on the conditional correlations. The closer the β_1 coefficient is to 1, the slower volatility decays. This results in a longer impact of volatility after structural breaks. From Table 2 we see that in the first set of indices OSEBX and FTSE have the lowest rate of volatility dissipation after a structural break, due to the β_1 coefficient being moderately close to 1. The $[joint]dcc\alpha_1$ and $[joint]dcc\beta_1$ coefficients are used to evaluate DCC as an assumption. All $[joint]dcc\alpha_1$ and $[joint]dcc\beta_1$ coefficients of Table 2 are significant below 5% and the combined estimate of $[joint]dcc\alpha_1$ and $[joint]dcc\beta_1$ is below zero. With this information in mind the DCC-GARCH(1,1) models in Table 2 are proven to capture the correlation effects between the indices. All fitted models in Table 2 are therefore accepted.

		OSEBX/DAX		OSEBX/CAC	
1		Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
2	[A]mu	0.000756	0.000000	0.000744	0.000000
3	[A]omega	0.000003	0.008616	0.000003	0.009459
4	[A]alpha1	0.116468	0.000000	0.115861	0.000000
5	[A]beta1	0.866843	0.000000	0.867635	0.000000
6	[B]mu	0.000635	0.000065	0.000475	0.001556
7	[B]omega	0.000003	0.095214	0.000003	0.156970
8	[B]alpha1	0.094125	0.000000	0.108505	0.000004
9	[B]beta1	0.894523	0.000000	0.880660	0.000000
10	[Joint]dcca1	0.024385	0.000001	0.034984	0.000000
11	[Joint]dccb1	0.971064	0.000000	0.951429	0.000000

Table 3: DCC results (2)

As for the fit of the next set of indices in Table 3; we see that α_1 and β_1 in both models are significant below 5%. $[A]\alpha_1$ and $[A]\beta_1$, and $[B]\alpha_1$ and $[B]\beta_1$ are combined below 1 in both models. This indicates that the data is deemed stationary. From $[B]\beta_1$ in OSEBX/DAX and OSEBX/CAC we see that the foreign indices have a lower rate of volatility dissipation after structural breaks compared to OSEBX in both models. The $[Joint]dcc\alpha_1$ and $[Joint]dcc\beta_1$ for both models are significant and combined below 1, which ensures positive unconditional variances. The fitted models in Table 3 are proven to capture the correlation and are thus accepted. The $[Joint]dcc\alpha_1$ and $[Joint]dcc\beta_1$ for both models are significant and combined below 1, which ensures positive unconditional variances. The fitted models in Table 3 are proven to capture the correlation and are thus accepted. The remaining DCC-models of OSEBX/OMX Copenhagen and OSEBX/OMX Stockholm are also accepted under the same criteria and can be found in Appendix A.

5.1.2 Rolling correlation of volatilities of DCC

To capture the correlation of volatilities of OSEBX and its trading partners over the past periods, the correlation of the residuals of the fitted DCC-models is presented for a portion of the indices in Figure 5.

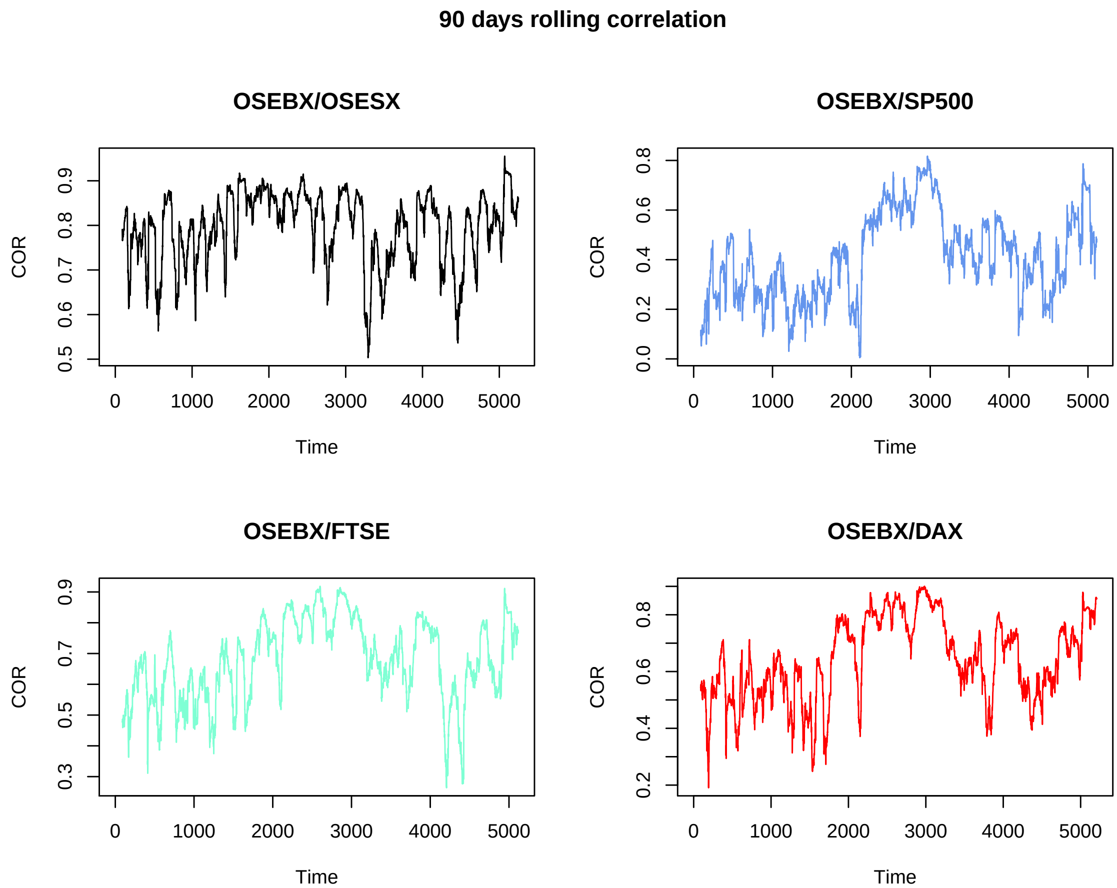

Figure 5: 90 days rolling correlation

The general impression from Figure 5 is that the correlation between OSEBX and its trading partners has heavily fluctuated over the past 20 years. With a three month rolling window most indices fluctuate between 0.20 (bottom) and 0, 8 (peak). The 3 month rolling correlation between the indices have never been negative over the 20 year period, OSEBX/SP500 being the closest to zero with a low of 0.004. This tells us that OSEBX, generally speaking, follows the trends of other indices. We see a period of declining correlation in the financial crisis from 2008-2009 in most cases with S&P500 being the most visible from Figure 5. The correlation of OSEBX and its trading partners have a slight upwards trend from 2000 to 2012 and a slight downwards trend from 2012 to 2020. The remaining graphs for correlation of volatility can be found in Appendix A.

5.2 ICSS

Through the application of the Iterated Cumulative Sums of Squares (ICSS) algorithm presented by Inclan and Tiao (1994), we present the number of structural breaks in OSEBX, OSESX, S&P500, FTSE, DAX, CAC, OMX Stockholm and OMX Copenhagen, both for daily and weekly data. Previous literature has shown that the ICSS algorithm tends to overstate the number of breaks, and it has been pointed out that the algorithm's behavior is questionable under the presence of conditional heteroskedasticity (Fernandez, 2020). This problem has been solved by filtering the return series through a GARCH (1,1) model, followed by applying the ICSS algorithm to the standardized residuals (Bachmann & Dubois, 2002). For each index we will illustrate the breakpoints using figures, in which we have used the aforementioned method of filtering. The time periods of each break for all indices can be viewed in the Appendix B.

