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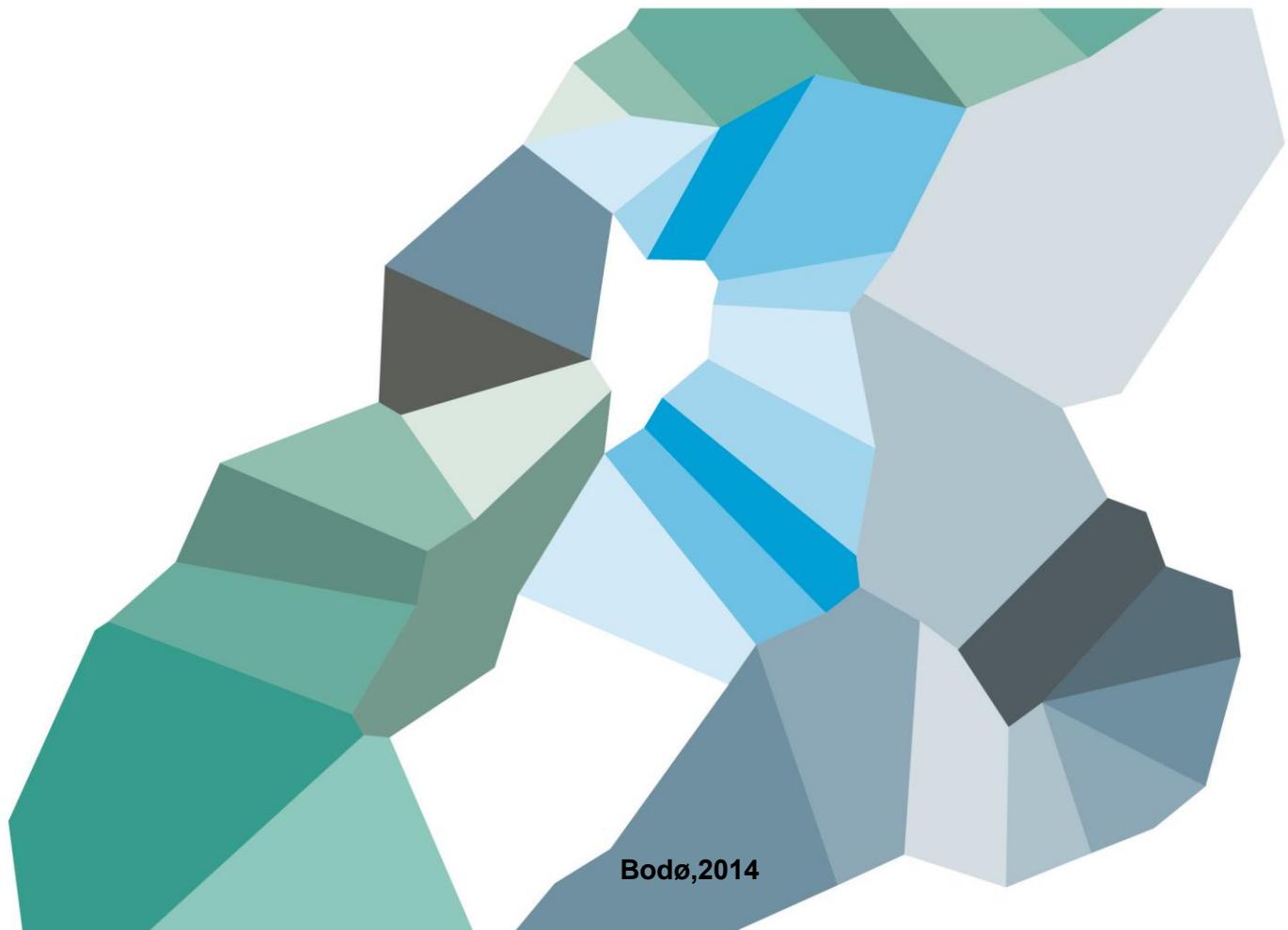
MASTER THESIS

**Optimal vs Naive diversification.
Introduction and testing a new model.**

Course code: BE305E

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Introduction.

In about the fourth century, Rabbi Issac bar Aha proposed the following rule for asset allocation: “One should always divide his wealth into three parts: a third in land, a third in merchandise, and a third ready to hand.”. 16th centuries later allocating wealth across risky assets has become a highly discussed issue. For the last 60 years there were plenty of complicated “optimal” rules and strategies developed in this field. However, as was noted by different researchers, the naive allocation rule, which assumes equally distributing wealth across N assets in proportion $1/N$ for each, still wasn’t beaten consistently by any of those sophisticated strategies(De Miguel et al, 2009). Basing on previously developed models, their advantages and disadvantages, I will try to find the optimal set of factors, which explain the returns the best. I will construct a new model that includes those factors. This “new” model will be tested using data from Russian and US stock markets. In order to test this model, I will compare its performance with $1/N$ strategy using Sharpe ratio as measure.

Personal motivation.

My personal motivation for writing thesis on this particular topic is determined by two factors. First, I got really interested in Finance, especially in portfolio theory and Econometrics. I wanted to combine those two subjects when writing my master thesis. So I chose the topic concerning portfolio construction simply because I like this subject. The second factor is that I’m eager to try to achieve the results that could give opportunities for further research. These results could serve as a starting point for making a contribution in science and/or useful for my future career.

Actualization, limitations of study and assumptions.

Since Henry Markowitz, there have been plenty of studies, developing an optimal model, which is able to provide the highest risk-return trade-off. Over the last years, so-called Smart-Beta strategies, risk factor investing are becoming more and more popular among investors, as an alternative to cap-weighted indexes. The purpose of this work is to try to capture as much common risk factors as possible by one single beta, and based on this beta, to construct a portfolio, which can consistently outperform the $1/N$ benchmark. The *limitations* of this study are following: this study only concerns stocks, included in S&P 500 index in the research period. Any effects of dividends, company merges, or overnight price changes are ignored. There are no restrictions on the number of stocks in any portfolio in any of the periods. There is only one *Assumption* in this Thesis: the distribution of all the returns in any given period is (or at least significantly close to) normal.

The structure of work

At the very beginning, there's a short section, named Literature review, where I give short notes, explaining which sources did I rely on each stage of the research project development.

This work consists of several sections. Section 1- provides the theoretical base for the research. The most popular models are discussed within this section, their major shortcomings are discussed. The first section is organized as follows: first, the model is presented, than it's shortcomings are discussed, then the way of solving them leads to the next model, which is presented. So the models are discussed one by one, with explaining the reasons for the next one appearing.

In Section 2 Methodology is discussed in details. The philosophical position, Data collection and analysis, research design and strategy are discussed in this section.

Section 3 explains how exactly the calculations were done and shows the results of calculations. In this section, the performance of portfolios, the factors and betas results are discussed and clarified.

Section 4 concludes the paper.

There is no Appendix because it is too enormous to be included in this particular paper. All the tables with raw data and calculations are available on request in “.xlsx”.

Literature review

In this very short section, I will provide some notes on the major literature sources, that I used during every step of the research.

1. The main books, I used for the Theoretical part were “Investments and portfolio management by Bodie, Kane, Marcus, Global edition, 2011 and Mark Grinblatt, Sheridan Titman Financial Markets and Corporate Strategy *International edition, McGraw-Hill, 1998*. This books provides the major theoretical aspects, concerning the existing optimal portfolio formation models as well as give some shortcomings of those.

The major articles that I used for Theoretical background, as well as later for clarifying and explaining what exactly has been done and why so:

Andrew Ang, Robert J. Hodrick, Yuhang Xing, Xiaoyan Zhang 2007 High Idiosyncratic Volatility and Low Returns: International and Further U.S. Evidence

2. Victor DeMiguel, Lorenzo Garlappi, Raman Uppal, Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy? 2009
3. Weimin Liu Liquidity Premium and A Two-factor Model Current Draft: July 2004

This articles underpin the basic Idea, the hypothesis of this work, as well as derive the actual calculations and specifications. I can definitely say, that this paper is inspired by and based on those particular articles.

The methodology part majorly refers to “Management research 4th edition” by Mark Easterby-Smith, Richard Thorpe and Paul Jackson, 2012. And to the second edition of “Introductory Econometrics for Finance” by Chris Brooks, 2008.

Later, in the Third Section of this work, I actively referred to the paper named “Smart Beta 2.0” developed by Noel Amenc, Felix Goltz and Lionel Martellini in June 2003. Basing on this paper, I defined and justified the way of computing the smart beta for my strategy and why the model proposed in this paper can be seen as the Smart-Beta strategy.

1. Theoretical Part.

1. Models overview

1.1 Markowitz model

Optimal capital allocation between the risky assets was a strongly discussed issue for a very long time. A lot of scientists and actual practicing investors in the field were racking their brain seeking the best formula of capital allocation from risk-return perspective. There were plenty of approaches in solving this problem developed during the last half of 20th century. In this section I will give a short overview of the major ones. It all started with Henry Markowitz in 1952. In his “pathbreaking work he derived the *optimal* rule for allocating wealth across risky assets in a static setting when investors care only about the mean and variance of a portfolio’s return.” (De Miguel, et al 2007).

In this work Harry Markowitz have “brought fancy math into economics” and due to this he stated and proved the fact that not correlated risks work best for diversification purposes, while those, that move together are riskier. This idea is taken for-obviously-granted now, though when Markowitz wrote his work, it was a novel. But the last mortgage crisis of 2008 shows how we still managed to fail to implement those basic ideas. (Crovitz, 2008)

Citating Jun Tu and Guofu Zhou’s article in The Journal of Financial Economics, 2011: “Although more than half a century has passed since Markowitz’s (1952) seminal paper, the mean-variance (MV) framework is still the major model used in practice today in asset allocation and active portfolio management despite many other models developed by academics.” The authors emphasize two main reasons for that: first is that “many real-world issues, such as factor exposures and trading constraints, can be accommodated easily within this framework with analytical insights and fast numerical solutions.” And the second reason explained in the same paper by the same authors is: “intertemporal hedging demand is typically found to be small.” (Tu, Zhou, 2011)

His model was later called “step one of portfolio management: the identification of the efficient set of portfolios, or the efficient frontier of risky assets.” (Bodie, Kane, Marcus, 2011). I will use those author’s book “Investments and Portfolio Management”, edition 9 pretty often during the work. So later, I would refer to these Authors as BKM.

1.2 Single-index model

But of course this model could not be left without any critics, which to my opinion is an engine of any science. Referring to BKM the first difficulty with Markowitz model is the huge amount of estimates. To apply the model for portfolio consisting of n stocks we need n estimates of variances, n estimates of expected return and $(n^2 - n)/2$ estimates of covariances. In the example provided in BKM's textbook, to find an optimal portfolio of 50 stocks, we need 1 325 total estimates. This amount is 4 times bigger for a set of 100 stocks! (Bodie, et al 2011)

Another issue with Markowitz model is the error in assessing correlation coefficients. As noted by BKM, such errors can lead to "nonsensical results" like, e.g. negative variance. (Bodie, et al 2011)

The same issue of Markowitz portfolio model was shown in the work of Tu and Zhou, that I've already mentioned above. In this work they used the statement by Jobson and Korkie (1980) who said: "Due to estimation errors "naive formation rules such as the equal weight rule can outperform the Markowitz rule." The naive diversification rule (equal weight rule) means that wealth is to be allocated between a set of N risky assets, in equal proportion $1/N$ for each. This rule was first academically studied by Brown(1976). (Tu and Zhou, 2011)

The amount of errors was tried to be reduced using new statistical tools and approaches, starting with "*Bayesian approach*" to estimation error, with its multiple implementations ranging from the purely statistical approach relying on diffuse-priors (Barry, 1974; Bawa, Brown, and Klein, 1979), to "shrinkage estimators" (Jobson, Korkie, and Ratti, 1979; Jobson and Korkie, 1980; Jorion, 1985, 1986), to the more recent approaches that rely on an asset-pricing model for establishing a prior (P'astor, 2000; P'astor and Stambaugh, 2000)"(De Miguel et al 2007)

But in this work I will try to stay focused on different models. So, going back to Markowitz model's critique, the new approach suggested was the Index models. It started with a single index model, which significantly reduced the amount of estimates needed to construct an optimal risky portfolio. This approach is based on a presupposition, that a broad market index can be a valid proxy for common factor. By common I mean a factor which affects all the companies in industry. So using the market index as a proxy, a simple single- index model was developed similarly to single-factor model. This model is linear and so the sensitivity of every stock to a common factor can be measured by using a single-variable linear regression. This coefficient is called β and the model basically looks like:

$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t)$$

R_m is for the market excess return, α is the intercept, showing the return on a stock when market return equals zero, and e is the error term or residual. In economics terms this residual represents

the firm-specific, surprise effect on return with a zero mean. If we ignore the constant (intercept) we can see that the uncertain return is decomposed into two parts: systematic or explained by market proxy through sensitivity coefficient beta and unsystematic, firm-specific or unexplained part described with the error term e . (Grinblatt, Titman, 1998)

To examine the advantages of a single-factor (single-index) model, I go back to the previous example of a 50 stock portfolio, and check the amount of estimates needed for that model. For single-index model we need the following estimates: n estimates of alfa (intercept), n estimates of beta, 1 estimate of market excess return and 1 estimate of common factor's variance. So in total we have $3n+2$ estimates which in our example is 152. This number is significantly less than 1 325 we needed for Markowitz model. (Bodie, et al 2011)

Another advantage pointed by BKM in 2008 and confirmed in 2011 is that the index models allow specialization of security analysis by industries simplifying the process of computing variances. On the other hand, index-model ignores the correlations, which might actually take place, assuming they're equal zero. While Markowitz algorithm actually takes into account covariances between every pair of stocks in the portfolio. The single-index model "oversimplifies the sources of real world uncertainties". So, BKM conclude that "optimal portfolio derived from the single-index model can be significantly inferior to that of the full-covariance (Markowitz) model when stocks with correlated residuals have large alpha values and account for a large fraction of the portfolio." (Bodie et al, 2008, 2011)

The last disadvantage of the single index model was also pointed out earlier by Grinblatt and Titman in 1998. They provided an example of influence of interest rate changes on GM's stock price, which would be counted as a part of market model's residual, a diversifiable risk, while indeed it's "a common factor". (Grinblatt, Titman 1998)

Referring to Don U.A. Galagedera's article in Managerial Finance, «when analyzing the risk of an individual security, however, the individual security risk must be considered in relation to other securities in the portfolio.» The risk of individual security have to be measured in terms of the additive risk in the portfolio as this security's contribution. He concludes that "a security's contribution to the portfolio risk is different from the risk of the individual security." The CAPM developed by Sharpe (1964) and Lintner (1965) relates the expected rate of return of an individual security to a measure of its systematic risk. (Galagedera, 2007)

1.3 CAPM

"The capital asset pricing model (CAPM) of William Sharpe (1964) and John Lintner (1965) marks the birth of asset pricing theory (resulting in a Nobel Prize for Sharpe in 1990). Four

decades later, the CAPM is still widely used in applications, such as estimating the cost of capital for firms and evaluating the performance of managed portfolios.”Fama and French explain such a popularity of a capital asset pricing model with it’s attractive suggestion of simple way of prediction the risk-return relations and risk measurement.(Fama and French, 2004)

I decided to shortly explain the basics of CAPM’s theory before analyzing its performance in implications. CAPM uses market portfolio instead of broader market index used in Single-factor model. Traditionally, CAPM is focused on risk- expected return relation, which under this model looks like:

$$E(R_i) = r_f + \beta_i(E(r_M) - r_f)$$

Where r_f is a risk-free return. So we can see that this model describes the dependence on expected security’s excess return to expected market portfolio’s excess return as a linear function. The CAPM needs a several amount of assumptions to be held.

