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Co-Developing Johan Castberg and Alta/Gohta: A Real Options Approach

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Abstract

This thesis describes the added value of co-producing the adjacent oil fields Johan Castberg and Alta/Gohta, with respect to capital expenditures. Limited research has been done on the effect of capital expenditures in co-producing fields using real options methodology. Co-developing adjacent oil fields is an important theme for the Norwegian Government and producers on the Norwegian Continental Shelf, where new discoveries tend to be smaller in size and unprofitable as individual developments. Both Johan Castberg and Alta/Gohta are large discoveries, but suffer from high cost from being located in the undeveloped Barents' Sea.

A central assumption in the valuation is that all benefits in co-producing the oil fields are reflected in investment costs, where developing Johan Castberg as a central production hub limits the need for invested capital on Alta/Gohta.

In order to determine added value, the total field value is compared with the value of the individually producing fields with flexibilities. For this purpose two real option methologies are used, the binomial option pricing model by Cox, Ross and Rubinstein (1979), and the Least Squares Monte Carlo proposed by Longstaff and Scwhartz (2001). The most suiting model is used to determine if co-production adds value.

The estimations show that including the option to decide when to invest in the fields adds significant value, as option methodologies take advantage of the highly volatile prices in the model. Benefits of being able to prematurely abandon field however, adds small value. This was the cases using both models.

The two models give different results already when estimating value in a base case. As decisions are introduced and limited to once a year, the level of granularity in the binomial model suppress values. Granularity can be introduced using a Monte Carlo simulation approach through increasing the number of simulations, and the Least Squares Monte Carlo is introduced as the model to determine the conditions where co-production adds value.

For co-development to be profitable, a large discount in capital expenditures on Alta/Gohta needs to be introduced, compared to the cost of extra capacity on Johan Castberg. Within assumptions made, nominal discount on the development on Alta/Gohta must exceed the extra capacity costs on Johan Castberg by 44 percent. The added value is also highly susceptible to sizes of the fields, where co-production is most beneficial when the reservoir size on Alta/Gohta is small.

Sammendrag

Denne avhandlingen beskriver merverdien av å samprodusere de nærliggende oljefeltene Johan Castberg og Alta/Gohta, med hensyn til investeringskostnad. Begrenset forskning er gjort på kapitalinvesteringers effekt på samkjørte oljefelt ved bruk av realopsjonsmetoder. Samarbeid mellom petroleumsfelt er et viktig tema for den norske stat og produsenter på norsk kontinentalsokkel, hvor nye olje-funn er mindre i størrelse og ofte ulønnsom om utviklet alene. Både Johan Castberg og Alta/Gohta er store funn, men er utsatt for høye kostnader gjennom å være lokalisert i det uutviklede Barentshavet.

En sentral forutsetning i verdsettelsen er at alle fordelene ved å samkjøre feltene er gitt ved kapitalkostnad, hvor Johan Castberg blir utviklet som et sentralområde for produksjon og dermed reduseres investeringsbehovet på Alta/Gohta. Estimering av merverdi gjøres ved å sammenligne totalverdien ved samproduksjon, med totalverdien av individuelt produserende felt inkludert fleksibilitet. Til dette formålet sammenlignes to realopsjonsmodeller, den binomiske opsjonsprisingsmodellen av Cox, Ross og Rubinstein og Minstekvadraters Monte Carlo-modellen av Longstaff et al. Modellen som beskriver verdiene av feltene mest presist blir så brukt til å bestemme merverdien av samproduksjon.

Estimeringene viser at å inkludere fleksibilitet til å bestemme investeringstidspunkt på feltet gir signifikant merverdi. Fleksibilitet til å legge ned feltene tidligere enn planlagt, gir derimot lite verdi. Dette var resultatet i alle casene.

Realopsjonsmodellene viser store forskjeller allerede fra estimering av en base case. Ved å inkludere tids-steg og beslutninger på årlig basis gir den binomiske modellen nedjusterte verdier når modellen ikke er granulær nok. Detaljnivå kan inkluderes i den Monte Carlo simulerings-baserte modellen ved å øke antall simuleringer, og blir brukt videre til å bestemme verdien av å produsere feltene sammen.

Merverdien på Johan Castberg + Alta/Gohta er avhengig av en stor reduksjon av kapitalkostnader på Alta/Gohta, sammenlignet med økningen i investeringsbehovet på Johan Castberg for å inkludere ekstra kapasitet. Innen gitte forutsetninger, må reduksjonen i kapitalkostnader på Alta/Gohta minimum være 44 prosent høyere enn de økte kostnadene på Johan Castberg for å være lønnsomt. Merverdien er videre påvirket av størrelsen av feltene, hvor samkjøring av feltene er mest lønnsom når reservene på Alta/Gohta er små.