5.2.1 Norway

5.2.1.1 OSEBX Through the use of the ICSS algorithm, we find 26 structural breaks in the OSEBX index through the last 20 years. Averaging 1.3 breaks per year, the number of breaks seems to be too high to associate with the term ´structural breaks´ as they appear on a regular basis instead of large global or domestic events. The number of structural breaks quite drastically differ with regards to using daily or weekly data. The results from using weekly data provides us with 4 breaks in total, averaging 0.2 breaks per year during a 20 year period. An interesting finding is that the ICSS algorithm did not detect any structural breaks from either the oil price crash in 2014, nor the COVID-19 crisis in 2020 compared to the daily data.

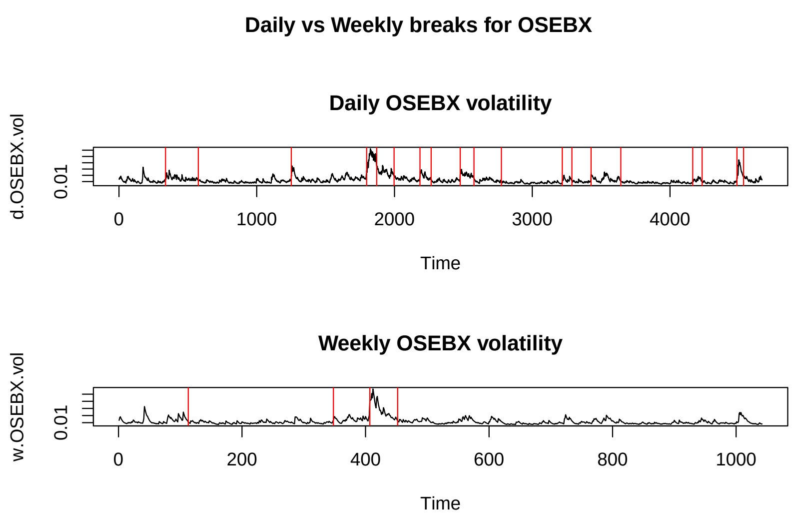

Figure 6: Structural breaks in OSEBX

Illustrated in Figure 6 above we see the volatility of OSEBX from 2000 until 2020, with the breaks illustrated in red vertical lines. An important remark is that for the weekly data, the ICSS algorithm detects a structural break around observation 350 and not at 1000, as both observations have relatively high spikes in volatility. We see from the figure that the increase in volatility at observation 1000 is higher than the one at 350, which initially should result in a break location here.

5.2.1.2 OSESX From the OSESX index we find there to be a total of 15 breaks in the daily time series. With an average of 0.75 breaks per year through 20 years, it seems to be quite too many also here. Comparing the OSESX and OSEBX index, a large proportion of all breaks are found to be in the same time period, which makes sense as both indexes are a representation of the Norwegian financial market. From the weekly data on OSESX we did not locate any structural breaks as indicated in the Figure 7. Through twenty years of weekly data, or 1040 observations, we see several substantial changes in the volatility and based on the OSEBX index there should be structural breaks here as they are highly correlated.

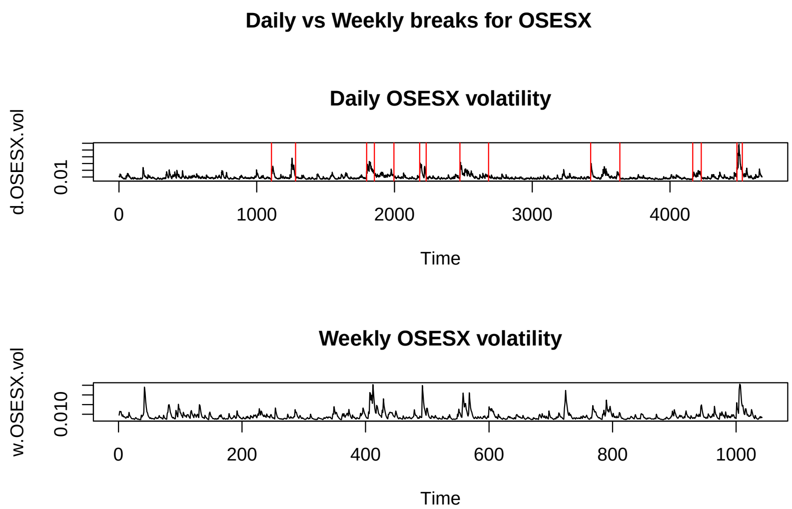

Figure 7: Structural breaks in OSESX

Illustrated in Figure 7 above we see the daily and weekly volatility for OSESX, and the structural breaks found through the ICSS algorithm.

5.2.2 The United States of America

For the Standard & Poor 500 (S&P500) index we located 24 breaks during the last twenty years, averaging 1.2 breaks per year. Through the years we see breaks located during both the financial crisis in 2007, the oil price crash in 2014 and the latest COVID-19 pandemic. The number of breaks located seems too high, as there are structural breaks located in seemingly low volatility periods. Compared to the daily time series, we found only 4 structural breaks in the weekly series. Through our literature review we found that Caporin & Malik (2020) found only 8 structural breaks in the S&P500 index. They were, however, using a modified ICSS-algorithm which are beyond the purpose of this thesis as of which we are using the standard algorithm.

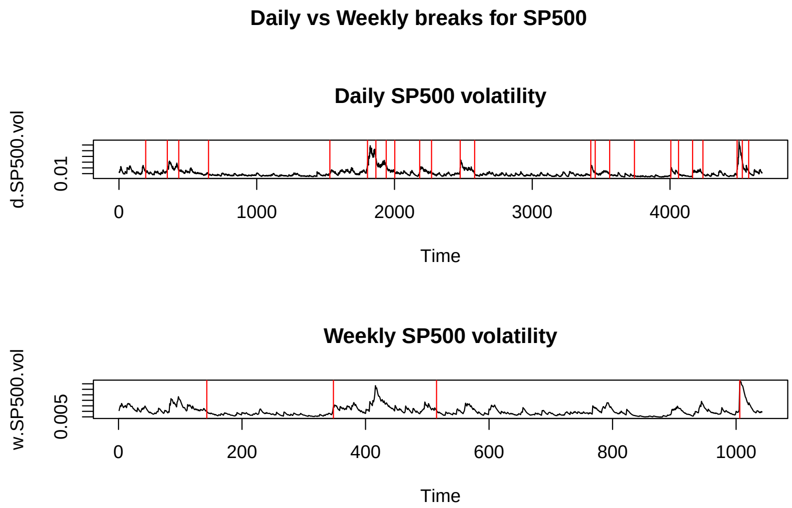

Figure 8: Structural breaks for SP500

Illustrated in Figure 8 above, we have plotted the volatility of S&P 500 from 2000 to 2020, with the structural breaks found from the ICSS algorithm in vertical red lines.

5.2.3 The United Kingdom

For the UK and the FTSE100 index, we find 22 breaks in the daily time series. Here we also see structural breaks captured in the major global happenings during the last twenty years in the financial crisis, COVID-19 etc.. With an average of 1.1 structural breaks per year, we located less breaks in the FTSE100 index than both Norwegian indices and the US S&P 500. In the weekly time series we located 5 structural breaks.

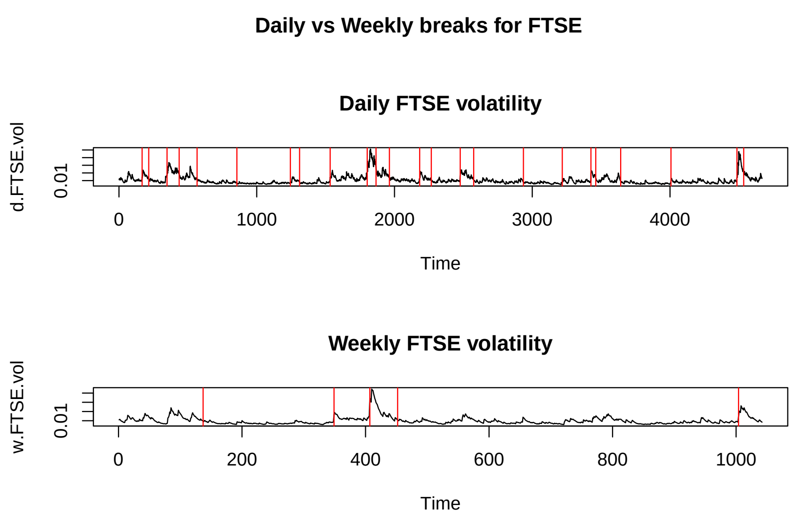

Figure 9: Structural breaks for FTSE

In Figure 9 we have illustrated the volatility of FTSE100 in both daily and weekly observations, with the structural breaks located from the ICSS algorithm indicated in red vertical lines.