1. Investors care only about the mean and variance of the portfolio’s returns
2. Markets are frictionless
3. Investors have homogenous beliefs, meaning that they share the same beliefs about means and standard deviations of portfolios.

(Grinblatt Titman, 1998)

Later, BKM have added a few more assumptions such as:

1. Investors are able to get risk-free loans
2. Myopic behavior- no concern about any changes after the end of single period horizon.

(Bodie, et al2011)

CAPM refers to the market portfolio as tangency portfolio. In CAPM it’s not the variance of the stock itself that is important, but it’s beta, that describes the covariance of the stock’s return with the return on market portfolio. According to Grinblatt and Titman, “market portfolio is a portfolio, where the weight on each asset is the market value of that asset divided by the market value of all risky assets. “(Grinblatt, Titman, 1998)

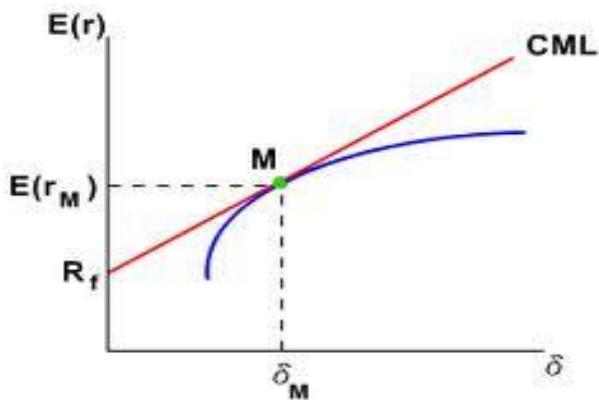
The CAPM is based on two fundamental relationships: Capital market line and Security market line. The former explains “the return an individual investor expects to receive on a portfolio”, while the latter “expresses the return an individual investor can expect in terms of a risk-free rate and the relative risk of a security or portfolio.”(Galagedera, 2007)

The capital market line is described by equation:

$$R_p = r_f + [(R_T - r_f) \times \sigma_p] / \sigma_T$$

Where R_T and σ_T are respectively return and risk (measured by standard deviation) of a tangency portfolio. Tangency portfolio is a unique optimal portfolio with no investments in risk-free assets. The name “Tangency” comes from a graphical interpretation of this unique portfolio. The line that connects this portfolio (denoted by M on Figure 1) with risk-free investments is tangent to the efficient frontier of risky investments. In other words there's no risky portfolio on efficiency frontier that provides a better risk-return tradeoff. This line is the CML, as shown on Figure 1. As mentioned before, in CAPM tangency portfolio is the market portfolio, often denoted by M.

Figure 1.



(Grinblatt and Titman, 1998)

The security market line is different from capital market line mainly for measuring risk through beta, not by using standard deviation. This line is represented with the formula which already was mentioned above.

$$E(R_i) = r_f + \beta_i(E(r_M) - r_f)$$

This formula describes relation between expected return and beta which is the familiar Sharpe-Lintner CAPM equation. Or in other words, classical Sharpe-Lintner CAPM model.

(Fama and French, 2004)

The security market line is showed on a graph on Figure 2. You can see comparing this graph with the graph of CML that while portfolios with the same expected return can have different variances, they will have the same beta. (Grinblatt and Titman, 1998)

In their article in Journal of Economic perspectives in 2004, Fama and French say that the “empirical record on the model is poor enough to invalidate the way it's used in applications.” They point out some reasons for this empirical failure. First of all, Fama and French points an

overwhelming set of unrealistic assumptions in this model. So these issues lead to idea of theoretical problems concerning CAPM that lead to such a bad empirical performance. Fama and French point two major unrealistic assumptions in their article. First is about unrestricted risk-free borrowing and lending. Fisher Black in 1972 has provided a version of CAPM where unrestricted borrowing and lending assumption was ruled out, instead he used an assumption of unrestricted short-sales. He shows, that using unrestricted short-sales would lead to the same “key result” as unrestricted risk-free borrowing and lending- the proof that market portfolio is mean-variance efficient. The only difference between Sharpe-Lintner and Black’s model of CAPM lies in their explanations of the expected return on assets, not correlated with the market. In classic Sharpe-Lintner model this return has to be equal to R_f - the risk-free rate, while in Black’s opinion this uncorrelated expected return just has to be smaller than market return to provide positive risk premium. But unrestricted short-sales assumption is as unrealistic as the first one. The lack of short sales and risk-free assets lead to the efficient portfolios formed not being typically efficient. This includes the market portfolio.(Fama and French 2004)

There were a lot of statistical tests run on CAPM .*Don UA Galagedera (2007)* provides a short overview of responses on statistical tests of single-factor CAPM. Considering the fact that CAPM uses past data to predict post data Galagedera poses a question if the past security returns actually conform to the CAPM. Summarizing the results of research provided in his arcticle I would state the following results:

- Miller and Scholes (1972) highlighted some statistical problems that appear when using individual securities in testing the validity of the CAPM.
- “Black *et al.* (1972) reported a linear relationship between the average excess portfolio return and the beta, and for $\beta > 1$ (< 1) the intercept tends to be negative (positive). Therefore, they developed a zero-beta version of the CAPM model where the intercept term is allowed to change in each period.”
- “Fama and MacBeth (1973) provided evidence (i) of a larger intercept term than the risk-free rate, (ii) that the linear relationship between the average return and the beta holds and (iii) that the linear relationship holds well when the data covers a long time period “
- The single-factor CAPM is rejected when the portfolio used as a market proxy appears to be inefficient (Roll, 1977; Ross, 1977).
- Beta is unstable over time (Bos and Newbold, 1984)

(Galagedera2007)

BKM clarify, that tests of CAPM were run in order to check the hypothesis that values are uniformly zero under the assumption that the market proxy actually is close to the true market portfolio, which is actually unobservable. BKM provide possible explanations on why did CAPM fail those tests. As main possible reasons for that they state:

- Failure of data
- Invalid market proxy
- Unsuitable statistical method

If the reason for these failures is one of those state above (or all of them) BKM make the following conclusion: “there’s no a better model out there but we measure alpha and beta values with unsatisfactory precision.”

(Bodie, et al 2011)

Grinblatt and Titman in Chapter 5 of their book “Financial Markets and Corporate strategy” provide time-series and cross-sectional tests of CAPM. They show that both those tests find evidence, that doesn’t support CAPM. The most important violations of CAPM, pointed in that chapter are following:

- “The relation between estimated beta and average historical return is much weaker than the CAPM suggests
- The market capitalization or size of a firm is a predictor of its average historical return
- Stocks with low market-to-book ratios tend to have higher returns than those with higher book-to-market ratios
- Stocks that performed well over the past six months tend to have high expected returns over the following six months”

Grinblatt and Titman offer two explanations of poor CAPM’s empirical performance. The first is about possible inability of a chosen market portfolio proxies don’t capture all of the important risk factors in the economy. The second one is that CAPM is “simply a false theory”. Such a strong statement was explained by authors by the “investor’s behavioral biases against classes of stocks that have nothing to do with the mean and marginal risk.” In other words, financial managers or brokers would not take the risk of being fired, investing in a firm which is close to bankruptcy is newspapers say that. Even if the returns on such companies’ stocks are high.

(Grinblatt, Titman 1998)

The performance of CAPM on emergingwas also tested, revealing pretty much the same results. For example, Debarati Basu in 2013 tested CAPM performance on Indian market. He studied 10 portfolios, covering 50 stocks. Intercept term appeared to be

significant for all 10 portfolios. He found a negative relationship between beta and excess return, which according to Basu «indicates inefficient capital market. So on example of Indian market, CAPM «fails tests completely.»

(Basu, 2013)

As already mentioned above, the possible problem of CAPM could be inappropriate beta. So the new- consumption beta was offered, representing the original model as Consumption CAPM or CCAPM. This model was tested in Taiwan by Ming-Hsiang Chen. The results of his work were pretty surprising. In fact, he showed a better performance of regular CAPM comparing to CCAPM. He concluded following: “the relationship between stock returns and beta is statistically significant and the coefficient of determination of the regression is high across all of seven industry sub-sectors. In comparison, the CCAPM fails to explain the Taiwan stock market although the consumption beta should offer a better measure of systematic risk theoretically.”(Chen, 2001)

There was another research strengthening CAPM’s position. The research of Swedish stock exchange market. This research was a master thesis of Rustam Vosilov and Nicklas Bergström under supervision of Anders Isaksson(2010). When doing a literature review, I mainly found evidence against CAPM. That’s why I find the research of Vosilov and Bergström interesting. In their research they compared the performance of CAPM, conditional CAPM, Carhart’s four factor model and three factor model by Fama and French with momentum as a fourth factor. As a sample they used returns on stocks of 366 firms listed on stock exchange between September 1997 and April 2010. They found that CAPM “explains stock return cross-section better than the other models suggesting that Beta is still a proper measure of risk. Furthermore, the conditional version of CAPM describes the stock return variation far better than the unconditional CAPM.»(Vosilov, et al. 2010)

Conditional CAPM allows time-varying beta. This model was well tested by Jonathan Lewellen Stefan Nagel(2003). Their empirical results differ from those obtained by Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Petkova and Zhang (2003), whose work resulted in a conclusion that a change from unconditional to conditional CAPM plays an important role in explaining returns. They explain this difference in following way: “they focus on cross-sectional regressions, instead of the time-series intercept tests that we emphasize.” Jonathan Lewellen and Stefan Nagel explain the work of Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Petkova and Zhang (2003) is actually an evidence against conditional CAPM in case we take into account the restrictions, that are imposed on cross-sectional slopes. The main conclusion of

Lewellen and Nagel's paper is that conditional CAPM do not explain such anomalies as book-to market or momentum. They show that that ““the conditionalCAPM performs nearly as poorly as the unconditional CAPM.(Lewellenand Nagel, 2003)

According to *Don U A Galagedera*“a growing number of studies found that the cross-sectional variation in average security returns cannot be explained by the market beta alone.» He mentioned the famous Fama and French work of 1993, which I will return to later in this work. The main point so far, is that there can be more than one factor, explaining stock returns and this fact led to creation of various multifactor models. For example, Arbitrage Pricing Theory (APT) multifactor models introduced by Ross (1976). Author refers to Groenewold and Fraser (1997) who examined validity of these models using Australian data and concluded that «APT outperforms the CAPM in terms of within-sample explanatory power.»(Galagedera, 2007)

Another important issue noticed by Galagedera is that «for the CAPM to hold, normality of returns is a crucial assumption.” He claims that there were several studies conducted, which had shown the non-normality of security returns.Of course, there were some extensions and augmentations of CAPM mentioned for example by Galagedera in the same article, but I'll skip their introduction and give a short overview on multifactor and APT models. After that I'll introduce Fama and French three factor model and take a closer look at this one.

1.4 Multifactor models and Arbitrage Pricing Theory (APT).

Grinblatt and Titman have emphasized that “interest rate risk generates correlation between the market model residuals.” Basing on this statement,” they concluded that more than one common factor generates stock returns.” Multifactor model can be represented as a linear equation, similar to the one used in Single-Index model, but with multiple number of factors, with different betas(sensitivity coefficient) for every factor.

$$R_i = \alpha_i + \beta_{i,1}F_1 + \beta_{i,2}F_2 + \dots + \beta_{i,N}F_N + e_i$$

F represents factors from $i=1 \dots N$.

These factors could be, for example, oil prices, GDP, expected future inflation and so on. According to Grinblatt and Titman, a common factor is an economic variable (or its proxy) that has a significant effect on the returns on broad market indexes rather than individual securities alone.”

(Grinblatt, Titman 1998)

The security market line (SML) of a multifactor model is determined by a several amount of factors and betas, associated with those factors. So the investors excess return for bearing risk is now composed of the excess returns of every factor, multiplied with respective beta

coefficient. BKM states that the difference between single- and multifactor models is that “a factor risk premium can be negative.”

(Bodie, et al, 2011)

Arbitrage pricing theory (APT), proposed by Ross(1976) is a theory that explains the risk- return relationship alternatively to CAPM. The amount of assumptions needed for APT not to be violated is significantly smaller than for CAPM. There are only three assumptions required:

1. Returns can be described by a factor model
2. There are no arbitrage opportunities
3. There are a large number of securities to diversify away the firm-specific risk.

(Grinblatt, Titman, 1998)

The arbitrage opportunity means that the securities with the same betas have different prices, which allows the investors to sell short the underpriced security and purchase the overpriced one. In that case an investor obtains riskless proceeds with zero initial investments. A benchmark portfolio(s) for security market line in APT is so-called factor or tracking portfolio, which is a “well-diversified portfolio constructed to have a $\beta=1$ on one factor and a beta if zero on any other factor.”(Bodie, et al. 2011)

I won’t dive too deep into theory. I’ll go directly to comments and empirical tests. I will give some results of empirical tests of this model alone, and comparing it with CAPM Marc Reinganum(1980) tested this model. Reingaum claimed that parsimonious APT fails the test. He found that portfolios of small firms earn “on average 20% more than portfolios of large firms, even after controlling for APT risk. This result was detected regardless of whether APT risk is measured with a three-, four-, or five-factor model.» Reinganum also mentioned that several hypotheses were tested simultaneously, so one cannot define which of those cannot be supported. He offers some reasons for «bad» test results, such as: «the stochastic process generating returns of financial securities may not be linear (for example, see Jarrow and Rudd [1980]). Or one may not be able to completely diversify away the idiosyncratic variances.» Another explanation for that could be existing arbitrage opportunities on the market in analyzed time period. He concluded that «average returns obtained by grouping portfolios on the basis of firm size is still not accounted for by empirical representations of capital market equilibrium.”(Reinganum, 1980)

Debarati Basu(2013) has stated that «increasing doubt about the validity of the one-factor Capital Asset Pricing Model in pricing financial assets, development of newer models or extensions has become the order of the day.»He tested the APT multifactor model in pretty much the same way, as he did with CAPM earlier.(I have mentioned his work on CAPM-testing above). He used the same sample as when testing CAPM application to Indian market. «The

regression results display accurate relationships that are significant for each of the 10 portfolios and moderate to high explanatory power. Thus, it concludes that APT is a good fit in India over the chosen sample period.» His research is an evidence of multifactor APT beating the single-factor CAPM.(Basu, 2013)

Jianping Mei (1993) tested a multifactor model with time-varying risk-premiums and constant beta. He found that such model “is capable of capturing the “size effect” and the “dividend yield effect, but is incapable of explaining the book-to-market effect and the earnings-price ratio effect. He concluded that a “constant-beta multi-factor model will not be able to explain the cross-sectional variation in expected returns.”(Mei, 1993)

APT multifactor model doesn’t provide any guide on how to look for factors. BKM refer to Chen, Roll and Ross who set a five factor model, including %change in industrial production, %change in expected inflation, %change in unanticipated inflation, return of corporate over government bonds and, finally, return on long-term government bonds over T-Bills. That was just one of infinite possible sets of variables, aiming to predict the expected return on a risky portfolio. Another approach uses not macro-factors, but firm-characteristics. An example of this approach is the Fama-French three-factor model, (which, by the way, resulted in a Noble prize for Fama this year). According to BKM, this model “has come to dominate empirical research and industry applications.”(Bodie, et al. 2011)

2. Fama and French three-factor model.

In 1993 in the article “Common risk factors in the returns on stocks and bonds”, Fama and French has introduced three common factors, explaining stocks returns. These factors were market, size and book to market ratio. These factors, influencing the stock return were determined empirically. Fama and French reference to works of Banz (1981).Bhandari (1988).Basu (1983).and Rosenberg, Reid, and Lanstein(1985). These factors were already mentioned in this work, and it has already been shown that none of models discussed or mentioned above account for them. Fama and French noted that 2 classes of stocks performed better as a market. These were small caps and firms with high book-to-market ratio. Fama and French used 6 portfolios formed basing on size (ME) and Book-to-Market raio (BE/ME). The factors used in the model were SMB (Small firms minus big firms) and HML (high BE/ME – low BE/ME). The equation, describing Fama and French three factor model looks like:

$$E(r_i)-r_f=a_i+b_i(r_M-r_f)+s_iE(SMB)+h_i(HML)$$

Fama and French have found that “for stocks, portfolios constructed to mimic risk factors related to size and BE/ME capture strong common variation

in returns, no matter what else is in time series regression.” Based on this finding, they stated that size and book-to-market is a good proxy for sensitivity for common risk factors in stock returns. They also found that intercepts in three factor regression are close to zero. So Fama and French claim that three factor model did a “good job in explaining the cross-section of average stock returns.”(Fama and French, 1993)

In 2000 Davis, Fama and French tested this model. They formed portfolios, by grouping stocks in dependence of sizes of the firms (two groups, big and small or B and S) and the book-to-market ratio(three groups, high, medium, low or H,M and L). During their work they form nine portfolios and use a regression:

$$R_i - R_f = a_i + b_i(RM - R_f) + s_iSMB + h_iHML + E_i$$

Analyzing the results, authors say: “all the regression R2 for July 1963 to June 1997 are at least 0.91, and the intercepts are small and statistically insignificant, except for S/L portfolio.” This indicates that returns are explained on 90% by the factors.(Davis, et al. 2000)

Liew and Vassalou tested whether the profitability of HML, SMB can be linked to futureGross Domestic Product (GDP) growth. Using data from ten countries, they found that HML and SMB contain significant information about future GDP growth. They noted that “even in thepresence of popular business cycle variables, HML and SMB retain their ability to predictfuture economic growth in some countries.”Their results supported a “risk-based explanationfor the performance of HML and SMB.”(Liew, Vassalou 2000)

Later, in 2003 they have continued their research in this field. As a result, they found that“a model that includes a factor that captures news related to future Gross Domestic Product(GDP) growth along with the market factor can explain the cross-section of equity returnsabout as well as the Fama-French model can.” They showed that SMB and HML contain mainly the news of future GDP growth. They concluded that: “when news related tofuture GDPgrowth is present in the asset-pricing model, HML and SMB lose much of their ability to explain the cross-section.” They proposed a model with stable parameters, which wasn’t reached by neither CAPM nor FF.(LiewVassalou, 2003)

BKM refer to Liew and Vassalou when introducing risk-based interpretation of test results shown in the work by Davis et al. in 2000. According to BKM this risk-based interpretation the two factors added to overall market risk “proxy for risks not fully captured be CAPM beta.” Another explanation offered by BKM is behavioral, mainly based on “overestimation of the value of firms with good recent performance.”(Bodie et al. 2011)

The example of research, comparing CAPM and FF performance on US stock market is a work by Dingquan Miao and Xin Yi, supervised by Clas Eriksson. They used the time-series regressions for the single-factor (market beta) and additional risk factors (ME, BE/ME). They showed that FF-model has much more explanatory power than CAPM, though the two factors introduced by Fama and French cannot explain the excess returns all by themselves. Their tests have shown that market beta is still a very important variable whether it's used in CAPM or FF-model.(Miao et al. 2013)

Though, need to mention, that pretty much the same result, as when using FF can be achieved in another ways. First way is to use a model, proposed by Liew and Vassalou, including news on future GDP growth (see discussion on page 12). The second way was suggested by Jangkoo Kang Tong Suk Kim Changjun Lee Byoung-Kyu Min in 2011. They have augmented the consumption based CAPM (or CCAPM) with a conditioning variable, they developed by themselves. Shortly, they generate the stationary trend deviation from the Johansen cointegration test. They "employed this stationary trend deviation as conditioning variable, since it is likely to incorporate information on the business cycle, as indicated by the four predictive variables." They have shown that after trend deviations are employed as a conditioning variable for the CCAPM, it performs almost as well as Fama and French's three-factor model.(Kang et al, 2011)

From being developed in 1993, Fama and French's three factor model has already been augmented at least once. This augmentation concerns an extra, fourth factor called momentum. According to BKM, Jegadeesh and Titman noticed a tendency for a certain (good or poor) performance of stocks to persist over several months, "a sort of momentum property." This "momentum" was added to three factor model by Carhart. He found that alpha of many mutual funds could be explained by sensitivity to market momentum. Such an augmented Fama and French model including four factors is being commonly used to evaluate "abnormal performance of a stock portfolio." This augmentation is not really relevant for my work, later I will show why.(Bodie, et al. 2011)

In 2003-2004 Weimin Liu has offered a new liquidity-measure, which as he has claimed can capture the liquidity premium, missed by CAPM and by Fama and French. He explained liquidity as "the ability to trade large quantities *quickly* at low cost with little price impact." Liu emphasizes that liquidity has a multi-dimensional nature, claiming that the previously developed measures have focused only on one dimension of liquidity. "For example, the bid-ask spread measure used in Amihud and Mendelson (1986) is related to the trading cost dimension; the turnover measure of Datar, Naik, and Radcliffe (1998) captures the trading quantity dimension; and Amihud (2002) and Pastor and Stambaugh (2003) construct their measures based on the

concept of price impact to capture the price reaction to trading volume. He also notes that “Although the evidence shows that liquidity risk plays an important role in explaining asset returns,² few studies have incorporated a liquidity risk factor into an asset pricing model, and those that do have had limited success in explaining cross-sectional variation in asset returns.” He divided his work into three parts: first was devoted to introducing liquidity measure, second was to liquidity risk role in asset pricing, and the third part was connecting liquidity risk and explaining anomalies. Basing on a new liquidity measure, Liu introduced a two-factor model (market and liquidity). He has shown that this two-factor model performs better than both CAPM and FF three factor model. He found that the two-factor model not only accounts for the liquidity premium, that CAPM and FF fail to capture, but also “subsumes documented anomalies associated with size, book-to-market, cashflow-to-price, earnings-to-price, dividend yield, and long-term contrarian investment. The model also accounts for price momentum after taking into account transaction costs.”(Liu, 2004)

Ang, Hodrick, Xingy and Zhang(2007), who will later be referred to as (AHXZ) have discussed another issue that was not captured by Fama and French model. They showed that “volatility of the market return is a priced cross-sectional risk factor. They reasoned that the idiosyncratic errors of a misspecified factor model would contain the influence of missing factors, and hence, by sorting on idiosyncratic volatility, they might develop a set of portfolios that would be mispriced by the Fama and French model, but that might be correctly priced by the new aggregate volatility risk factor.” AHXZ found that U.S. stocks with high lagged idiosyncratic volatility had indeed been mispriced by the Fama-French model. “The average return on the first quintile portfolio of stocks with the lowest idiosyncratic volatility exceeds the average return on the fifth quintile portfolio of stocks with the highest idiosyncratic volatility by over 1% per month. AHXZ also demonstrated that their findings could not be explained either by exposure to aggregate volatility risk or by other existing asset pricing models.” They refer to work of Merton(1987) who states, that in presence of lack of information, stocks with high idiosyncratic volatility have high expected returns because the investors cannot fully diversify away the firm-specific risk. AHXZ show the result, supporting the opposite statement. Though, they claim that the lack of theoretical framework, prevents from confirming idiosyncratic volatility as priced risk factor. In their work, AHXZ focused on finding a relationship between past idiosyncratic volatility (which is easily observable and calculated) and expected returns in the cross-section of international stock returns. Need to mention, that firstly they have examined the phenomena of low-volatility stocks have higher returns in the scale of 23 developed markets to avoid the small sample bias. They measure idiosyncratic volatility using Fama and French three-factor model. In their work they used the following regression:

$$R_i = \alpha_i + \beta_i MKT + s_i SMB + h_i HML + e_i$$

MKT is a market factor, which is a value-weighted return on market portfolio over the one-month US T-Bill rate. SMB and HML are the two factors added by Fama and French and discussed earlier. The idiosyncratic volatility for stock i is measured as the standard deviation of the residuals e_i after estimating equation above using daily excess returns over the past month. More notes on the methodology of this particular work will be discussed in “Methodology” part of my MOPP.(Ang et al., 2007)

Introducing the factors, benchmark and measures.

Before introducing and shortly explaining my choice of variables, I first need to discuss on why naive diversification strategy should be used as a benchmark. This section is dedicated to Naive diversification. DeMiguel, Garlappi and Uppal(2009) (later referred to as DMGU) have defined naive portfolio diversification rule as “one in which a fraction $1/N$ of wealth is allocated to each of the N assets available for investment at each rebalancing date.” They mention several optimal rules, with different approaches to estimating errors, and then test the optimal models against this naive allocation rule. They claim that this rule is easy to implement and “despite the sophisticated theoretical models developed in the last 50 years and the advances in methods for estimating the parameters of these models, investors continue to use such simple allocation rules for allocating their wealth across assets.” They tested 14 different models in 7 different empirical datasets and showed that “none of them is consistently better than the naive $1/N$ benchmark in terms of Sharpe ratio, certainty-equivalent return, or turnover. They derived an *analytical* expression for the *critical length* of the estimation window that is needed for the sample-based mean-variance strategy to achieve a higher CEQ return than that of the $1/N$ strategy. This critical estimation window length is a function of the number of assets, the *ex ante* Sharpe ratio of the mean-variance portfolio, and the Sharpe ratio of the $1/N$ policy. They found that the critical length of the estimation window is 3000 months for a portfolio with only 25 assets, and more than 6000 months for a portfolio with 50 assets. They also stated that in practice, these portfolio models are typically estimated using only 60 or 120 months of data. DMGU stated that to implement the mean-variance model, both the vector of expected excess returns over the risk-free rate and the variance-covariance matrix of returns have to be estimated. They refer to Merton, (1980) asserting that a very long time series of data is required in order to estimate expected returns precisely. According to DMGU “the estimate of the variance-covariance matrix is poorly behaved (Green and Hollifield, 1992; Jagannathan and Ma, 2003). The portfolio weights based on the sample estimates of these moments result in extreme positive

and negative weights that are far from optimal “allocation mistakes” caused by using the 1/N weights can turn out to be *smaller* than the error caused by using the weights from an optimizing model with inputs that have been estimated with error.”(De Miguel, et al, 2009)

Tu, Zhou (2011) also confirm that naive diversification rule outperforms those sophisticated strategies. In their paper, Tu and Zhou combined 1/N allocation rule with sophisticated strategies, and showed that these combined rules work much better than their “uncombined counterparts”. But their research was based on the statement that initially naive diversification strategy outperform the complicated theories about allocating wealth across risky assets.(Tu, Zhou, 2011)

Another argument for naive diversification can be found in the work of Pflug, Pichler and Wozabal(2011). They also referred to naive diversification strategy as to “uniform investment strategy”. Furthermore, they showed that this strategy is actually rational to follow “in stochastic portfolio decision problems where the distribution of asset returns is ambiguous, and the decision maker adopts a worst case approach taking into account all measures in an ambiguity set.” During their study they have demonstrated how even small level of ambiguity cause diversification in the optimal portfolios. (Pflug et al, 2011)

Basing on everything written above, it's possible to assert that Naive allocation strategy can be used as a benchmark.

Sharpe ratio.

BKM define Sharpe ratio as the excess return for an asset divided by its standard deviation. They also refer to this ratio as to “reward-to-volatility” ratio. This is simply a measure of risk-return tradeoff.

$$S = E(r_i - r_f) / \sigma_i$$

(Bodie, et al, 2011)

Sharpe ratio was introduced by William Sharpe (as its name demonstrates) in 1966 and it's still widely used in investments decision making. The important feature of Sharpe ratio is that it captures idiosyncratic as well as systematic risks, which was shown for example by Kavita Sriram in a short article in Economic Times (2011). That's why I will use this ratio as an appropriate measure of a model's performance. By using this ratio, I will compare the performance of my model with the naive 1/N model's performance. Just the same way it was done by Demiguel et al (2009). The only issue I need to mention here is that a Sharpe ratio is an appropriate measure of portfolio performance only if standard deviation is a compete measure of risk, which assumes normality of excess return's distribution.