5.2.4 Germany

In the German DAX index we located 19 structural breaks in the daily observations, averaging 0.95 breaks per year. Again, the ICSS algorithm has located breaks in the major global events, but also in relatively low volatility periods. In the weekly time series we located 4 structural breaks, however we did not locate any in the latter COVID-19 pandemic.

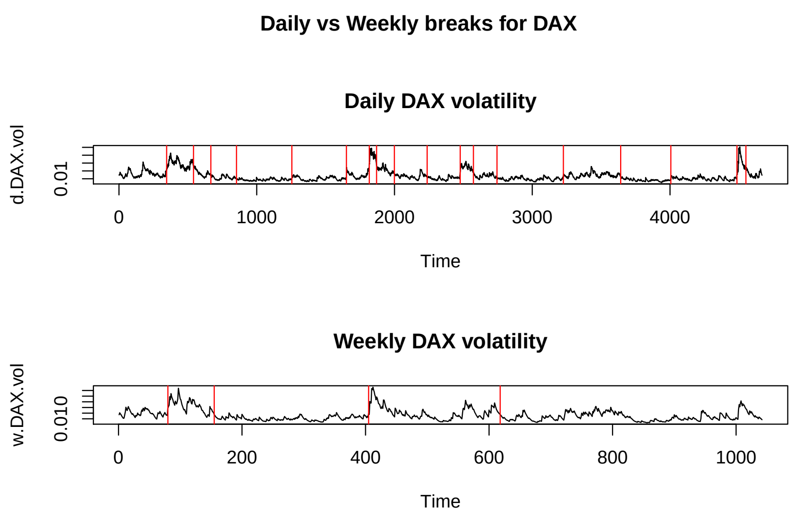

Figure 10: Structural breaks for DAX

In Figure 10, the volatility of the German DAX index is illustrated both in daily and weekly observations, with the structural breaks located in the ICSS algorithm indicated in red vertical lines.

5.2.5 France

For the French CAC100 index we located 23 breaks averaging 1.15 structural breaks per year. As illustrated in the plot below, the structural breaks for the daily time series fits the spikes of volatility in the CAC index well. In the weekly time series we located 4 structural breaks, seemingly the same number as the previous indices.

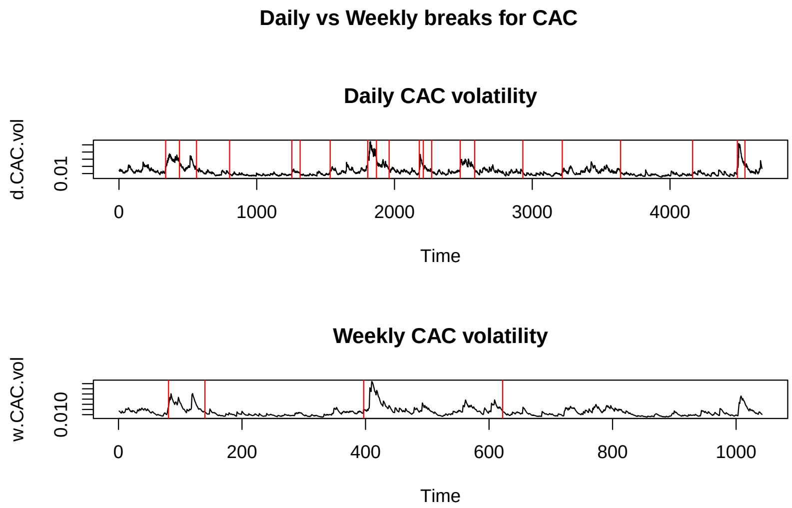

Figure 11: Structural breaks for CAC

Illustrated in Figure 11 above, we see the daily and weekly volatility time series of the CAC100 index, with the structural breaks found from the ICSS algorithm indicated in red vertical lines.

5.2.6 Sweden

From the Swedish OMX index we found 20 structural breaks during the period 2000 to 2020, averaging 1 structural break per year. For the weekly time series we located 3 structural breaks. For the weekly series, illustrated in Figure 12 below, the ICSS algorithm seems to not have captured any large global happenings through the last twenty years.

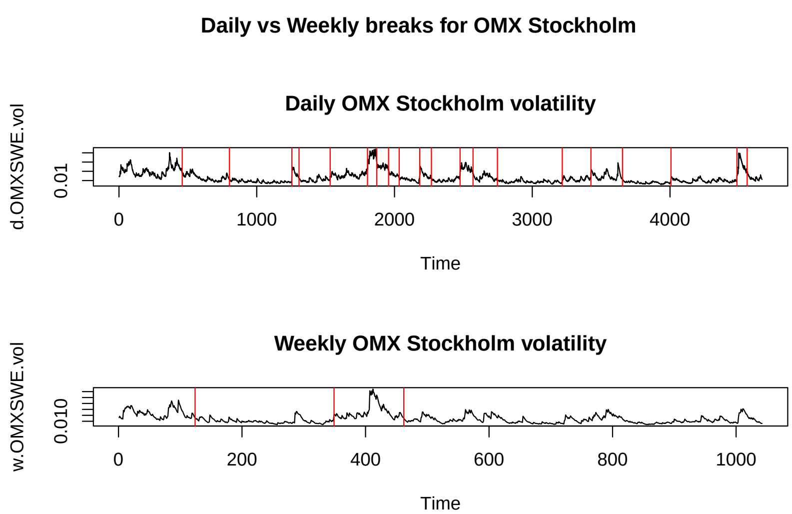

Figure 12: Structural breaks for OMX Stockholm

In Figure 12 we have illustrated the volatility of the Swedish OMX index in both daily and weekly observations, with the structural breaks found from the ICSS algorithm given in red vertical lines.

5.2.7 Denmark

From the Danish OMX index, the ICSS algorithm located 21 structural breaks for the daily time series, and 4 breaks for the weekly series. With an average of 1.05 structural breaks per year in the daily time series, it seems like the breaks follow the global happenings. Illustrated in Figure 13 below we see the daily and weekly observations for the Danish OMX index.

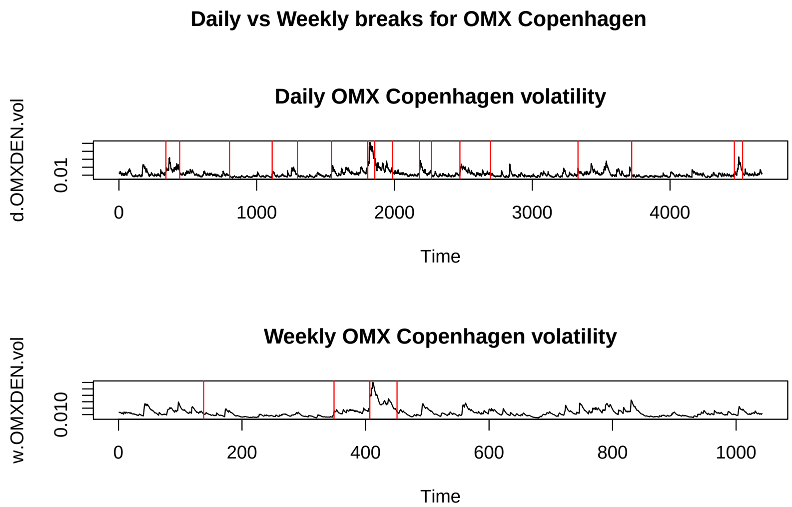

Figure 13: Structural breaks for OMX Copenhagen

Illustrated in Figure 13 we see the daily and weekly observations of volatility in the Danish OMX index, with the structural breaks found from the ICSS algorithm given in red vertical lines.