(Bodie, et al. 2011)

It was already mentioned in this work, that security returns were proved to be non-normal. In order to overcome this issue lower partial standard deviation (LPSD) can be used. Lower partial standard deviation is computed like the usual SD but using only negative excess returns. The excess return divided by LPSD is the non-normal alternative of Sharpe ratio and is called the Sortino ratio.(Bodie, et al. 2011)

Referring to Schuhmacher, Eling(2011) “The most popular reward-to-risk performance measure is the Sharpe ratio.” In their paper, they provided scientific justification of using other risk-return measures like Sortino ratio for example. An interesting finding in their work was that “under the location and scale property, any admissible performance measure is a strictly increasing functionin the Sharpe ratio. This means that for these distributions, performance ranking will be the same regardless of whether it is conducted via an admissible performance measure or via the Sharpe ratio.” They claimed that the same conditions that provide a decision-theoretic foundation for the Sharpe ratio also provide a decision-theoretic foundation for admissible performance measures.(Schuhmacher, Eling, 2011)

In my work, I will not test the distribution of excess returns to find their type of distribution. But still I’m going to use Sharpe ratio as well as Sortino ratio because I believe that use of both these ratio would provide a more detailed comparison and probably more opportunities for further research. So in the model I’m going to test, I will use three variables. Market factor (MKT), idiosyncratic volatility factor (VT) and liquidity factor (LIQ). Considering everything mentioned above, I believe that these three factors are able to explain the excess returns, with respect to all the phenomena mentioned so far. I will summarize and give more details to this statement in conclusion.

Idiosyncratic volatility factor.

I decided to introduced this factor, inspired by work of AHXZ discussed above. They found that stocks with lower idiosyncratic volatility have higher returns. This phenomena was demonstrated on the example of 23 developed markets. I remind that the measure of idiosyncratic volatility proposed by AHXZ based on Fama and French three factor model is following:

$$R_i = \alpha_i + \beta_i MKT + s_i SMB + h_i HML + e_i$$

The idiosyncratic volatility for stock i is measured as the standard deviation of the residuals e_i after estimating equation above using daily excess returns over the past month. They didn’t claim that there is a idiosyncratic volatility factor which explains a part of excess returns. But on the other hand, they didn’t state that this factor is absent. Basing on AHXZ results I decided that checking the existence of this factor make sense. So in my work, I will construct this factor in the following way. The deriving of the factor will be similar to that of Fama and French in their

famous three-factor model. Just like they estimated Small minus Big with respect to size and High minus Low in respect to value, I will derive Low minus High with respect to idiosyncratic volatility i.e. the return on portfolio of stocks with low idiosyncratic volatility minus the return on portfolio of stocks with high idiosyncratic volatility. But, first, of course I will have to divide the stocks into two groups with respect to their volatility (high or low). I will shortly name this factor (VT).(Ang et al. 2009)

Liquidity factor.

The second factor I wish to consider is the liquidity factor, introduced by Liu in 2003 when he was describing his two factor model. I will simply “take” this factor as it is introduced by Liu. First I will show how the measure of liquidity by Liu actually looks like. The new liquidity measure used in this study is defined as the turnover-adjusted number of zero daily trading volumes over the prior 250 trading days, which is denoted as $No0V1y$. In algebraic form, the new measure is given by

$$No0V1y = \frac{1/Toly}{\text{number of no-trading days over the prior 250 trading days} + 1000000}$$

where $Toly$ is the average daily turnover over the prior 250 trading days, daily turnover is the ratio of the number of shares traded on a day to the number of shares outstanding at the end of the day, and

$$\frac{1/Toly}{0 < 1000000 < 1} \text{ for all stocks in the sample.}$$

Both the turnover measure ($Toly$) and the new liquidity measure ($No0V1y$) are constructed at the end of each month for each individual stock based on daily data, and their calculations require 250 non-missing trading volumes over the prior 250 trading days. Liu claims that the new liquidity measure captures multiple dimensions of liquidity with particular emphasis on trading speed that existing research has largely ignored. His research has shown that illiquid stocks tend to be small and value. He claims that liquidity risk factor can proxy for financial distress to a certain extent. Referring to results of several previous works and the fact that liquidity risk is a state variable, he develops a two factor model with liquidity risk factor (LIQ) replacing SMB and HML factor of Fama and French. He constructed this factor in similar way that SMB and HML were constructed. He sorted stocks based on the new liquidity measure introduced above. Then he formed two portfolios: LL(low liquid) and HL(high liquid). The liquidity factor LIQ was calculated as the monthly arbitrage profits from buying one dollar of equally-weighted *LL* and selling one dollar of equally-weighted *HL*. I will focus more on deriving the factors in methodology. The point important to reveal now, however, is that Liu showed the negative

correlation between liquidity and market factors, and positive correlation between LIQ and SMB and LIQ and HML. He pointed the inability of Fama and French model to explain liquidity premium, though he claimed that two factor model captures the size, value characteristics as well as momentum. By the way, that's the reason why I didn't pay much attention to this augmentation of three-factor FF model. I remind, that the result of Liu's work has shown that his two-factor model performs better than Fama and French three-factor model. The third factor I will include in the model, that will be tested in MOPP is MKT or market factor, similarly like in Liu's or Fama and French models. I will dedicate a section in methodology to choosing appropriate method of beta estimation.

(Liu,2003)

One can say,that basically I'm going to introduce and test the Liu's two-factor model, augmented with volatility factor, based on AHXZ's paper.

The empirical tests which I referred to when writing this work showed that the same model can perform differently on different markets, especially if one compares developed and emerging markets. That's why I've decided to test the new model using the data from Russian market as well.

Summary

In this work the most popular portfolio strategies were discussed. It has been shown that each of them is commonly used in practice, but the most popular is CAPM, classical and with modifications. There are several problems connected with application of CAPM. The most important of them can be summarized as follows: too much unrealistic assumptions and too much is expected to be explained by a single market factor (see, for instance, Galagedera, 2007 or Grinblatt and Titman 1998). Later some of the issues were overcome by improving the original Sharpe-Lintner CAPM to conditional CAPM with time-varying beta or changing the beta itself, providing a more accurate estimate of it (for example, consumption beta and respectively CCAPM). The empirical results on these models appeared to be only slightly better than those of original CAPM. (see Lewellen and Nagel, 2003). Still this model, even the basic one is still commonly used worldwide (see Bartholdy and Peare, 2004). Another possible reason for CAPM to fail the empirical tests which is often mentioned by researchers (e.g. Bodie, et al, 2011) is that statistical methods and techniques used when estimating beta and the model itself were possibly inappropriate. This issue will be discussed in more details in MOPP. Still, the result is that mainly CAPM performs worse than its counterparts. But of course, there are some tests, providing the opposite result! (see for example Vosilov, et al. 2010).

Some of the issues were overcome by Arbitrage Pricing Theory (APT). (In this work only multifactor version of this model was discussed) First, this theory significantly reduced the set of assumptions. Another important features of APT was that it replaced the market portfolio with the factor portfolio and the factor risk premium could be negative. (see Grinblatt and Titman, 1998).

The multifactor APT model has shown better performance than CAPM, as was documented by, for example Basu in 2011. Empirical tests on this model have shown better results, though there were still some phenomena not explained or captured anyhow. The effects of size (or market capitalization) and value (book-to-market equity) were still not accounted for. (see for example, Reinganum or Mei).

To overcome that issue, Fama and French decided to use firm-specific factors instead of macroeconomic ones. They introduced the famous three factor model, consisting of market factor (MKT), size or market capitalization factor (SMB) and value factor (HML). (see Fama and French, 1993). This model's performance was particularly successful! Those factors together with market beta, explained 90% of total changes in excess return. (See Davis et al, 1993). Though there were some arguments showing that in certain cases, CAPM can show better results than Fama and French (see Miao et al, 2012) most part of empirical evidence supports Fama and French's three factor model as the superior one to CAPM.

The critics on this model reveals the relation of SMB and HML factors to future GDP news (see Liew, Vassalou, 2003). Also it was shown by Kang et al in 2011, that using consumption CCAPM augmented with conditional beta developed by them can lead to pretty much the same results as FF model.

Later, Liu (2003) showed that liquidity factor premium was not captured neither by CAPM nor by FF. He developed a new measure of liquidity, capturing its multi-dimensional nature. Based on this measure, he developed a two-factor model which, as was shown performed better than FF. The reason for that was that this liquidity measure has a strong correlation with FF factors and momentum, capturing liquidity premium as well. (see Liu, 2003)

Another issue, connected with using Fama and French was shown by Ang et al. 2009. They have built a set of portfolios, based on idiosyncratic volatility that would be mispriced by the Fama and French model, but that might be correctly priced by the new aggregate volatility risk factor. (see Ang et al)

Summarizing everything mentioned above, it's possible to conclude that in different situations different models can be justified to apply. There is no single "right" solution, when choosing a way to allocate wealth across risky assets. Using the theory, I have shown for the last 60 years of these strategies developing, there has always been something to discuss, improve, or

argue about. That makes it possible to assert, that the issue of finding the best portfolio strategy is still challenging.

Considering everything discussed above, a proposition of a new model seems to be a reasonable thing to do. In the third part of work I justified the choice of the properties, that this model should have. Trying to capture all the possible issues considered in this work the three factor model, including market factor (MKT), volatility factor (VT) and Liquidity factor (LIQ). In other words, that would be the model, introduced by Liu augmented with volatility factor, based on idiosyncratic volatility measure developed in the work of Ang et al. Referring to Liu, his liquidity measure captures everything that was explained by SMB and HML in the works of Fama and French and liquidity premium. He also claims, that this premium is captured better than with the previously introduced liquidity measures. I believe that adding volatility factor, based on a measure described by AHXZ will improve this model. The deriving of this last factor was shortly explained in the last section of this work, and will be described in more detail in MOPP. The model that will be tested will look like:

$$R_i = \alpha_i + \beta_i(MKT) + l_i(LIQ) + v_i(VT) + e_i$$

Also in the third section I justified the choice of naive allocation rule as benchmark and Sharpe and Sortino ratios as measures of the new model's performance. The data for the test will be taken from US stock exchange..

Here, there's also a need to mention following. The strategy I aim to present in this paper can be referred to as a form of "Smart Beta" strategies, whose popularity increased lately. This is an alternative to widely accepted cap-weighting indexations. I will give more details on Smart Beta later in this paper. An example of such strategy could be the US Minimum volatility strategy. In their work, Amenc, Goltz and martellini stated that such strategy could include too many low volatile stock, ignoring the other measures and factors, that should be accounted for in portfolio formation. This problem could be overcome by the model I suggest. Because the strategy, proposed in this work aims to account for all the relevant factors simultaneously.

2. Methodology.

This section clarifies the following methodological aspects of this particular research:

- Definition and criteria of research
- Philosophical position
- Research design
- Types of data used
- Sources of data collection
- Data analysis
- Validity and reliability

In the conclusion of this section there is a short summary of everything mentioned above.

Definition and criteria of research.

According to Dawson, 2002, a research has to be the process that: is undertaken within a framework of set of *philosophies* and approaches; uses *valid and reliable* procedures, methods and techniques, is designed to be *unbiased and objective*.

(Dawson, 2002)

Before getting started, I need to make sure that the research conducted replies to all of the above criteria.

Philosophical aspects will be discussed later in this section.

Validity implies that the procedures used are correct and applied properly. The reliability refers to quality of measurements. Unbiased and objective technically implies that the conclusions are not majorly influenced by a researcher's opinion, the results are not fake or deliberately incorrectly treated and interpreted. The bias can be referred to as a deliberate attempt to conceal or highlight something.

(Dawson, 2002)

The Hypothesis is a specific statement. In my example the hypothesis is that on given market during a certain period of time, the model I suggest will outperform the benchmark model. The aim of this work is not to prove this statement or not, but just to find the evidence, supporting or refuting the hypothesis.

The financial engineering is a very probabilistic endeavor, that's why the word "prove" will be too loud and strict. The evidence, that my hypothesis has it's right to exist at least in given conditions- would be the aim, closer to reality.

Philosophical position.

In order to establish a philosophical position, underpinning this particular research, I firstly need to give a short observation of those. The philosophy underpins the four following dimensions of any research project:

- Ontology
- Epistemology
- Methodology
- Methods and techniques

Each former element leads to a choice of each latter element. I'll start analyzing them one by one.

Ontology is a philosophical assumption about the nature of reality. The major ontologies are Realism, Internal Realism, Relativism and nominalism.

(Easterby-Smith, 2012)

In simple words, Ontology is about relation to the truth and facts. According to Easterby-Smith, Realism supposes that the truth is single, concrete and absolute, the facts exist and can be observed directly and measured reliably. Internal realism ontology states that truth exists, but it's obscure, not certain. The facts are certain but cannot be accessed directly. The supporters of relativism positions state that there are many truths depending on the time period and the observer's background and viewpoints. The same dependence they believe concerns the facts. Finally, Under Nominalistic point of view, there is no truth at all and any facts are simply human creations. (easterby-Smith, 2012)

Basing on the classifications given above, I can state, that my research is underpinned with Internal realistic ontology. Namely, I believe that there are true facts concerning the stocks returns and their sensitivity to factors, but these facts are certainly very difficult to be accessed directly. (I will give more details on this, when discuss about smart-beta strategy). This can lead to a simple conclusion, that the truth is obscure.

Another important element of philosophical position chosen for the research is Epistemology.

Following Easterby-Smith, epistemology “is a general set of assumptions about ways of inquiring into the nature of the world”. (E-S, 2012)

The two extreme points of epistemology are Strong positivism and social Constructionism. The positivism supposes, that the world exists externally, so the observer has to be objective, independent on the object of observations and rely on numbers. The social constructivist view suggests that the observer is directly involved in the process he studies, or, in other words becomes a part of what is observed. The research based on this Epistemology, relies more on qualitative data and small samples. This approach doesn't provide certain numbers and the measurable results that can only be treated in one way. There is no any theory or hypothesis as starting points. As the choice of Ontology has an impact at the choice of Epistemology, the Epistemology directly influences the Methodology of research. According to Easterby-Smith, 2012, there are following implications of Positivism and Social constructionism, demonstrated in the table below:

Comparison of social constructionism and positivism

Element	Positivism	Social Constructionism
The observer	independent	Part of what is observed
Human interests	irrelevant	The main drivers of science
Explanations	Must demonstrate causality	Aim to increase general understanding of the situation
Research progresses through	Hypothesis and deductions	Gathering rich data from which ideas are induced
Concepts	Are measurable	Should incorporate stakeholder's perspective
Units of analysis	Should be reduced to simplest terms	May include the complexity of whole situations
Generalization through	Statistical probability	Theoretical abstraction
Sampling requires	Large numbers selected randomly	Small numbers or cases chosen for specific reasons

(Mark easterby-Smith, 2012)

Referring to my choice of Ontology and the actual field of study, using common sense I can hardly imagine how the Social Constructivist approach could be helpful for me.

I am clearly independent of the object of observations, there are no any humans interests that can be relevant for my study, I will try to demonstrate causality in explanations and to generalize through statistical probability. Further in this Section, I will discuss the data issues, so it will be easily seen, that I use large numbers and use mathematical measures, but technically they are not selected randomly.

Following the epistemological classifications, proposed by Easterby-Smith, 2012, the Epistemology used in this particular research is Positivism. This epistemological choice leads to certain choices of methodologies, discussed below.

Methodology can be explained as “a combination of techniques used to inquire into a specific situation”. (Easterby-Smith, 2012)

The methodological implication, in dependence of epistemology, and it concerns the following elements:

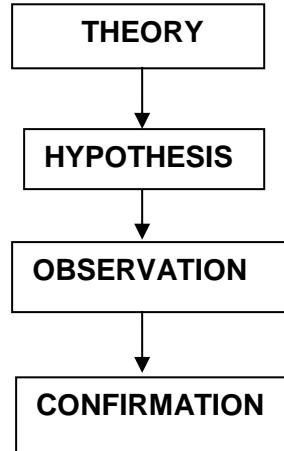
- Aims
- Starting points
- Designs
- Data types
- Analysis/interpretation
- Outcomes

I have already stated, that my starting point is a hypothesis. The aim of this research is to try to discover a new model, showing a better performance than the existing ones. In the worst case, I hope to bring any contribution to further studied, concerning “what can corrections should be made to succeed?”, or “what should not be done?” issues. Easterby-Smith provides ‘Experiment’ as a very broad type of positivistic research designs. The details of research design for this particular paper will be discussed further in this section. But at this point, I can already confirm that it will be some sort of experiment, because I am basically doing something, that hasn’t been done before, make it fit the existing findings and analyze the results to make the conclusion.

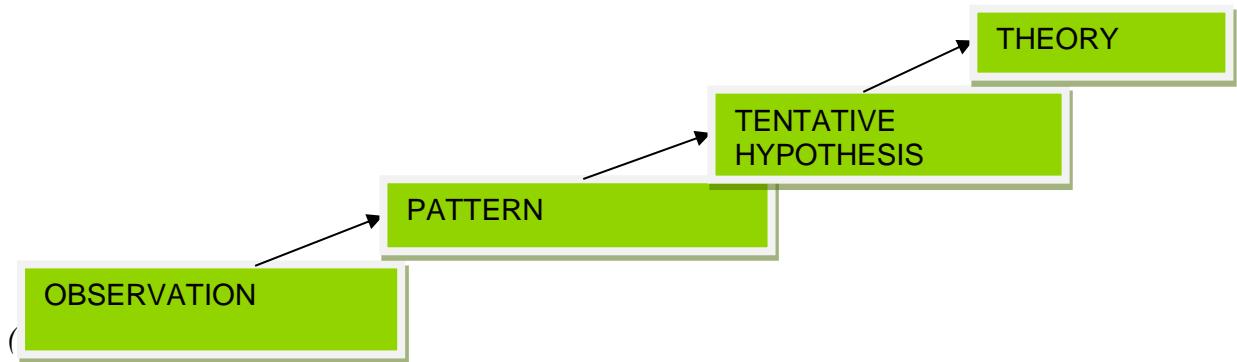
The epistemology that perfectly fits the dimensions of my research is a mix of Strong Positivism and Positivism. So I can name it just Positivism.

Research design and strategy.

Research approach. According to A. Burney, 2008, there are two broad classification of research types: *deductive and inductive*. Deductive approach works from more general to more specific, it’s also called the “top-down” approach. The scheme of such type of research looks as following:



The inductive approach is moving from specific to general, (“Bottom up”) approach. Under this approach the theories are generated based on specific observations result. This approach can be schematically shown as:



So what is the approach, used in this particular research? I would say, that it's some sort of mix of deductive and inductive reasoning. The research starts with observing the existing theories and facts. Then the Hypothesis is formed. The basic hypothesis was the conclusion of theoretical part- there is an actual need for an improvement in optimal portfolio construction, and as a result- a new model (a kind of combinations of old models) is suggested. The tentative hypothesis of this model supremacy is formed on this stage.

Then this particular model is observed and tested. Basically, that's not the model, but the tentative superiority hypothesis which is tested. And then, the results lead to conclusion, which, in turn, becomes the tentative theoretical knowledge.

Burney, (2008) argues that “Observations tend to be used for inductive arguments”. All the information used in this Thesis can be defined as observations and results of tests. As can be seen in Theoretical Part, most part of the statements and hypothesizes discussed, have counter-arguments and evidence against their validity. (*See Section 1*)

Anyway, there is some logical sense, that prevents me from calling this research purely inductive. For example CAPM is generally accepted and widely used, though dozens of scientists and researchers argue for its inability to predict the returns well. (*See Section 1*)

So what I used in order to come to my specific inductive starting point, was generally accepted ideas, principles, theories and models. That's why I believe, this research is a mix of two approaches.

Research design

Following Esterby-Smith, considering the Positivist epistemology it's possible to construct the research design. The research design provides a set of underlying principles of what questions should be answered and what issues have to be resolved or what should be the main points of attention during every stage of research. I used the template, proposed by easterby-Smith, 2012 to construct this project's research design.

In the background- which is the Theoretical Section, I provide the specification of a problem, review the currently existing researches on this topic and the shortcomings of existing solutions, then I conclude, that the problem exists and needs improvements in solutions techniques.

Basing on this findings, I propose a new model, and state the hypothesis, that it would perform better than the chosen benchmark.

The aim of research is to test the hypothesis and specify it's right for existence. The sample size is justified in the further part of the work. In order to test the hypothesis I use the statistical procedures, as well as mathematical and logical reasoning.

Concluding the background, I specified which factors I will calculate, and the measures I will use. Further in this work I will explain the choice of the dataset, the formulas used to calculate the factors, and the portfolios weights as well as the choice of measures.

Then, the benchmark portfolio will be constructed, responding to the mathematical principles. After that, to test the resulting portfolio against the benchmark I will use the measures, defined in Section 1 and calculated in section 3. The statistical analysis will be used in this work, not to generalize the findings, but to construct the testable portfolios, in other words, this will be a part of data analysis procedures. All the processes, which underpin the conclusions directly are to be purely mathematical.

Types of data used.

In this research the data concerning a set of samples of stocks over a certain period will be used.

There are several classifications of data, that can be used in a research project. First of all, according to Easterby-Smith the data I am going to use is secondary, because it's collected from the archive, not directly observed. (Easterby-Smith,2012)

There are some other classifications of data (cross-sectional, or time-series, quantitative or qualitative, etc.) I will pick a classification of the data, I am going to use on a one-by one basis.

Time series data are data collected over a period of time on one or more variables. Such data are associated with a certain frequency of observation or collection. Such data also require the same frequency of observation, meaning that all the factors are to be observed with the same frequency (say, monthly). The data may be quantitative (e.g. rates, prices) or qualitative (a survey of financial products purchased by private individuals over a period of time).

(Brooks, 2008)

Cross-sectional data are the data collected on one or more variables at a single point in time. The major difference in treating cross-sectional and time-series data is that in cross-section the chronological order of data is irrelevant, while it's quite important in time-series context. The data, having the dimensions of both time-series and cross-sectional is called panel data.

(Brooks, 2008)

In this work, I use the data across different companies in different time periods, so it's both time-series and cross-sectional, or panel data.

Another classifications of data divides them into continuous and discrete. Continuous data can take on any value limited by precision only, while discrete data can only take certain values (usually integers).

The data can also be cardinal, ordinal or nominal. This classification concerns natural ordering. I will provide an example of two numbers: 2 and 4 treated differently in dependence on classification. For cardinal numbers a measure of 4 is “twice as good as” 2. For ordinal numbers, 4 is still “better” than 2 but cannot be considered “twice as good”. Finally for nominal numbers the measures 4 and 2 cannot be anyhow ranked or ordered. In that case, that would be just figures.

(Brooks, 2008)

So, basing on the classification as was discussed by Brooks (Introductory Econometrics for Finance, 2008), the data I seek for constructing the risk factors are:

- time-series
- continuous

- cardinal
- quantitative
- panel

Sources of Data.

The data on US market, namely on S&P 500 index stocks is taken from CRSP.

Data analysis

In this particular work, the data analysis is done through a set of steps:

The first step is to obtain the raw data. On the second step, this data is restructured in order to make the formulas for calculations work properly. On the third step the factors are computed. Then, using statistical instruments, the beta of every stock is found. The beta is then used to determine the weights. After that, the portfolio is formed, and its annual return and standard deviation is obtained. Then, the resulting Sharpe ratio is compared with the one of Naive (Benchmark) portfolio.

Validity and reliability of research design

Following Easterby-Smith, 2012, The validity, reliability and generalizability of the work, based on positivist epistemology are the following questions to be answered.

Validity is concerned about how closely the measures correspond to reality or do they provide a good approximation for the variables of interest.

To prove that the findings are reliable one has to answer the following questions: has the design eliminated the alternative explanations or will the measures yield the same results in other occasions?

Generalizability of the research is defined by the questions; to what extent this study confirms or contradicts with existing ones in the same field or how probable is that patterns observed in the sample will be repeated in the general population?

These questions are to be answered in the conclusion to this work.

Summary

Summarizing everything, discussed in this section it's possible to make the following statements:

1. The underlying ontology of this work is Internal Realism
2. The underpinning epistemology is positivism
3. The research is completed through observation of a dataset, dividing this dataset into a certain amount of element, calculating the measures and comparing the results.
4. The research uses secondary panel quantitative data obtained from CRSP database.

3. Specifications, Calculations and analysis of the results.

In this section I will give the detailed description of the data samples, time intervals, factor calculations, measure calculations, weighting stocks in the portfolio. I will also analyze the results obtained.

The calculation of Factors.

In this section I will give the detailed description of the way, the factors used to construct portfolios will be calculated. I chose to calculate the factors annually, to use annual returns and one year as a holding period. Below, I will explain why it's possible to do so, without sufficiently changing the results.

The volatility Factor.

Following Ang et al. 2007, in order to construct the volatility factor I first need to regress the equation based on F-F factors, which (as mentioned in section data sources) are taken from Kenneth French website. The equation looks as follows:

$$R_i = \alpha + \beta_i MKT + \gamma_i SMB + \delta_i HML + \epsilon_i$$

This is the original regression, used by Fama and French in their pathbreaking work in 1992. The idiosyncratic volatility is the standard deviation of the residual ϵ_i after estimating equation above using daily excess returns for the past month.

That's the way, this measure was constructed originally in the work of Ang et al. In this particular work, I will use another one. This will be the measure of annualized daily returns deviation. This measure also used by Ang et al, and they found a high correlation between this measure and the others they used in their paper!

(see Ang et al, 2007)

The holding period, chosen in this work is one year. The factors are calculated annually, and the returns observed are also annual. That's why the standard deviation also has to be annualized. The standard deviation of stock's return is claimed to be a reliable measure of idiosyncratic (non-systematic) stock volatility. When annualized, this measure is called the historical (annual) volatility. The historical volatility is calculated in the following way:

- Stock's annual return is calculated as the change in price over the year, divided by the opening price for the year
- The standard deviation of stock's daily returns over the year is calculated (in Excel, using "stDev" function)

- Then, to annualize this, the daily standard deviation is multiplied with square root of 250

Annual historical volatility= std_daily*sqrt(250), in Excel.

(<http://www.istockanalyst.com/finance/story/5193863/historical-volatility-part-5-how-to-annualise-standard-deviation>)

Then, this measure is calculated for each stock in the sample. The stocks are sorted based on this measure. The higher is the measure- the higher is the volatility. In the paper by Ang et al., 2006, it was demonstrated, that stocks with higher past idiosyncratic volatility, earn low future average returns. (see, AHXZ, 2006). So in order to sort the companies, I will use the prior year volatility measure. And then, simply repeat the technique, as by Fama and French for factor construction. I will take the return of 30% of stocks with lowest past idiosyncratic volatilities against 30% of stocks with highest past volatilities. The resulting figure, would be the volatility factor, or VT in this paper.

Liquidity factor (LIQ)

Following Liu (2004) the liquidity factor (LIQ) is constructed in the following manner, pretty “similar to Fama and French.” (Liu, 2004). Liu’s liquidity measure looks like:

$$No0V1y = \text{number of no-trading days over the prior 250 trading days} + (1/ To1y) : 1000\ 000$$

Liu has claimed that this measure captures the multidimensional nature of liquidity. He showed, that this measure highly correlates with the previously used measures (like Bid-Ask Spread, P/B, dividend yield, etc. and also takes into account the F-F factors and the momentum. Using this measure, Liu constructs the factor in the following way:

First, the two portfolios have to be constructed, basing on their liquidity. The first portfolio is called low liquid or LL and is constructed of the lowest liquidity stocks. The second portfolio is similarly constructed of the highest-liquidity stocks and called respectively High liquid or HL. The LIQ factor is calculated in the following way: the two portfolios described above are held for 12 months. The LIQ is the monthly arbitrage profit of buying one dollar of equally weighted LL and selling one dollar of equally-weighted HL. The lowest liquidity border

was 15% from one subperiod and 35% for another. The turnover measure (T0ly) used by Liu is similar to the one used by Datar et al.

(Liu, 2004)

This measure is calculated as the average daily turnover over the prior three months. The daily turnover is simply number of shares traded on a day divided by the number of shares outstanding at the end of the day. Datar et al. claimed, that there was no significant impact on the calculations, when they used three-month, one-month, 6,9 or 12-month average T0ly.

(Datar et al. 1998)

That's why in my work I decided to use a 12-month average T0ly.

This allows me to calculate the liquidity premium as the annual arbitrage profit of selling one portfolio and buying another. Mathematically, this would just be the difference between the returns on two target portfolios.

I have made some other changes in the original measure introduced by Liu.

First of all, I noticed, that when there are 0 shares at the end of the day, T0ly would not be calculated, and the liquidity on that day will be estimated as 0 when actually this would not be true. That's why the measure I use is following:

$$T0ly = \frac{ST}{SO+ST}$$

ST- Shares traded on a day

SO shares outstanding at the end of the day.

This number will never be more than 1 (mathematically) and the bigger it is- the more liquid is the stock!