5.3 BEKK-GARCH (1,1)

Table 4 summarizes the first set of estimation results of the bivariate BEKK-GARCH model. Due to the fact we estimate the effects of structural breaks and spillovers in OSEBX with 7 other indices, we had to restrict our tables to make them more comprehensible. We see from Table 4 the presence of ARCH- and GARCH-effects since the estimated diagonal elements of A11, A22, B11 and B22 are all highly significant below the threshold of 5%. This in return shows that the conditional variance of OSEBX and the stock indices in Table 4 are affected by their previous breaks and volatility spillover. The estimate of the coefficient A11 and A22 shows the indices ability to present shock persistence. For example, an estimate in A11 of 0.233 in OSEBX/OSESX implies that 23.3% of the effects of a structural break persists the next day. The estimate of the coefficient B11 and B22 shows the indices ability to present volatility persistence. For example, an estimate in B11 of 0.975 in OSEBX/OSESX implies that 97.5% of the volatility spillover effect persists the next day.

		OSEBX/OSESX		OSEBX/SP500		OSEBX/FTSE	
1		Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
2	mu1	0.001027162	4.5541e-13 ***	0.000853592	4.1149e-09 ***	0.000646234	6.4618e-06 ***
3	mu2	0.000928741	7.1942e-14 ***	0.000712105	6.8973e-10 ***	0.000204259	0.0832206 .
4	A011	0.002848085	< 2.2e-16 ***	0.002850062	1.3349e-12 ***	0.002854520	< 2.2e-16 ***
5	A021	0.003392419	< 2.2e-16 ***	0.001168384	6.6288e-09 ***	0.001922625	< 2.2e-16 ***
6	A022	0.001339275	< 2.2e-16 ***	0.002238432	< 2.2e-16 ***	0.001669102	< 2.2e-16 ***
7	A11	0.232834255	< 2.2e-16 ***	0.307511339	< 2.2e-16 ***	0.282139783	< 2.2e-16 ***
8	A21	0.006616033	0.763174	0.048358314	0.00029455 ***	0.038467972	0.0254244 *
9	A12	0.180294587	1.8702e-10 ***	(-)0.039112262	0.06186676 .	0.055495069	0.0129042 *
10	A22	0.452136264	< 2.2e-16 ***	0.308461047	< 2.2e-16 ***	0.306124820	< 2.2e-16 ***
11	B11	0.974817628	< 2.2e-16 ***	0.922827994	< 2.2e-16 ***	0.934553153	< 2.2e-16 ***
12	B21	0.019890013	0.048139 *	(-)0.013162162	0.09766431 .	(-)0.015134957	0.0538158 .
13	B12	(-)0.106074980	1.2381e-09 ***	0.012688184	0.24921463	(-)0.027231911	0.0036685 **
14	B22	0.806708581	< 2.2e-16 ***	0.916133826	< 2.2e-16 ***	0.918260159	< 2.2e-16 ***

Table 4: BEKK results (1)

As for the cross-market effects of structural breaks and spillover effects between OSEBX and other indices are estimated in the off-diagonal elements of matrices A and B, i.e. A21, A12, B21 and B12. As with the diagonal elements of the matrices, the coefficient of A considers the effects of structural breaks and the coefficient of B considers the effects of volatility spillover. The off-diagonal elements of A21 and A12 show the ability for structural break transmission between the indices. The off-diagonal elements of B21 and B12 show the ability for volatility transmission between the indices. As our thesis mostly consists of exploring the effects of shock and volatility between OSEBX and seven other indices; we will look deeper into the off-diagonal elements of OSEBX and the other indices.

5.3.1 OSEBX/OSESX

The results from OSEBX/OSESX indicate evidence of a uni-directional relationship regarding the transmission of structural breaks. This means that the transmission of volatility during structural breaks between the indices only goes one way, from A to B, but not B to A. We can see from Table 4 that A12 is significant below the threshold of 5%, while element A21 is not significant. This tells us that structural breaks originating in OSEBX will affect the volatility in OSESX. This is as mentioned a uni-directional relationship, which in return means that we can not find evidence of this happening in reverse, e.g., shocks in OSESX affecting the volatility in OSEBX. As for the coefficients of B21 and B12 we see evidence of a bi-directional relationship regarding transmission effects of volatility spillover due to B21 and B12 both being significant to the threshold of 5%. We do however see a higher level of significance to B12 which indicates that the transmission from OSEBX to OSESX is stronger. As an example, to visualize the effects of B21 and B12 we see that the estimates have a value of 0.0199 and (-)0.1061, respectively. This implies that 1% increase in returns of OSEBX transmits 10.61% of volatility to OSESX. From OSESX to OSEBX we see a 1% increase in returns resulting in a volatility spillover of 1.99%.

5.3.2 OSEBX/S&P500

Furthermore, the results from OSEBX/S&P500 indicate evidence of a uni-directional relationship regarding transmission of volatility to another index when a structural break is occurring in the originating index. We see from Table 4 that A21 is highly significant and A12 is not significant to 5%. Regarding the transmission effects of volatility spillover, we see a no relationship from S&P500 to OSEBX or in reverse, since B21 and B12 are not significant to 5%. This means that there is no clear evidence of volatility spillover between OSEBX and S&P500.

5.3.3 OSEBX/FTSE

The results from OSEBX/FTSE indicate evidence of a bi-directional relationship in both regarding structural breaks and volatility spillover. We see from the coefficients A21, A12 and B12 that all are significant to the threshold of 5%. This in return shows that we have evidence that a shock originating in either OSEBX or FTSE will affect the volatility in the other index. This also shows that we have evidence of volatility spillover happening from OSEBX to FTSE. From A21 and A12 we see however that the relationship is marginally stronger from OSEBX to FTSE. From B21 and B12 we see that the relationship is unidirectional from OSEBX to FTSE. We find these results puzzling since FTSE is one of the largest indices in the world and OSEBX is smaller in comparison. We can see from the Table that the percentage of volatility transmitted is higher from FTSE to OSEBX in each off-diagonal matrix, but due to the lower p-value in both instances the relationship itself is considered stronger. The reasoning for this occurrence is the oil price, and its effect on macroeconomic conditions in general and Norway in particular. We explore this in detail in chapter 5.3.7.

		OSEBX/DAX		OSEBX/CAC	
1		Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
2	mu1	0.000805415	5.8075e-09 ***	0.000695812	4.0011e-07 ***
3	mu2	0.000676557	6.6122e-06 ***	0.000297427	0.0365763 *
4	A011	0.002847890	< 2.2e-16 ***	0.002848038	< 2.2e-16 ***
5	A021	0.001973875	< 2.2e-16 ***	0.002073254	< 2.2e-16 ***
6	A022	0.002237877	< 2.2e-16 ***	0.002039670	< 2.2e-16 ***
7	A11	0.333710077	< 2.2e-16 ***	0.358406540	< 2.2e-16 ***
8	A21	0.044541010	0.037754 *	0.071080875	0.0014823 **
9	A12	0.032102560	0.122320	0.013982228	0.5259046
10	A22	0.299725405	< 2.2e-16 ***	0.325752913	< 2.2e-16 ***
11	B11	0.918410156	< 2.2e-16 ***	0.911478076	< 2.2e-16 ***
12	B21	(-)0.006638155	0.581316	(-)0.022645071	0.0483728 *
13	B12	(-)0.017810120	0.034052 *	(-)0.009387993	0.3625270
14	B22	0.923106737	< 2.2e-16 ***	0.918811126	< 2.2e-16 ***

Table 5: BEKK results (2)

The next set of indices is in Table 5. Here we see a strong presence of GARCH- and ARCH-effects due to all diagonal elements (A11, A22, B11 and B22) being significant at the 5% significance level. This means all indices in Table 5 are affected by their previous shocks and its own lingering volatility.

5.3.4 OSEBX/DAX

As for the results regarding OSEBX/DAX, we see evidence of a unidirectional relationship regarding the effects of structural breaks. The p-value of A21 is significant to 5%, while the p-value of A12 is not. This signalizes that structural breaks occurring in DAX affects the volatility in OSEBX. From the coefficients of B21 and B12 we see a unidirectional relationship regarding volatility spillover. B12 is significant to 5%, while B21 is not. This means that there is evidence of volatility spillover from OSEBX to DAX occurring in our dataset. This in line with the results from OSEBX/FTSE puzzles us and will be addressed in chapter 5.3.7.

5.3.5 OSEBX/CAC

Regarding the results from OSEBX/CAC we encounter evidence of a unidirectional relationship in the effects of structural breaks. A21 is significant to the threshold of 5%, while A12 is not significant. This means structural breaks in CAC affect the volatility in OSEBX, but not in reverse. In the off-diagonal elements of B21 and B12 we also see evidence on a unidirectional relationship between the indices. B21 is significant and B12 is not significant. This means volatility spillover is occurring from CAC to OSEBX and not the other way.

		OSEBX/OMX.S		OMX.C	
1		Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
2	mu1	0.000818153	4.1587e-09 ***	0.01013033	0.00176616 **
3	mu2	0.000626997	4.4393e-05 ***	0.01028961	0.00101801 **
4	A011	0.002848337	0.00028291 ***	0.01201810	0.05662947 .
5	A021	0.002064996	< 2.2e-16 ***	0.01626835	0.30023561
6	A022	0.002124405	0.21640496	0.01516651	0.21191958
7	A11	0.365188841	4.7518e-14 ***	0.29658607	0.01884573 *
8	A21	0.076619755	0.11939170	0.00942289	0.93843016
9	A12	0.003462951	0.90979404	0.16769187	0.43524692
10	A22	0.313707702	0.00435128 **	0.40212473	0.01327690 *
11	B11	0.909218771	< 2.2e-16 ***	0.99999900	< 2.2e-16 ***
12	B21	(-)0.024850284	0.62206582	0.11869805	0.31876267
13	B12	(-)0.005394107	0.54981728	(-)0.16707874	0.30981557
14	B22	0.923331006	< 2.2e-16 ***	0.70193006	0.00010198 ***

Table 6: BEKK results (3)

The last set of indices is shown in Table 6. There is evidence of GARCH and ARCH effects on the diagonal elements of A11, A22, B11 and B22. However this evidence is significantly less strong than the previous sets of indices. This can be an indication of a bad fit to the model.

5.3.6 OSEBX/OMX Stockholm & OSEBX/OMX Copenhagen

The results from both indices are the same where we see no evidence of a relationship regarding effects from either structural breaks or volatility spillover between the scandinavian indices and OSEBX. A21, A12, B21 and B12 all fail to be significant to the threshold of 5%. We believe this to be the result of a bad model fit or converging-problems in the BEKK-GARCH(1,1) because of the similarity of the datasets.

5.3.7 The FTSE and DAX conundrum

One explanation to the results from the BEKK estimated from OSEBX/FTSE and OSEBX/DAX could be anchored in the price of oil. As mentioned earlier Norway's economy is heavily influenced by the oil price. So in an attempt to explain the results from OSEBX/FTSE we would also run BEKK models on OSEBX and Brent Oil, FTSE and Brent Oil, and DAX and Brent Oil.

In Table 7 we see the results from the BEKK models on OSEBX, FTSE and DAX compared with the spot price of oil. We see strong evidence of GARCH and ARCH effects as the diagonal estimates are all significant to the threshold of 5% (A11, A22, B11 and B22).

		OSEBX/BRENT		FTSE/BRENT		DAX/BRENT	
1		Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimater	Pr(>|t|)
2	mu1	0.000849019	5.9253e-09 ***	0.000849019	5.9253e-09 ***	0.000628834	3.2282e-05 ***
3	mu2	0.000599359	0.02061871 *	0.000599359	0.02061871 *	0.000473774	0.0599536 .
4	A011	0.002867320	< 2.2e-16 ***	0.002867320	< 2.2e-16 ***	0.002981991	< 2.2e-16 ***
5	A021	0.001619049	2.1654e-05 ***	0.001619049	2.1654e-05 ***	0.001008946	0.0078688 **
6	A022	0.004312594	1.8474e-13 ***	0.004312594	1.8474e-13 ***	0.004484250	< 2.2e-16 ***
7	A11	0.330958626	< 2.2e-16 ***	0.330958626	< 2.2e-16 ***	0.342200118	< 2.2e-16 ***
8	A21	0.117781595	2.4532e-05 ***	0.117781595	2.4532e-05 ***	0.033069793	0.1418271
9	A12	(-)0.023225230	0.00582457 ***	(-)0.023225230	0.00582457 ***	0.014843443	0.0927820 .
10	A22	0.241630043	< 2.2e-16 ***	0.241630043	< 2.2e-16 ***	0.311287019	< 2.2e-16 ***
11	B11	0.914709705	< 2.2e-16 ***	0.914709705	< 2.2e-16 ***	0.914322822	< 2.2e-16 ***
12	B21	(-)0.047405557	0.00033841 ***	(-)0.047405557	0.00033841 ***	(-)0.018013289	0.1070681
13	B12	0.006972905	0.05037575.	0.006972905	0.05037575 .	(-)0.002634665	0.4928694
14	B22	0.944371392	< 2.2e-16 ***	0.944371392	< 2.2e-16 ***	0.928352490	< 2.2e-16 ***

Table 7: BEKK results (4)

As for the off-diagonal elements we see evidence of a bidirectional relationship in the effects of structural breaks in OSEBX/BRENT, FTSE/BRENT, but not in DAX/BRENT. A21 and A12 are both significant in OSEBX/Brent and FTSE/BRENT, which results in evidence that structural breaks in the spot price of oil affects OSEBX. A21 and A12 are not significant in DAX/Brent. This means that we can not find any evidence that structural breaks in DAX or Brent affect the volatility in each other. As for B21 and B12 we see a unidirectional relationship in the models of OSEBX/Brent and FTSE/Brent from Brent to OSEBX/FTSE. This results in evidence that volatility spillover occurs from Brent to OSEBX or FTSE. Although this is probably not the singular reason for our results in the BEKK of OSEBX/FTSE, we believe this could be a logical reason for our surprising results from that model. This is in line with Malik and Hammoudeh (2007) where they found similar results regarding to volatility spillover and the transfer of structural breaks. They investigated other oil dependent countries in the middle east. They found that the oil price shocks transmitted volatility to stock indices in Kuwait, Saudi Arabia and Bahrain. Interestingly in the case of Saudi Arabia they observed a significant volatility spillover from the national index to the oil price. We failed to find any results like Saudi Arabia in our research. From DAX/Brent however, neither B21 or B12 is significant. This means that we can not find any evidence in daily data that DAX and Brent affect each other with regards to structural breaks or volatility spillover. This means that there are other forces affecting OSEBX and DAX which may affect the results from the BEKK-model in Table 5. It is worth a mention that in weekly data we can see a relationship between DAX and Brent in both off-diagonal matrices, but we are certain that the oil price is not the singular reason for this occurrence.

5.3.8 BEKK on weekly data

All previous BEKK models are all run on daily data. To avoid the non-synchronous trading problem, all BEKK models were also computed on weekly data. From the BEKK models on weekly data we encountered problems with the models for OSEBX and the scandinavian indices (OSESX, OMX Stockholm and OMX Copenhagen). These failed to converge resulting in incomplete data. As for the rest, the results were similar. The findings from OSEBX/S&P500 gave us similar results regarding the relationship between the indices. As for OSEBX and FTSE/DAX/CAC we saw an increase of significance in influence from OSEBX in both structural breaks and volatility spillover. OSEBX/FTSE now has a unidirectional relationship in both off-diagonal elements from OSEBX to FTSE. OSEBX/DAX and OSEBX/CAC now have a bi-directional relationship on both off-diagonal elements. A possible explanation to this can be the influence the spot price of oil has on the european economy. All BEKK-models on weekly data can be found in the appendix.