Another change I made concerns the non-trading days over the prior year. If N- the number of zero trading days, then the bigger it is, the lower is the stock's liquidity. I use 1/N so that, the higher this number is- the less non-trading days the stock had over the prior year, meaning, the more liquid it is! Under these corrections, the sum of T0ly and 1/N will be directly proportional to liquidity. The bigger is this sum, the more liquid is the stock. These numbers are just mathematical rebranding of Liu's measure, so I will still name it No0vly.

To summarize, annually I will calculate the measure of liquidity in the following way:

$$No0vly = \frac{ST}{SO+ST} (\text{Annual average}) + 1/N, \text{where } N-\text{number of non-trading day over the prior year.}$$

The market factor MT

The market factor is constructed as the monthly excess return of the value weighted market portfolio (in this case- S&P 500) over the one month T-Bill. (see for example, Ang et al, 2006 or Liu, 2004 or Fama and French, 1993)

But in this particular work, I use annual returns, which leads me to the need of finding the one-year analogue for the one-month T-Bill. This analogue is the 10-year constant maturity T-Bond.

And analogically, the factor will be calculated for Russian equivalents.

Specification on Data used.

Referring to all the factors described above, it's possible to outlay the set of necessary data needed in order to conduct the research. First of all, I need to remind the factors, which will be considered. These factors are:

- Idiosyncratic volatility factor (VT)
- Liquidity factor (LIQ)
- Market factor (MT)

Now, when the factors are established, its needed to find out, which data is needed to actually construct those factors.

1. Idiosyncratic volatility factor.

The idiosyncratic volatility measure is simply the annual (historical) volatility. The factor is constructed using the annual returns of the constituents. The data needed for this is following: for each stock in the S&P 500, for every year I need the daily returns. Then, I will use the stDev function to compute the standard deviation of daily returns and then simply annualize it, multiplying by square root of 250 (because on average, there are approximately 250 trading days in a year). To calculate the factor itself, I will need the annual returns of each stock in a sample. To calculate this, I will take the year closing and opening price for every stock for every year within the research interval.

2. Liquidity premium factor (LIQ)

Considering the way, this factor is constructed, I will need the daily volumes and number of shares outstanding at the end of the day, for every company, for every year. Then, I will take the average annual measure. To calculate annual returns, I will need the prices of each stock at the beginning and end of every year.

3. Market factor.

In order to calculate the market factor for the period, I need the overall S&P 500 annual return and the return of 10-Year, constant maturity T-Bond, the return on which is taken as risk-free in this paper.

To summarize, there is the following set of data for each stock in the sample, needed to construct the chosen factors:

- return of the value weighted market portfolio (S&P 500)
- 10-Year constant maturity T-Bond
- Number of non-trading days over the year
- Number of stocks traded during the day
- Number of stocks, outstanding at the end of the day
- Daily returns for each year

In order to run a regression one first needs to understand what type of data is imputed. In this section I will give a short description of major types and classifications of data used in statistics, and give a full description of the data that will be used in this work.

The sample

The companies included in SP 500 index will form a sample of data for the US market. The S&P 500 index represents an index of 500 stocks chosen for market size, liquidity and industry grouping, among other factors. The S&P 500 is designed to be a leading indicator of U.S. equities and is meant to reflect the risk/return characteristics of the large cap universe.

The S&P 500 index is constructed in the following way:

The first step in this methodology is to compute the market capitalization of each component in the index. This is done by taking the number of outstanding shares of each company and multiplying that number by the company's current share price, or market value.

Thus, S&P 500 is a value-weighted index, in which the weight of every stock is proportional to its market capitalization.

Companies included in the index are selected by the S&P Index Committee, a team of analysts and economists at Standard & Poor's. The S&P 500 is a market value weighted index - each stock's weight is proportionate to its market value

Time interval.

In this work there are several simplifications, such as the time interval. The time interval, used in this paper is: Year 2000- Year 2007. As mentioned above, I will need the prior year information in order to construct the factors, so technically there will be portfolios constructed for years 2001-2007 inclusively. There are several reasons for taking such a short time period:

First and obvious: simplification! Considering my lack of experience in fieldwork, I need to assure that I don't get lost in the huge bulk of numbers. This will provide the possibility for valid analysis.

Second reason is the estimation window length. In paper by DeNiguel et al, 2007, they stated that when the estimation window is huge, the optimal mean-variance strategy normally outperforms the Naive allocation rule. This fact, led me to assume, that there may be a correlation between the length of testing time interval and the tendency of optimal portfolio to outperform the naive one. That's why I believe, that if the portfolio is able to perform better than the benchmark within these seven years, that would be the empirical evidence of its supremacy. Otherwise, that would be really useless in practice.

The returns on S&P 500 during this period were following:

Year	Percent
2000	-10.1
2001	-13.0
2002	-23.4
2003	26.4
2004	9.0
2005	3.0
2006	13.6

2007	3.5
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The enormously huge drop in 2002 reflects the market downturn of 2002- the internet bubble bursting. This downturn can be viewed as part of a larger bear market or correction, after a decade-long bull market had led to unusually high stock valuations.

On 2003, the index grew significantly

Then, until 2007 the index was in the stage of stable, moderate growth. But the 2007 was claimed to be one of the highest quotes for S&P 500 for the last few decades.

Measures, Benchmark, And portfolios to be tested construction.

The measures of performance, referring to POPP are following: Sharpe ratio and/or Sortino ratio. The Sharpe ratio is claimed to be a reliable measure of risk-return trade-off, is the standard deviation (σ) is a reliable measure of risk. The latter is true, if the normality of returns assumption is held.

(Brooks, 2011)

The normality of returns is the assumption, under which the distribution of returns is normal.

The Sharpe ratio is calculated like:

$$SR = \frac{R}{\sigma}$$

Where R- annual return of the value-weighted portfolio

σ – standard deviation of the portfolio.

The portfolios return is the sum of each stock returns, multiplied by the weight of the stock in the portfolio. So that:

$$R_{\text{portfolio}} = \sum_{i=1}^N r_i \times w_i$$

Where r_i - the return on i-stock, w_i - the weight of i-stock in the portfolio, N - the number of stocks

For the purpose of this work, not the returns but relative price change will be used to estimate the portfolios. Basically, that's the relation of the year-end price to the price at the beginning of the year. The return on the portfolio was seen as the relative increase of its value, assuming that the beginning portfolio value was 1000. (That was the sum invested in all the assets available).

The value of each portfolio is a weight of each stock multiplied with relative price change of the stock. And then, the SD of such smart-beta portfolio is the standard deviation of returns over the seven years- the research period.

And the standard deviation is the volatility measure for each stock (also called historical volatility). It's calculated as $\text{std_daily} * \sqrt{250}$.

To calculate the standard deviation of a portfolio,

The benchmark chosen in this work, to which I will compare my self-constructed portfolios is the portfolio, constructed based on $1/N$ naive allocation strategy. This means, that I simply divide the wealth into N parts, where N - is the number of stocks (in my case the number of stocks in the S&P 500 index). $1/N$ share is invested in every stock, that's the weight of every stock in portfolio. And then, this weight is used to determine the portfolio return and standard deviation, in the same manner as described above. The return and SD of the resulting portfolio is then used to calculate the Sharpe Ratio of a benchmark.

As I chose a one-year holding period, and estimated factors annually, I will rebalance all the portfolios every year. As far, as the time interval is short enough, this will be simply done manually.

The portfolio construction.

There are few portfolios that are to be annually constructed in this work. First of all, I will need to construct the target portfolios for estimating liquidity and volatility factors. Those portfolios will be equally weighted. An equally-weighted portfolio is a portfolio, in which every stock gets the same weight, which is equal to $1/N$. E.g. the target portfolios for liquidity (volatility) factor construction are constructed as 30% lowest and 30% highest liquidity (volatility) stocks

combined into an equally-weighted portfolio. The benchmark (naive allocation portfolio) is constructed in the pretty much the same manner.

Such way of constructing a portfolio is also called “Max Deconcentration portfolio”. Such equally weighted benchmarks are widely used. Their particular attractiveness can be explained by avoiding the concentration and trend-following of cap-weighted indices, or, (for example, minimum-volatility portfolio). That’s why naive portfolios produce higher returns, than their cap-weighted or optimal counterparts.

(Amenc, Goltz, Martellini, 2013)

The purpose of this work, as mentioned before is technically to test, whether some sort of combination of smart-beta strategies can outperform such portfolio or not.

Smart beta is basically investing in factors- the weighting scheme is based on the stock’s exposure to a certain macro factor, rather than capitalization.

The critique of smart beta strategy, according to Amenc et al, proves that sometimes, taking into account only one factor- factor of interest when using smart beta strategy leads to a risky situation when certain stocks are underpriced and as a result- their weight in the portfolio is too big, while some other stocks tend to be out of value.

(see Amenc et al, 2013)

I decided to construct and compare the performance of several portfolios. According to Amenc et al, and the techniques they demonstrated, all of those portfolios can be referred to as Smart Beta portfolios. The first one, is the portfolio, which weights are fully based on its liquidity measure.

The weights are constructed in such a way, that the lower is liquidity- the higher is the weight

The second one is based on volatility measure. And the third portfolio’s weights aim to account for three macro factors at the same time: liquidity, volatility and market factor, which is normally presented as the excess return of the risky assets over the riskless T-Bond. In order to construct the portfolio, which is to capture the three risk factors at the same time, I will need to estimate the beta coefficient for every risk factor.

The beta estimation

As I already mentioned, the data I have is Panel. In other words, at this particular stage I have a cross-section of stocks, over a certain period of time. The annual data is regressed over seven years.

The panel data I am going to analyze basically has the following form:

Company	Year	Y	X1	X2
1	2000	0.5	0.9	-0.75
1	2001	-0.6	0.8	0.3
2	2000	1.4	-1.1	0.25

In such form, it can be directly analyzed in ‘R’. The panel data can be balanced or unbalanced. If the data is balanced, every company is supposed to have the observations over the same number of years. Otherwise, the data is unbalanced.

In my case, the data is totally unbalanced- so many companies got in and out of S&P 500 index.

The software I use for regression (“R-studio”) automatically detects the unbalanced data and treats it in a proper way.

After that, I need to specify, what exactly am I trying to establish and what regression am I going to run.

There are several ways to estimate and to analyze the panel data. The first one- is to use the pooling model. This model supposes, that all the id-s (the companies) is the same one and checks, whether there’s one variable, having a common influence for all the dependent variables.

The pooled estimator refers to the case, when one obtains a different subsample every year- which is exactly my case. But there are some other dimensions and characteristics, that can lead to a need to apply another estimation technique.

Following Dougherty, 2010, the fixed effects models support inference about the group of measurements. A random effect model, can allow the observer to make inferences about the population, from which the sample was drawn. The other distinction between those two is that, the fixed effect assumes that the individual specific effect is correlated to the independent variable, while random effect assumes the opposite.

At this stage of analysis, I would claim, that for the particular dataset, I use in this paper, the fixed effect model will be appropriate, though R provides the certain functions that allow us to check which model better fits the given data.

In order to test if fixed or random model is preferable- Haussman test is used. To compare the validity of pooling and random models, the Breusch-Pagan test is run in R. I will not deepen in mathematical details of those now. The only output I will download from R is the model, that fits the data best according to R.

Summary

To complete this Thesis I will download the following daily data, for each stock in S&P 500 and RTS over the period 2000-2007. Due to the fact, that I use prior year data to construct the current year portfolio, the research period is limited to years 2001-2007 inclusively. The data are:

- Daily returns
- Daily prices (though only year-closing and opening of those are to be used)
- Daily trading volumes
- Daily amounts of shares outstanding at the end of the trading days

Then, using this data I will construct the three factors:

1. Liquidity Factor (or LIQ):

$$No0vly=1/N+T0ly$$

N- the number of non-trading days over the prior year,

T0ly- average daily turnover over the prior year.

$$\text{Daily turnover} = \frac{ST}{ST+SO}$$

ST- shares traded on a day,

So- shares outstanding at the end of the day.

The companies are sorted on ascending order in the beginning of each year. The two target portfolios are formed: low liquid(LL) and high liquid(HL) . The former is

comprised of 30% stocks with lowest No0vly, the latter- 30% stocks with highest No0vly. The annual excess return of LL over HL is the liquidity factor or LIQ.

2. *Market factor*

The overall S&P 500 index annual return less the return on 10-Year constant maturity T-Bond is computed every year and represents the Market factor or MT.

3. *Volatility factor*

The companies are sorted in ascending order in the beginning of each year, based on their volatility measure, which is the annual (historical) volatility over the prior year, calculated as: Daily returns standard deviation multiplied with $\sqrt{250}$. The two equally weighted target portfolios are formed: the Low volatile portfolio, representing the stocks with lowest past historical volatility and the High volatile portfolio, comprised of the 30% highest past historical volatility stocks. The volatility factor or VT is calculated as the excess return of the Low volatile portfolio over High volatile portfolio.

The portfolios are rebalanced annually. Every year, the constituents and their weights in the portfolio are reviewed. The factors and betas are also reestimated every year.

Then, the multivariate regression is run and the betas for each stock in the sample are obtained. The beta of each stock is used to construct the weight of each stock in the final portfolio. This weight is then multiplied by annual return of each stock to determine the overall portfolio return. And then, using weights and historical volatility of each stock, the standard deviation of the portfolio is determined. After that, by dividing the portfolio return by the portfolio standard deviation, I obtain the Sharpe ratio which is to be compared with the benchmark.

The benchmark is the portfolio, constructed using the Naive allocation rule. The weights are found as $1/N$ for each stock, where N is the number of stocks in S&P 500 for the current year. After the weights are estimated, the procedure for calculating the Sharpe ratio is the same as described above.

Calculations.

In this section I will provide a detailed description of how exactly did I do the calculations and which software did I use. The first part of work- before running the multivariate regression- was completed in Microsoft Excel.

1. The data restructuring.

Before doing actual calculations I did a few manipulations of data. I have obtained the data downloaded from CRSP database. Firstly I introduced the column “Year” which derived the year from the full date in the second column of a spreadsheet. I used MS Excel function “LEFT”. After that I have set up the column named “Year change” and set it to be equal 1 whenever the row j number in “year” column is different from the row $j-1$ number in the same column. I used the “IF” function for that purpose. I had two different datasets for the entire period: the first- from year 2000 til 2004, the second- from year 2005 to 2007. I combined them in one spreadsheet, and used a “Custom sort” option, sorting first by ticker from a to z, then by the year in ascending order. When calculating the measures, there can appear a problem that the list of constituents changes from year-to year. And that’s why, it’s possible that two different companies data will be widely used for computing one measure. This problem will be resolved in further steps.