5.4 Economic implications

Estimating volatility spillover across markets has serious economic consequences regarding optimal portfolio allocation (Kroner & Ng, 1998), risk management (Christoffersen, 2009) and dynamic hedging (Haigh & Holt, 2002). An analysis of all the economic implications is beyond the purpose of this paper, but should at least be discussed. Demonstrated by Haigh and Holt (2002), a bivariate GARCH model that accounts for volatility spillover between spot and futures markets results in a better hedging strategy compared to a GARCH model that ignores these effects. Through the research of Lien and Yang (2010), Caporin and Malik (2020) state that daily currency risk can be better hedged with currency futures when controlling for unconditional variance breaks using a bivariate GARCH model. Caporin and Malik (2020) also find that more variance in hedge ratios results in an substantial increase in portfolio rebalancing costs due to traders having to make more adjustments. With regards to the findings in this thesis, investors with positions in Britain, Germany, France and Norway, should be aware of the spillover effects with regards to hedging for volatility spillover.

6 Limitations

There are advantages of applying lower frequency data with weekly versus daily data, as it avoids the non-synchronous trading problem. The trading hours are almost the same within the Scandinavian region, but partially overlapping with the US market. Therefore, higher frequency data might generate the asymmetric information sharing issue (Ng, 2000, Baele, 2002, Skintzi & Refenes, 2004, Christiansen, 2007, etc.). The structural breaks from the ICSS algorithm seems to include a large portion of noise in the time-series, providing us more breaks than the actual case in the daily dataset. In the weekly set we find a shortage of breaks, which are to be considered a flaw of the algorithm. Comparing the 24 structural breaks we detect in S&P500, Caporin and Malik (2020) locate only 8 breaks in approximately the same time span. An explanation of this is that Caporin and Malik used a modified ICSS algorithm, and due to this not being the main goal of our thesis we applied an unmodified version.

The recent paper from McAleer (2009) focus on the caveats of the DCC-GARCH(1,1) model. The paper analyzes the widely used DCC-model and criticizes the terms of algebraic existence, mathematical regularity conditions and asymptotic properties of consistency of the Dynamic Conditional Correlation Model. McAleer further persists that the DCC-model should be avoided at all costs and only be used with extreme caution in empirical practice (McAleer, 2009). If we could start all over again we would have considered the use of another model, but since the DCC-GARCH(1,1) model is widely used in the academic community, we stand by our results from our use of the DCC model.

7 Conclusion

The initial purpose of this thesis has been to investigate the structural breaks and volatility spillover from surrounding financial markets affects the Norwegian financial market with the OSEBX index in specific. In total we construct three different multivariate GARCH-models to explore the effects of structural breaks and volatility spillover to and from the Norwegian market. To locate the specific dates of each structural break we apply the ICSS-algorithm by Inclan and Tiao (1994).

From the ICSS-algorithm we find several breaks on each time series. We do, however, find that the algorithm works better for daily observations than for weekly observations. Even though some of the findings in daily data are to be considered noise in the series, the algorithm does not detect breaks in clear changes in volatility.

From conducting an analysis of the correlation of our dataset, we find strong evidence for time variability in the conditional correlation. Our intention was to use the CCC-GARCH(1,1) model by Bollerslev (1994), however, due to the dynamic nature of the conditional correlation, the DCC-GARCH(1,1) model by Engle (2002) was a better fit for our dataset. From the DCC-GARCH (1,1) we find that in periods of structural breaks, the correlation between the indices has a tendency to decline. This is most notable during the financial crisis of 2008-2009.

From the BEKK-GARCH (1,1) model by Engle and Kroner (1995) we find strong evidence that structural breaks and volatility spillover affects the volatility to and from OSEBX. The majority of indices have a stronger effect on the volatility to OSEBX, but interestingly we found evidence of the relationship being reversed when exploring OSEBX and FTSE. Here we see an indication of structural breaks and spillover affecting the returns in FTSE. We also find evidence of a unidirectional relationship of volatility spillover from OSEBX to DAX. Through more thorough analysis we find that in the instance of OSEBX and FTSE this was due to the Norwegian economy's strong dependence on oil, as well as its effect on macroeconomic conditions in general. As for the instance of OSEBX and DAX, although it can be explained to an extent in weekly data, we can not explain the volatility spillover from OSEBX to DAX in daily data.

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Appendix

Appendix A

		OSEBX/OMX.S		OMX.C	
1		Estimate	Pr(>$\lvert t\rvert$)	Estimate	Pr(>$\lvert t\rvert$)
2	[A]mu	0.000771	0.000000	0.000790	0.000000
3	[A]omega	0.000003	0.011219	0.000003	0.008658
4	[A]alpha1	0.116348	0.000000	0.114389	0.000000
5	[A]beta1	0.866588	0.000000	0.868140	0.000000
6	[B]mu	0.000567	0.000244	0.000699	0.000003
7	[B]omega	0.000002	0.475991	0.000005	0.002899
8	[B]alpha1	0.091102	0.009383	0.103188	0.000000
9	[B]beta1	0.902144	0.000000	0.859745	0.000000
10	[Joint]dcca1	0.035144	0.000000	0.040868	0.000000
11	[Joint]dccb1	0.953720	0.000000	0.928816	0.000000

Table 8: DCC results (3)

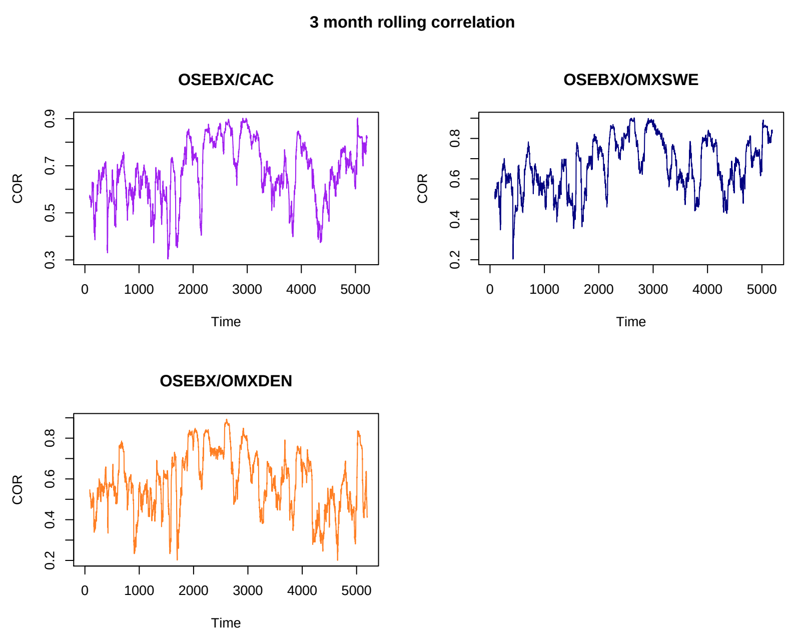

Figure 14: Structural breaks for OMX Copenhagen

Appendix B

	Series	Break points	Time period
1	OSEBX	26.00	Jan 4, 2000 - May 30, 2000
2			May 31, 2000 - Sep 12, 2001
3			Sep 13, 2001 - Nov 21, 2001
4			Nov 22, 2001 - Jun 07, 2002
5			Jun 08, 2002 - Jun 25, 2003
6			Jun 26, 2003 - May 2, 2005
7			May 3, 2005 - Nov 23,2005
8			Nov 24, 2005 - May 15, 2006
9			May 16, 2006 - Jul 31, 2006
10			Aug 1, 2006 - Aug 9, 2007
11			Sep 10, 2007 - Sep 15, 2008
12			Oct 16, 2008 - Dec 23, 2008
13			Dec 24, 2008 - Jul 20, 2009
14			Jul 21, 2009 - May 5, 2010
15			May 6, 2010 - Sep 8, 2010
16			Sep 9, 2010 - Aug 4, 2011
17			Aug 5, 2011 - Dec 28, 2011
18			Dec 29, 2011 - Nov 2, 2012
19			Nov 3, 2012 - Oct 2, 2014
20			Oct 3, 2014 - Jan 27, 2015
21			Jan 27, 2015 - Aug 24, 2015
22			Aug 25, 2015 - Jul 29, 2016
23			Jul 30, 2016 - Oct 11, 2018
24			Oct 12, 2018 - Jan 24, 2019
25			Jan 25, 2019 - Feb 24, 2020
26			Feb 25, 2020 - May 7, 2020