2. Computing the measures

Now I can compute some of the measures, I need for further calculations of factors. First of all, I insert the column “Zero trading days”. Using “IF” function I set it equal to 1 whenever, the number in column “Volume” is equal to zero. Then, I have calculated the sum of such days for every 250 trading days. Then I insert the column $1/N$, in which I divide 1 by the number of non-trading days over the prior period. This number is put on the first day of the following year. So it’s then easier to use. To clarify: I use the prior year data in order to construct the portfolio for the current year, that’s why I have those measures right in the beginning of the year, so I don’t get confused when start analyzing the numbers. So here I have the number of non trading days over the prior year. Then I calculate daily turnover. I divide the number in column “Volume” by the sum of the cells “Volume” and “Shares Outstanding”. The formula is repeated over the whole dataset. Then, at every year-end I take the average of this measure. It’s also calculated at every beginning of the year and put in the first day of the year cell.

To compute annual return, I calculate the percentage increase in price during every year by dividing the increase in price over the year by the opening year price. This number is put in the first year-day cell for every year for each stock.

The daily standard deviation of each stock is calculated, by using “StDev” function for the daily returns over the prior year. This is measure for every stock in the sample. Then, to annualize this daily standard deviation is simply multiplied by square root of 250 by using “sqrt(250)” in Excel.

3. Constructing factors

Liquidity factor

In order to construct the liquidity factor I copy all the necessary measure from the spreadsheet described above and paste them as values on the new spreadsheet. Then I run a test on constituents, using “TYPE(VLOOKUP(....))<>16”. This function returns “FALSE” for all the companies, that are not included in both years in the S&P500 index. So, my final sample is achieved by sorting this column on “TRUE”. That’s the way of resolving the problem of “merging” data of two different companies, when computing one measure. Though the portfolio will not include all the stocks, currently included in S&P 500, but this is within the frames of limitations of study. The same stocks will later be included in the Benchmark portfolio.

After that, I simply sort all the numbers by No0vly from smallest to largest, then take the average 30% lowest No0vly stocks returns less the 30% highest No0vly stocks returns. The given amount is the liquidity premium for the current year. The presence of this factor, was evidenced every year, except for Year 2007. The details, in section below.

Market Factor.

To construct the market factor, I simply take the overall S&P 500 return for the current year, less the return on constant maturity 10-year T-Bond. This factor is also calculated annually.

Volatility factor.

I need to mention, that this factor wasn't previously observed. The only thing stated and proved by AHXZ was that stocks with higher past volatilities tend to have lower future returns. The factor I constructed, is only tested to actually persist. The design of calculations is explained above. Now, I will show how exactly I derived the factor.

At the beginning of each year, I have calculated the standard deviation based on daily returns over the prior year, then multiplied it by $\sqrt{250}$. This measure is consistent with the one used by Ang et al, 2007.

Then I tested the constituents in the same way, as when computing the liquidity factor (using “TYPE(VLOOKUP(...))>16”). This was done in order to have the same stocks for all the portfolios and all the factors.

Then I sorted the stocks, but this time based on volatility in ascending order. The excess return of the 30% lowest volatile stocks over 30% highest volatile stocks is the liquidity premium.

I need to remind, that the presence of the factor itself, wasn't tested and approved yet. There was only noticed a tendency of stocks with low past idiosyncratic volatilities to have higher returns. When doing my calculations, I found an opposite trend. Namely the 30% most volatile stocks, with annual historical volatility as a measure, tend to outperform the lowest 30% volatile stocks in terms of annual returns. All the details and analysis is performed in the next section.

The results and analysis

Factors calculations results

First of all, I need to argue for the volatility factor I constructed, based on the findings of Ang et al, 2007.

Ang et al claimed that stocks with higher past idiosyncratic (non-systematic) volatility tend to have lower returns. This phenomenon was demonstrated across 23 developed countries, after controlling for market size and value factors. In this work, I didn't control for any of those factors, which fits my study limitations and assumptions. In this current work, I simply tried to find out if there's any common movement or trend, concerning non-diversifiable risk. And there is one! In this paper, the calculations have evidenced the presence of the trend, opposite to Ang et al. The volatility factor or VT under Ang's assumptions is negative for all subperiods in the testing interval. In other words, I found that stocks with higher past idiosyncratic volatility tend to have higher future returns. This seems common sense: the holder of a high volatile stock, bears higher non-diversifiable risk, which means he demands higher return- a premium for that.

These findings are not revolutionary. They are consistent with the results of works, obtained before the paper by Ang et al. was issued (see for e.g. Merton, 1987). He showed that "the presence of market frictions where investors have limited access to information, stocks with high idiosyncratic volatility have higher expected returns."

(Ang et al. 2007)

This relationship between firm-specific risks and annual returns is clearly seen in my results. The 30% stocks with higher past annual volatility outperform those 30% with lowest past volatility. The difference, is assumed to be the VT factor in this research. Please, see the factor, recalculated annually in the table below.

Table 1. Volatility factor

Year	Factor, %
2001	23,5%
2002	8,71%
2003	312,67%

2004	3,027%
2005	50%
2006	0,405%
2007	49%

Have to notice, that volatility factor, as calculated in this work tends to be ironically volatile itself. The actual presence of such factor, and its probable influence on the stocks returns will be discussed later, with the results of regression.

Just to be sure, I have divided the stocks into 10 deciles, where 1 decile is the lowest volatile, and the 10th decile contains the stocks with highest past idiosyncratic volatility. I have checked the presence of the factor for 3 Years: 2001,2004 and 2007. The tenth decile has always outperformed the first one. Still, sometimes, the lower volatile deciles outperformed the top ones.

To test the deciles, I choose the following way, which I believe is reasonable logically and mathematically. I took the 5 lowest volatile deciles and 5 highest volatile deciles and put them against each other. The 30% border includes three deciles from each extreme and accumulates those. The detailed deciles analysis can show where is the main reasons or drivers of a resulting factor.

Please, see the details in table below. The numbers represent the percentage difference between the return on higher volatile deciles and lower volatile deciles.

Table 2. The volatility decile portfolios performance comparing.

Year		2001	2004	2007
Deciles				
10-1	81,5%	22,18%	124%	
9-2	17,72%	-12%	41,2%	
8-3	-18%	4,56%	-5,56%	
7-4	-4,83%	-2%	18,45%	
6-5	-4,24%	0,5%	-56%	

As can be seen from the Table above, the 10th (the highest volatile decile) consistently outperforms the 1st (The lowest volatile) decile. And though, I believe I can't simply write-off the negative factor values for the 9-2, 8-3 or 7-4 deciles to random noise, I believe, these

numbers are not inconsistent with the results of 30% highest-30% lowest. Especially considering, that 5 and 6-th decile are almost the same set of stocks, mainly driven by a few outliers. (pls see my w/p)

So I assume, that the results on 30% border are correct and use those numbers as the VT (Volatility factors) for respective years.

The Liquidity factor, I have obtained, using the measures and procedures described above have following values:

Table 3. The liquidity factor.

Year	Factor, %
2001	20,15%
2002	32,35%
2003	244,2%
2004	2,29%
2005	17,5%
2006	10,85%
2007	-29,7%

As can be seen, the factor persists during all the years, except for year 2007. I have constructed ten deciles for this year, in the same manner as I did with volatility factor. This time, the low liquid decile is to outperform the top decile (the most liquid). The results of deciles performance is shown in the table below.

Table 4. Liquidity deciles comparison

Deciles (H-L)	Difference, %
10-1	9,8%
9-2	1,44%
8-3	-32,9%

7-4	18,37%
6-5	22,26%

I used the same manner of testing the deciles. As can be easily seen from the table above, there's only one negative number which represents the difference between the 8th and the 3rd deciles. All the other results are fully consistent with the common trend presupposition. So I can assume, that there's a small number of outliers, which have a significant impact on the overall picture. These outliers sit somewhere in the 3rd or the 8th deciles, providing a huge negative result. Considering, that the other deciles do not provide any arguments against the factor persistence and that all the other years' factors are positive, I will simply exclude this difference from the calculations, use only first two and last two deciles to estimate the factor, to be used in regression analysis. In that case, the Liquidity factor for Year 2007 will be: 8,5%.

The market factor, calculated as the excess return of S&P 500 index over the constant maturity 10-Year T-Bond is following:

Year	Annual return S&P 500	10 Year Constant Maturity Bond rate	Market Factor
2001	-11,85%	5,05%	-17%
2002	-21,97%	3,82%	-26%
2003	28,36%	4,25%	24%
2004	10,74%	4,22%	7%
2005	4,83%	4,39%	0%
2006	15,61%	4,7%	11%
2007	5,48%	4,02%	1%

The market premium appeared to be negative in 2001 and 2002. This reflects the overall market falling, when the constant maturity 10-Year Bonds still provided positive returns. During those years, market was unable to provide any excess return for the higher risk.

It's noticeable, that all the three factors increase significantly, by 200% in 2003. The Market factor in 2002, could be that low as a result of the “internet bubble bursting”. Then, in 2003, the market went bullish again, which is reflected not by the returns only, but also by every one of the three factors computed. This allows me to assume, even before the actual regression is run, that there is a correlation between the factors and the returns. So I believe, that till this point at least, I was moving in the right direction.

The regression results and multifactor beta.

In order to run a regression, I have structured the data in Excel using the following template:

Ticker	Year	Return	Factor 1....
A	2000		
A	2001		
A	2002		
AA	2001		
AA	2002		
AA	2003		

This is the structure, needed for R in order to treat the data as panel. (of course it's not enough, I also had to use `y<-cbind(Return)` and allocate the `xs` in the same manner.

As factors, I used the annual liquidity, volatility and market factors, computed before.

After running the hausman test (`phtest()` in R) and the Breusch-Pagan test (`plm()` in R) I found that the random effect models is the best in terms of fitting to my data.

The results of the regression were following:

```
> summary(random)
```

Oneway (individual) effect Random Effect Model

(Swamy-Arora's transformation)

Call:

```
plm(formula = y ~ x, data = pdata, model = "random")
```

Unbalanced Panel: n=7, T=488-520, N=3519

Effects:

	var	std.dev	share		
idiosyncratic	73.9422	8.5990	0.995		
individual	0.3875	0.6225	0.005		
theta :					
Min.	1st Qu.	Median	Mean 3rd Qu.	Max.	
0.4698	0.4706	0.4756	0.4755	0.4793	0.4819

Residuals :

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
	-2.36	-0.71	-0.29	0.00	-0.01	388.00

Coefficients :

	Estimate	Std. Error	t-value	Pr(> t)
(Intercept)	0.30901	0.30082	1.0272	0.3044
xmt	1.56187	1.25993	1.2396	0.2152
xlt	0.72765	0.90607	0.8031	0.4220
xvt	-0.38430	0.75308	-0.5103	0.6099

Total Sum of Squares: 260250

Residual Sum of Squares: 259740

R-Squared : 0.0019649

Adj. R-Squared : 0.0019627

F-statistic: 2.30669 on 3 and 3515 DF, p-value: **0.074681**

I have emphasized the necessary values. The Pr(>|t|) is much higher than 5% which leads to the conclusion that all three independent variables are insignificant. The residual sum of squares is almost as high as the total and this is also reflected by Adjusted R-Squared, which is almost 2%. The Adjusted R-square demonstrate how well the variety in dependent variable is explained by

the independent variables. Anyway, this test's results show that the factors by itself do not explain the returns.

I decided to change the independent variables and run one more regression. Everything is the same, but this time I will use the individual measures, related to factors. And see what will be the result then.

```
> summary(random)
```

Oneway (individual) effect Random Effect Model

(Swamy-Arora's transformation)

Call:

```
plm(formula = y ~ x, data = pdata, model = "random")
```

Unbalanced Panel: n=7, T=488-520, N=3518

Effects:

	var	std.	dev	share		
idiosyncratic	0.5756194	0.7586959	0.999			
individual	0.0006724	0.0259308	0.001			
theta :						
	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
	0.2019	0.2025	0.2063	0.2063	0.2092	0.2113

Residuals :

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
	-0.627	-0.056	-0.012	0.000	0.032	42.900

Coefficients :

	Estimate	Std. Error	t-value	Pr(> t)
(Intercept)	0.70778837	0.09444081	7.4945	8.375e-14 ***
xmt	0.99097657	0.00148385	667.8418	< 2.2e-16 ***
xliq	-0.79257140	0.11469843	-6.9100	5.728e-12 ***
xvt	0.00050092	0.00527259	0.0950	0.9243

Signif. codes:	0 ‘***’	0.001 ‘**’	0.01 ‘*’	0.05 ‘.’
	0.1 ‘ ’	1		

Total Sum of Squares: 260610

Residual Sum of Squares: 2022.3

R-Squared : 0.99224

Adj. R-Squared : 0.99111

F-statistic: 149775 on 3 and 3514 DF, p-value: < 2.22e-16

Again R has picked the random model as the most suitable one. The results this time are crucially different! All the explanatory variables are significant. The F-test p-value demonstrates the same. The adjusted R-square is now 99,2%. Though I need to mention that considering the significance level, it's mainly the volatility factor that has strong influence on the returns.

The random effects assume that the entity's error term is not correlated with the predictors. According to Torres-Reyna, 2013, that the main reason to use random effects is a belief, that differences across entities have some influence on the dependent variable.

So, that's the model I used, and based on the results, it's possible to say that every explanatory variable has an effect on dependent one.

This effect is measured by the coefficients (betas) over seven year concerning the unbalanced cross-section. It's noticeable that the liquidity measures coefficient is negative. This means, that the increase in No0vly leads to decrease in returns, which is fully consistent with the findings of Liu. This confirms the existing of the liquidity factor in the way it was calculated. Also, all the coefficients significantly exceed their standard errors, which also supports the fact, that the independent variables in the model have an influence on the dependent one.

So in order to weight the stocks in the three-factor portfolio, I will use those betas.

Portfolios construction.

First, I have constructed the smart beta portfolio, based on volatility measures and the one, based on liquidity measures. This was done in Excel. To calculate the weights, I created the Year-Measure matrices. The total value of the measure was computed using sumif() function over seven years. Then, the contribution of each stock's measure was found and used as a weight of the stock. This weight was then multiplied with the relative price change. Then, the resulting figure was multiplied with the portfolio value of the prior year. In 2000 it's value was 1000. Then, the returns for every year were found, as following:

The return was found as the $(\text{current portfolio value} - \text{prior year portfolio value}) / \text{prior year portfolio value}$.