Table 9: Break periods for OSEBX

	Series	Break points	Time period
1	OSESX	17.00	Jan 4, 2000 - May 4, 2000
2			May 5, 2000 - May 12, 2000
3			May 13, 2000 - Oct 6, 2005
4			Oct 7, 2005 - Jul 4, 2006
5			Jul 5, 2006 - Sep 9, 2008
6			Sep 10,, 2008 - Nov 26, 2008
7			Nov 27, 2008 - Jul 21, 2009
8			Jul 22, 2009 - May 4, 2010
9			May 5, 2010 - Jul 19, 2010
10			Jul 20, 2010 - Aug 2, 2011
11			Aug 3, 2011 - Jul 5, 2012
12			Jul 6, 2012 - Aug 19, 2015
13			Aug 20, 2015 - Jul 19, 2016
14			Jul 20, 2016 - Oct 11, 2018
15			Oct 12, 2018 - Jan 16, 2019
16			Jan 17, 2019 - Feb 24, 2020
17			Feb 25, 2020 - Apr 22, 2020

Table 10: Break periods for OSESX

	Series	Break points	Time period
1	S&P 500	24.00	Jan 4, 2000 - Apr 23, 2001
2			Apr 24, 2001 - Jun 3, 2002
3			Jun 3, 2002 - Oct 3, 2002
4			Oct 4, 2002 - Aug 29, 2003
5			Aug 30, 2003 - Jul 2, 2007
6			Jul 3, 2007 - Jun 19, 2008
7			Jun 20, 2008 - Sep 19, 2008
8			Sep 20, 2009 - Jan 15, 2009
9			Jan 16, 2009 - Apr 22, 2009
10			Apr 23, 2009 - Jan 15, 2010
11			Jan 16, 2010 - May 25, 2010
12			May 26, 2010 - Apr 6, 2011
13			Apr 7, 2011 - Sep 9, 2011
14			Sep 10, 2011 - Mar 13, 2015
15			Mar 14, 2015 - Apr 30, 2015
16			May 1, 2015 - Oct 2, 2015
17			Oct 3, 2015 - Jun 9, 2016
18			Jun 10, 2016 - Aug 4, 2017
19			Aug 4, 2017 - Oct 25, 2017
20			Oct 26, 2017 - Apr 5, 2018
21			Apr 6, 2018 - Jul 23, 2018
22			Jul 24, 2018 - Aug 2, 2019
23			Aug 3, 2019 - Sep 24, 2019
24			Sep 25, 2019 - Dec 4, 2019

Table 11: Break periods for SP500

	Series	Break points	Time period
1	FTSE	22.00	Jan 4, 2000 - Jul 24, 2001
2			Jul 25, 2001 - Oct 2, 2001
3			Oct 3, 2001 - Apr 18, 2002
4			Apr 19, 2002 - Aug 22, 2002
5			Aug 23, 2002 - Feb 27, 2003
6			Feb 28, 2003 - Dec 10, 2003
7			Dec 11, 2003 - Nov 14, 2005
8			Nov 15, 2005 - Feb 22, 2006
9			Feb 23, 2006 - Jan 23, 2007
10			Jan 24, 2007 - Mar 3, 2008
11			Mar 4, 2008 - Jun 6, 2008
12			Jun 7, 2008 - Oct 27, 2008
13			Oct 28, 2008 - Feb 2, 2010
14			Feb 3, 2010 - Dec 15, 2010
15			Dec 16, 2010 - May 10, 2011
16			May 11, 2011 - Nov 20, 2012
17			Nov 21, 2012 - Jan 21, 2014
18			Jan 22, 2014 - Nov 27, 2014
19			Nov 28, 2014 - Oct 20, 2015
20			Oct 21, 2015 - Apr 27, 2017
21			Apr 27, 2017 - Apr 26, 2019
22			Apr 27, 2019 - Jul 8, 2019

Table 12: Break periods for FTSE

	Series	Break points	Time period
1	CAC	23.00	Jan 4, 2000 - Apr 16, 2002
2			Apr 17, 2002 - Sep 11, 2002
3			Sep 11, 2002 - Mar 4, 2003
4			Mar 5, 2003 - Sep 4, 2003
5			Sep 5, 2003 - Jul 29, 2004
6			Jul 30, 2004 - Jan 10, 2006
7			Jan 11, 2006 - Apr 19, 2006
8			Apr 19, 2006 - Mar 16, 2007
9			Mar 17, 2007 - May 9, 2008
10			May 10, 2008 - Aug 7, 2008
11			Aug 8, 2008 - Dec 30, 2008
12			Dec 31, 2008 - Nov 27, 2009
13			Nov 28, 2009 - Jan 15, 2010
14			Jan 16, 2010 - Apr 20, 2010
15			Apr 21, 2010 - Mar 1, 2011
16			Mar 2, 2011 - Aug 2, 2011
17			Aug 3, 2011 - Jan 31, 2013
18			Feb 1, 2013 - Apr 7, 2014
19			Apr 8, 2014 - Dec 18, 2014
20			Dec 19, 2014 - Jan 29, 2016
21			Jan 30, 2016 - Mar 19, 2018
22			Mar 20, 2018 - Aug 26, 2019
23			Aug 27, 2019 - Oct 10, 2019

Table 13: Break periods for DAX

	Series	Break points	Time period
1	DAX	19.00	Jan 4, 2000 - Jul 31, 2001
2			Aug 1, 2001 - Oct 16, 2001
3			Oct 17, 2001 - May 8, 2002
4			May 9, 2002 - Mar 6, 2003
5			Mar 7, 2003 - Mar10, 2004
6			Mar 11, 2004 - Jan 10, 2006
7			Jan 11, 2006 - Sep 10, 2007
8			Sep 11, 2007 - May 29, 2008
9			May 30, 2008 - Aug 12, 2008
10			Aug 13, 2008 - Jan 8, 2009
11			Jan 9, 2009 - Mar 8, 2010
12			Mar 9, 2010 - Mar 10, 2011
13			Mar 11, 2011 - Jul 29, 2011
14			Jul 30, 2011 - Apr 23, 2012
15			Apr 24, 2012 - May 8, 2014
16			May 9, 2014 - Feb 9, 2016
17			Feb 10, 2016 - Aug 2, 2017
18			Aug 3, 2017 - Aug 7, 2019
19			Aug 8, 2019 - Nov 22, 2019

Table 14: Break periods for CAC

	Series	Break points	Time period
1	OMX Stockholm	20.00	Jan 4, 2000 - Aug 17, 2000
2			Aug 18, 2000 - Apr 4, 2003
3			Apr 5, 2003 - Aug 9, 2004
4			Aug 10, 2004 - Jul 5, 2006
5			Jul 6, 2006 - Jun 29, 2007
6			Jun 30, 2007 - Aug 4, 2008
7			Aug 5, 2008 - Nov 7, 2008
8			Nov 8, 2008 - Mar 25, 2009
9			Mar 26, 2009 - Jul 27, 2009
10			Jul 28, 2009 - Mar 10, 2010
11			Mar 11, 2010 - Jul 19, 2010
12			Jul 20, 2010 - Jun 1, 2011
13			Jun 2, 2011 - Oct 20, 2011
14			Oct 21, 2011 - Jul 24, 2012
15			Jul 25, 2012 - Jul 25, 2014
16			Jul 26, 2014 - Jun 10, 2015
17			Jun 11, 2015 - May 26, 2016
18			May 27, 2016 - Nov 13, 2017
19			Nov 14, 2017 - Nov 19, 2019
20			Nov 20, 2019 - Mar 16, 2020