Then, the average of these returns over 7 years represented the return for the portfolio over seven years. And the standard deviation of the portfolio's returns over seven years was calculated.

$$\text{Sharpe ratio} = \frac{r_p}{\sigma}$$

So here are the results of volatility and liquidity portfolios, constructed in such manner.

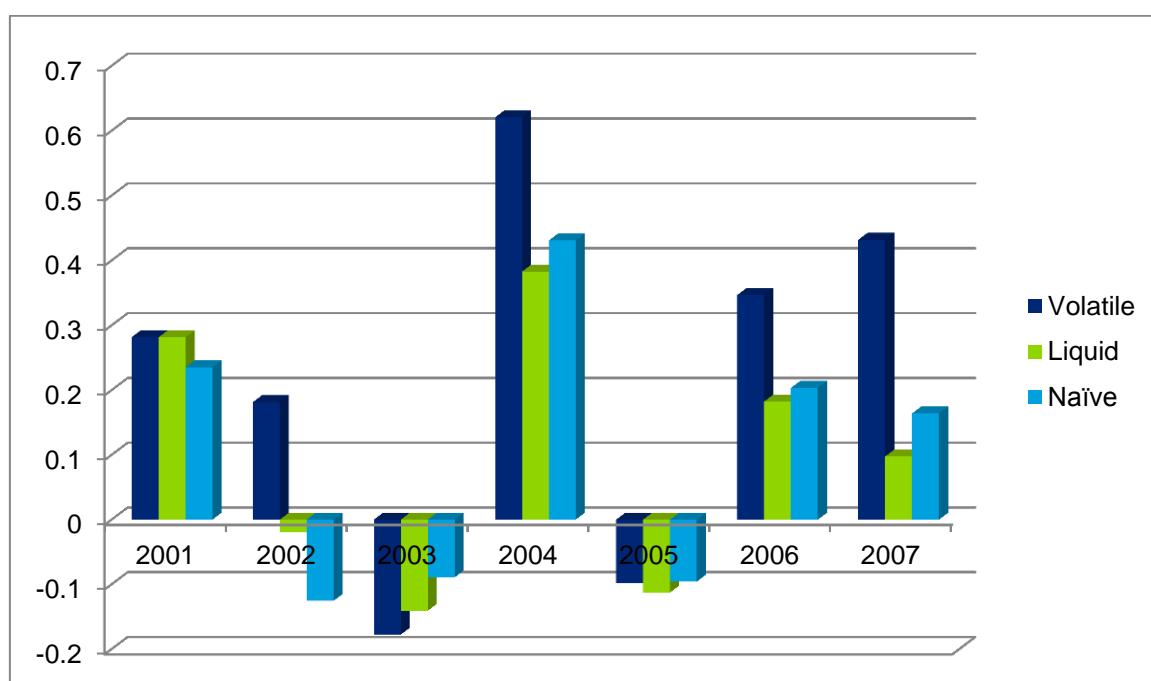
Returns	
Volatility	liquidity
0.2819	0.2817
0.1815	(0.0189)
(0.1776)	(0.1408)
0.6205	0.3828
(0.0980)	(0.1128)
0.3467	0.1824
0.4319	0.0976
0.2267	0.0960
0.2845	0.1988
0.796903	0.482957

It's easily seen, that the volatility portfolio provides higher Sharpe ratio than its liquidity-based counterpart. The reason for this could probably be the logical mistake in constructing the liquidity-based portfolio. This is a No0vly based portfolio, but constructed in such a way, that the higher is the liquidity- the higher is the stock's weight, when in terms of maximizing the returns it should have been the opposite.

Then, I have constructed the benchmark- The naive portfolio in order to compare the performance of these three.

1000 Return	
1235.062	0.235062
1080.056	-0.1255
983.7549	-0.08916
1407.55	0.430793
1272.852	-0.0957
1531.921	0.203534
1783.221	0.164043
Average	0.103295
St Dev	0.195427
Sharpe	0.528562

The Sharpe ratio of the benchmark portfolio is 0.528562, which is lower then the volatility-based portfolio measure, but higher then the liquidity-based one. Below, the histogram of annual returns for each portfolio is given.



From the Histogram it's easily seen, that the volatility-based portfolio's returns have the highest spread. Except for Year 2002, liquidity-based and the benchmark-naive portfolios are moving really closely together in terms of the returns. This is also reflected by the Sharpe ratios. The liquidity-based portfolio has a Sharpe ratio very close to the one of naively allocated. (it's 0.48 against 0.52)

Now, I need to adjust the weights of the stocks to capture the three factors simultaneously, by using the betas from the estimated equation.

I need to mention, that the way, I am going to adjust the weights of the stocks is purely experimental. I will multiply the factor corresponding measure of each stock with this measure's coefficient from the random effect panel data regression. Then, I will take the average of the three. Sum them up- and that's how I get the annual weights of every stock in the portfolio. This is some kind of value weighting. Namely, I take the factor-related measures of every stock, weight them with the coefficients and get the weighting base! The technique is pretty similar to the one, used for standard cap-weighted indices. That's why what I've done can be technically called the smart-beta strategy.

Then, the procedure is pretty much the same to the one, applied before when weighting the smart-beta portfolios.

The average measures I took often appeared to be of different sign. Technically, the negative eight of a stock in a portfolio means, that the investor has to have this stock short.

As far as the point of weighting like that was to capture the effect of the risk factors on the portfolio of stocks, the sign doesn't really matter. I am not trying to build a model, that predicts the future stocks returns, I simply try to construct the portfolio, that would provide the best risk-return trade-off. So, I skipped the sign differences, by simply taking the ABS() of the average measure. Every stock was then weighted as a proportion to its individual measure to the total sum of those.

Here are the results of this three-factor smart-beta portfolio:

Year	Return
2001	0.2787
2002	0.0349
2003	0.1856

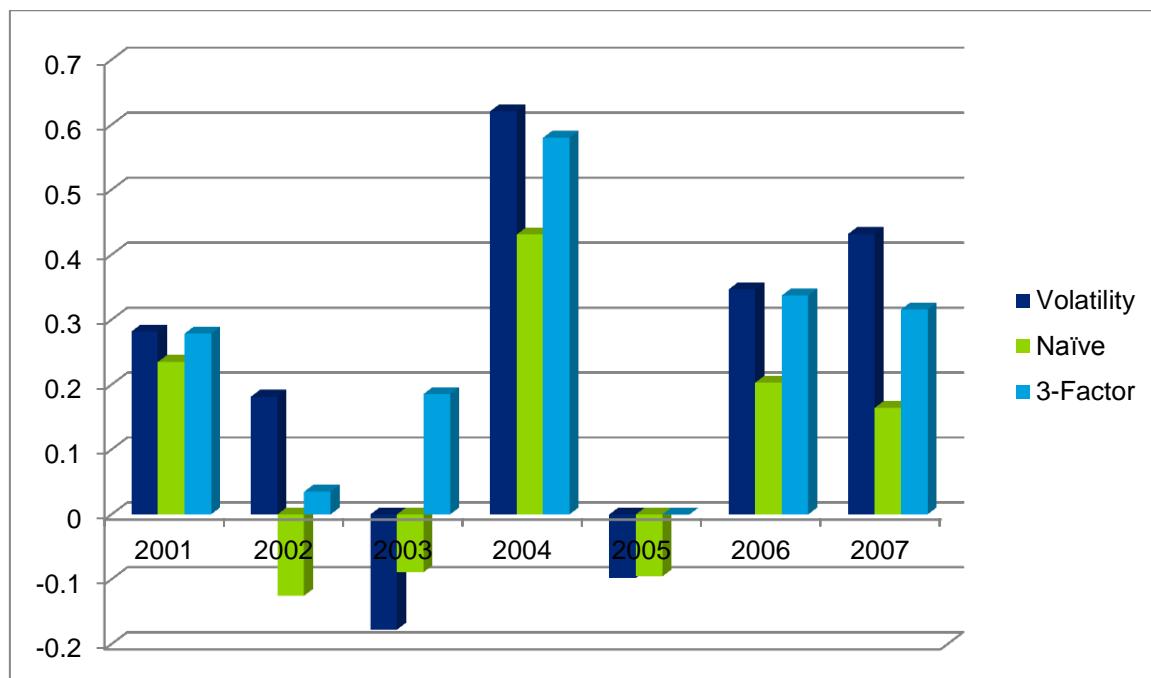
2004	0.5800
2005	-0.0008
2006	0.3372
2007	0.3158

The standard deviation and the average return on this portfolio over the seven years are 0.1979 and 0.2473 respectively.

These numbers provide the Sharpe ratio of 1.250059, which is the highest among the portfolios compared in this work.

Below, I've put two histograms: one represents the returns on Volatility, Naive and Three-factor smart-beta portfolios, the other- represents the portfolios value over years

The returns



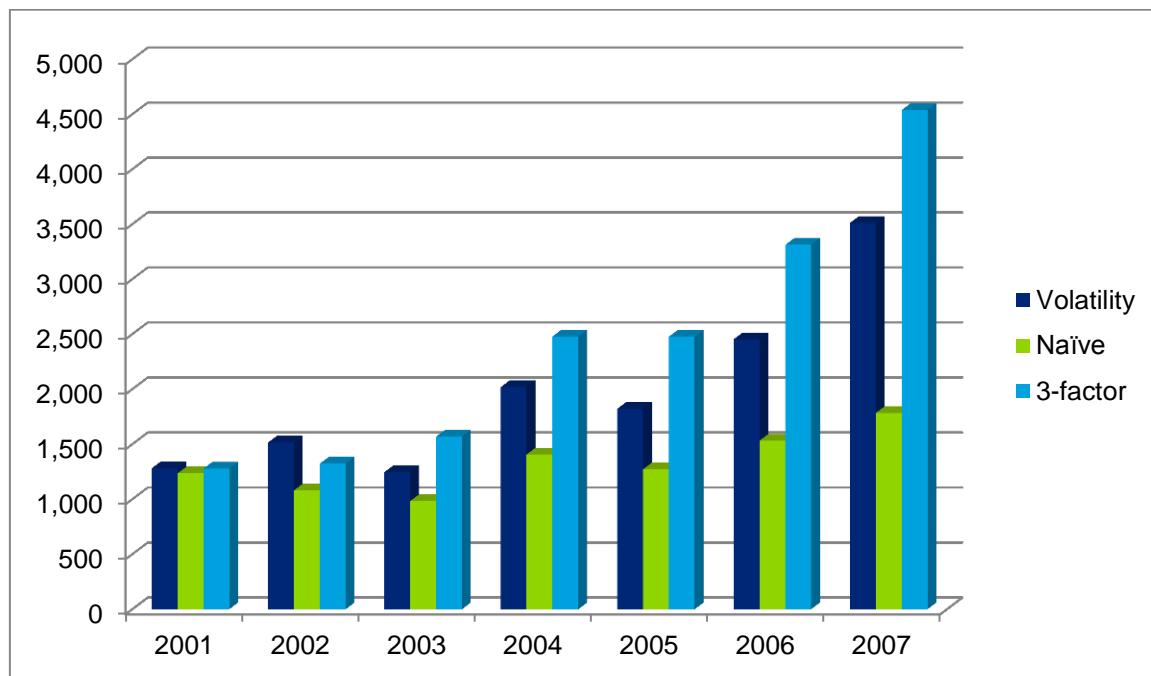
The return on the 3-factor smart-beta portfolio tends to be positive more often than it's counterparts. The spread of this return is smaller- the measure is more stable and it's always higher than the benchmark.

The portfolio values

Year	Volatility	Naive	3-factor
2001	1,281	1,235	1,278
2002	1,515	1,080	1,323
2003	1,246	983,75	1,569
2004	2,019	1,407	2,479
2005	1,821	1,272	2,477
2006	2,452	1,531	3,312
2007	3,511	1,783	4,538

The histogram of portfolio values is presented below

The portfolio values



Now the growth of the portfolios value over time is clearly seen. The 3-factor portfolio represents the most stable growth over the period. You can draw a line more closely to this one, while the other portfolios trend is not that easily detectable. To summarize, there are two portfolios showing better performance over given 7 years period than the naive benchmark. The volatility based smart beta portfolio and the 3-factor smart beta portfolio. The corresponding Sharpe ratios are given below.

Volatility	Liquidity	Naive	3-factor
0.796903	0.482957	0.528562	1.250059

Conclusions.

The purpose of this work was to find a model, that will show better performance in terms of Sharpe ratio, than the widely used naive allocation model. Basically, I tried to find an approach to weighting stocks in the portfolio, that would give me the best possible risk-return trade-off.

In order to do that, the stocks included in S&P 500 index were chosen as a sample, the period was from year 2001-to 2007. I also obtained data for 2000 in order to calculate the measures based on the prior year results.

The Smart-Beta strategy was chosen to be tested against the Naive portfolio. Based on the idea of such strategies, 2 risk-factor smart beta portfolios were constructed: the volatility-based and liquidity-based portfolio. All the portfolios were rebalanced annually. The volatility-based portfolio showed better Sharpe ratio than the benchmark did. But according to Amenc, et al. , there was a huge probability, that this portfolio is overwhelmed with volatile stocks and ignores the other important characteristics and factors. This provides an idea to improve the strategy.

The three factors are constructed every year: Liquidity factor, based on the adjusted-No0vly measure, volatility factor, based on annual historical volatility measure and market factor, which is the excess return of the S&P 500 value-weighted index over constant maturity 10-year T-Bond.

The factors constructed are consistent with the currently existing findings in the area and support the hypothesis that there is an influence of those factors on the stock returns.

But the regression results provided no statistically significant evidence on the factors, having the effect on the returns. Still, the regression, that used measures as independent variables provided 3 significant coefficients. These coefficients were used to find the weights of a smart-beta-three factor portfolio. The aim was to capture all the relevant factors. The resulting portfolio was the best in terms of Sharpe ratio among all the other portfolios tested in this work.

Still there is a shortcomings in this paper that should be accounted for and probably resolved in later works on this topic:

The time interval is too small and the sample is not big enough to make valid conclusions, though the random effect model gives the generalization opportunities.

Still, I believe that the results of this work justified the right for existence of the main hypothesis: there can be a model that would consistently outperform the naive allocation strategy. I believe, that the smart-beta strategies deserve further investigation. Such strategies can be the future of modern financial engineering. The fact, that the two of them outperformed the benchmark over given 7 years provides the base for further research. For example, one could test if the amount of risk factors included in the smart-beta strategy has a direct impact on the resulting risk, return, Sharpe ratio.

The raw data was taken from CRSP Database. All the calculations and tables available upon request in (.xlsx, csv, R-workspaces). I could not include those in the Appendix because of size and number formats.

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