Table 15: Break periods for OMX Stockholm

	Series	Break points	Time period
1	OMX Copenhagen	21.00	Jan 4, 2000 - Jun 10, 2002
2			Jun 11, 2002 - Nov 5, 2002
3			Nov 6, 2002 - May 12, 2004
4			May 13, 2004 - Sep 29, 2005
5			Sep 30, 2005 - Jul 7, 2006
6			Jul 8, 2006 - Jul 23, 2007
7			Jul 24, 2007 - Sep 2, 2008
8			Sep 3, 2008 - Nov 25, 2008
9			Nov 26, 2008 - Jun 8, 2009
10			Jun 8, 2009 - Mar 22, 2010
11			Mar 23, 2010 - Aug 6, 2010
12			Aug 7, 2010 - Jun 27, 2011
13			Jun 28, 2011 - Oct 10, 2011
14			Oct 11, 2011 - Jun 29, 2012
15			Jun 30, 2012 - Aug 26, 2014
16			Aug 27, 2014 - Jul 9, 2015
17			Jul 10, 2015 - Feb 5, 2016
18			Feb 6, 2016 - Oct 12, 2016
19			Oct 13, 2016 - Dec 13, 2017
20			Dec 14, 2017 - Jan 3, 2020
21			Jan 4, 2020 - Mar 2, 2020

Table 16: Break periods for OMX Copenhagen

	Series	Break points	Time period
1	Brent Crude oil	14.00	Dec 11, 2000 - Feb 19, 2001
2			Feb 20, 2001 - Aug 20, 2001
3			Aug 21, 2001 - Jan 30, 2002
4			Jan 31, 2002 - Feb 8, 2005
5			Feb 9, 2005 - Jan 15, 2008
6			Jan 16, 2008 - Aug 1, 2008
7			Aug 2, 2008 - Feb 16, 2009
8			Feb 17, 2009 - Jan 14, 2011
9			Jan 15, 2011 - Sep 9, 2013
10			Sep 10, 2013 - Jul 27, 2015
11			Jul 28, 2015 - Apr 10, 2017
12			Apr 11, 2017 - Jun 20, 2018
13			Jun 21, 2018 - Aug 23, 2018
14			Aug 24, 2018 - Oct 4, 2018

Table 17: Break periods for Brent Crude Oil

Appendix C

		OSEBX/OSESX		OSEBX/SP500		OSEBX/FTSE	
1		Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
2	mu1	0.000757826	0.013372 *	0.001354001	4.8368e-06 ***	0.001109977	0.00014690 ***
3	mu2	0.002078158	1.0658e-14 ***	0.000130479	0.6186607	0.000455071	0.062722795 .
4	A011	0.002655093	< 2.2e-16 ***	0.003603573	1.7319e-14 ***	0.003040039	6.5243e-11 ***
5	A021	0.001683377	NA	0.001210184	0.0027790 **	0.002174536	1.4984e-09 ***
6	A022	0.001224696	NA	0.002055652	6.6781e-07 ***	0.001560008	4.0738e-07 ***
7	A11	0.335897576	4.4409e-16 ***	0.436545243	< 2.2e-16 ***	0.304413870	1.1819e-09 ***
8	A21	0.102176234	< 2.2e-16 ***	0.176712421	8.3552e-07 ***	0.052386436	0.23149685
9	A12	(-)0.043040697	0.413168	(-)0.037055544	0.5706462	0.226515773	1.4147e-05 ***
10	A22	0.019621330	NA	0.294119141	8.8383e-12 ***	0.421351828	< 2.2e-16 ***
11	B11	0.898041960	< 2.2e-16 ***	0.865562596	< 2.2e-16 ***	0.920835916	< 2.2e-16 ***
12	B21	(-)0.025088321	NA	(-)0.057402677	0.0063838 **	(-)0.015697032	0.51985739
13	B12	0.039637961	NA	(-)0.009411851	0.7366106	(-)0.104507014	0.00011997 ***
14	B22	0.987779398	NA	0.916062064	< 2.2e-16 ***	0.871810563	< 2.2e-16 ***

Table 18: Bekk results (5)

		OSEBX/DAX		OSEBX/CAC	
1		Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
2	mu1	0.001125600	0.00014833 ***	0.001190234	5.6364e-05 ***
3	mu2	0.000208661	0.53090192	0.000453733	0.174349
4	A011	0.003578595	< 2.2e-16 ***	0.003556290	< 2.2e-16 ***
5	A021	0.002385496	1.1771e-11 ***	0.002971404	8.7999e-10 ***
6	A022	0.002151739	4.7151e-12 ***	0.002003059	1.2531e-09 ***
7	A11	0.357117565	< 2.2e-16 ***	0.389355395	7.5415e-12 ***
8	A21	0.067226346	0.06996878 .	0.127419179	0.041206 *
9	A12	0.138963600	0.00011096 ***	0.109218702	0.019913 *
10	A22	0.325954403	< 2.2e-16 ***	0.319967086	4.3521e-14 ***
11	B11	0.866906289	< 2.2e-16 ***	0.872222544	< 2.2e-16 ***
12	B21	(-)0.044522618	0.03634960 *	(-)0.061679469	0.021663 *
13	B12	(-)0.039112308	0.00381421 **	(-)0.046354375	0.029712 *
14	B22	0.926933712	< 2.2e-16 ***	0.915791627	< 2.2e-16 ***

Table 19: Bekk results (6)

		OSEBX/OMX.S		OSEBX/OMX.C	
1		Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
2	mu1	NA	NA	0.001476064	1.3585e-05 ***
3	mu2	NA	NA	0.000885616	0.00487989 **
4	A011	NA	NA	0.003432224	1.6353e-05 ***
5	A021	NA	NA	0.001564779	0.00030973 ***
6	A022	NA	NA	0.001823754	5.3784e-07 ***
7	A11	NA	NA	0.333310930	0.03090576 *
8	A21	NA	NA	0.093958177	0.31644121
9	A12	NA	NA	0.107035873	0.49684631
10	A22	NA	NA	0.206603209	0.01260595 *
11	B11	NA	NA	0.878844495	< 2.2e-16 ***
12	B21	NA	NA	(-)0.029409373	0.44669552
13	B12	NA	NA	(-)0.016961333	0.65838067
14	B22	NA	NA	0.957054588	< 2.2e-16 ***

Table 20: Bekk results (7)

		OSEBX/Brent		FTSE/Brent		DAX/Brent	
1		Estimate	Pr(>|t|)	Estimate	Pr(>|t|)	Estimate	Pr(>|t|)
2	mu1	0.001406976	2.7210e-06 ***	0.000540952	0.0322040 *	0.000264945	0.430234
3	mu2	0.001157419	0.03063993 *	0.001048413	0.0471939 *	0.001119190	0.036970 *
4	A011	0.003106563	<2.22e-16 ***	0.002234016	1.3523e-13 ***	0.002811894	2.8616e-09 ***
5	A021	0.002266679	0.00510588 **	0.002416363	0.0085388 **	0.001354847	0.082429 .
6	A022	0.003897829	2.0488e-06 ***	0.003969334	1.3249e-05 ***	0.004078765	1.5253e-06 ***
7	A11	0.440534937	<2.22e-16 ***	0.378134625	<2.22e-16 ***	0.339386928	<2.22e-16 ***
8	A21	0.233692563	2.4415e-06 ***	0.280575054	1.1657e-07 ***	0.173799725	2.1590e-05 ***
9	A12	(-)0.025767222	0.13104950	(-) 0.013486133	0.3887025	0.015185335	0.515128
10	A22	0.255459861	<2.22e-16 ***	0.270356528	5.0626e-14 ***	0.315756823	<2.22e-16 ***
11	B11	0.874175784	<2.22e-16 ***	0.908109566	<2.22e-16 ***	0.918727413	<2.22e-16 ***
12	B21	(-)0.091762903	0.00044013 ***	(-) 0.093218466	0.0017256 **	(-) 0.052633369	0.036750 *
13	B12	(-)0.004176609	0.66301206	(-) 0.007894308	0.03239597	(-) 0.007568953	0.499641
14	B22	0.933696877	<2.22e-16 ***	0.924144511	<2.22e-16 ***	0.924572247	<2.22e-16 ***

Table 21: Bekk results (8)