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Acronym List

ACF	Autocorrelation Function
ARIMA	Autoregressive, Integrated, Moving Averages
ARIMA(0,1,0)	First Order Integrated ARIMA Model (See ARIMA, GBM)
AR(1)	First Order Autoregressive Process
B&S	Black & Scholes
CAPEX	Capital Expenditures
d	Down Factor in Binomial Lattices
DCF	Discounted Cash Flow
EMM	Equivalent Martingale Measure
E&P	Exploration and Production
FPSO	Floating Production, Storage and Unloading
GBM	Geometric Brownian Motion
LSM	Least Squares Monte Carlo
NPD	Norwegian Petroleum Directorate
NPV	Net Present Value
MRM	Mean Reverting Model(s)
OPEC	Organization of the Petroleum Exporting Countries
OPEX	Operating Expenditures
p	Risk Neutral Probability
PACF	Partial Autocorrelation Function
ROV	Real Option Valuation
t	Period t (Usually Refers to Investment Period)
T	Final Period in t (see t)
TPA	Third Party Access Agreement
u	Up Factor in Binomial Lattices
v	Period v (Usually Refers to Production Period)
V	Final Period in v (see v)
WACC	Weighted Average Cost of Capital

Preface

This thesis concludes a Master of Science in Business at Nord University Business School, with the specialization Energy Management.

Building on a Bachelor’s thesis in valuation, and motivation to further my knowledge in real asset valuation in the energy industry - real options application in petroleum was a challenging and valuable experience.

I would like to thank my supervisor Thomas Leirvik for guidance and constructive comments. I would also like to thank Svein Sundallsfoll for important feedback during the final period, and to Kristian Støre at Nord University for interesting discussions and an essential introduction to financial option modelling.

1.0 Introduction

This thesis examines the added value of co-producing oil fields Johan Castberg and Alta/Gohta. Co-producing adjacent fields is an important theme for the Norwegian Government (2005) and players on the Norwegian Continental Shelf. New-found fields tend to be small, where many are unprofitable as stand-alone. In order to increase the efficiency on maturing fields, these smaller fields are coupled together with nearby licenses, to limit expenses and increase efficiency. This is especially an important theme in the current climate with inflated costs and low oil prices (Norwegian Petroleum Directorate (NPD), 2005).

Johan Castberg, Alta and Gohta (Alta/Gohta) are currently the most prospective fields located adjacently in the mild part of the Barents' Sea. Following Goliat, the two areas can be the second and third two oil fields developed in the Norwegian High North. Lacking supporting infrastructure, and being far from markets in an unexplored basin – Johan Castberg and Alta/Gohta are challenged by high expenditures. To reduce costs, there has been discussions on developing Johan Castberg as a central hub, with capacity to include nearby licenses - as Alta/Gohta (Bjørsvik, 2015).

This paper aims to estimate added value of co-producing Johan Castberg and Alta/Gohta, using effect on capital expenditures (CAPEX) and real options as value measurement tool. Real option methods are increasingly being favored as a valuation framework with focus on the value of flexibility in uncertain environments. Petroleum exploration and production projects often serve as example projects in academic research in real option valuation, as the projects contain both.

In order to estimate the value of co-producing the fields, two cases will be described; the case of individually producing fields, and the case of co-producing fields. In the case of co-production, the relatively more mature Johan Castberg will serve as a central hub, with expanded production capacity to include oil for Alta/Gohta, the tie-in field. This comes at a cost. Alta/Gohta could then be developed at a discount, reflecting that the field will be connected to Johan Castberg. A central assumption is that all benefits from coproduction are reflected in the capital expenditures.

Both fields have added flexibilities, the option to prematurely abandon the field, and the option to defer investment. To incorporate these flexibilities, two central real options methodologies will be assessed. The models are based on the binomial option pricing model by Cox, Ross and Rubinstein, and the Least Squares Monte Carlo (LSM) method proposed by

Longstaff and Scwhartz (Cox et al., 1979; Mun, 2006; Longstaff & Scwhartz, 2001). Based on a discussion on precision and ease of modelling to derive new values – the most suiting model will be used to determine the effect of co-production. An underlying theme will therefore be to analyze value added by including these flexibilities.

Limited research has been done on the effect of capital expenditures in co-producing fields in real options methodology. With Alta/Gohta and Johan Castberg as the case to study, the problem statement is defined as:

"With respect to capital expenditures, does co-producing Johan Castberg and Alta/Gohta add value? ".

With high uncertainty in the estimates on capital expenditures if the fields are developed conjunctly – the problem statement will be answered in terms of the discount needed on Alta/Gohta compared to the expenditures for extra capacity on Johan Castberg. The paper strives to explain the modelling process in detail for understanding of the process in order to assess the precision, and replicability.

1.1 Limitations

Due to confidential information, this thesis is based on both public information and own estimations. This is a highly simplified description of a complex real life process, and central parameters are assumed to be static and inflexible.

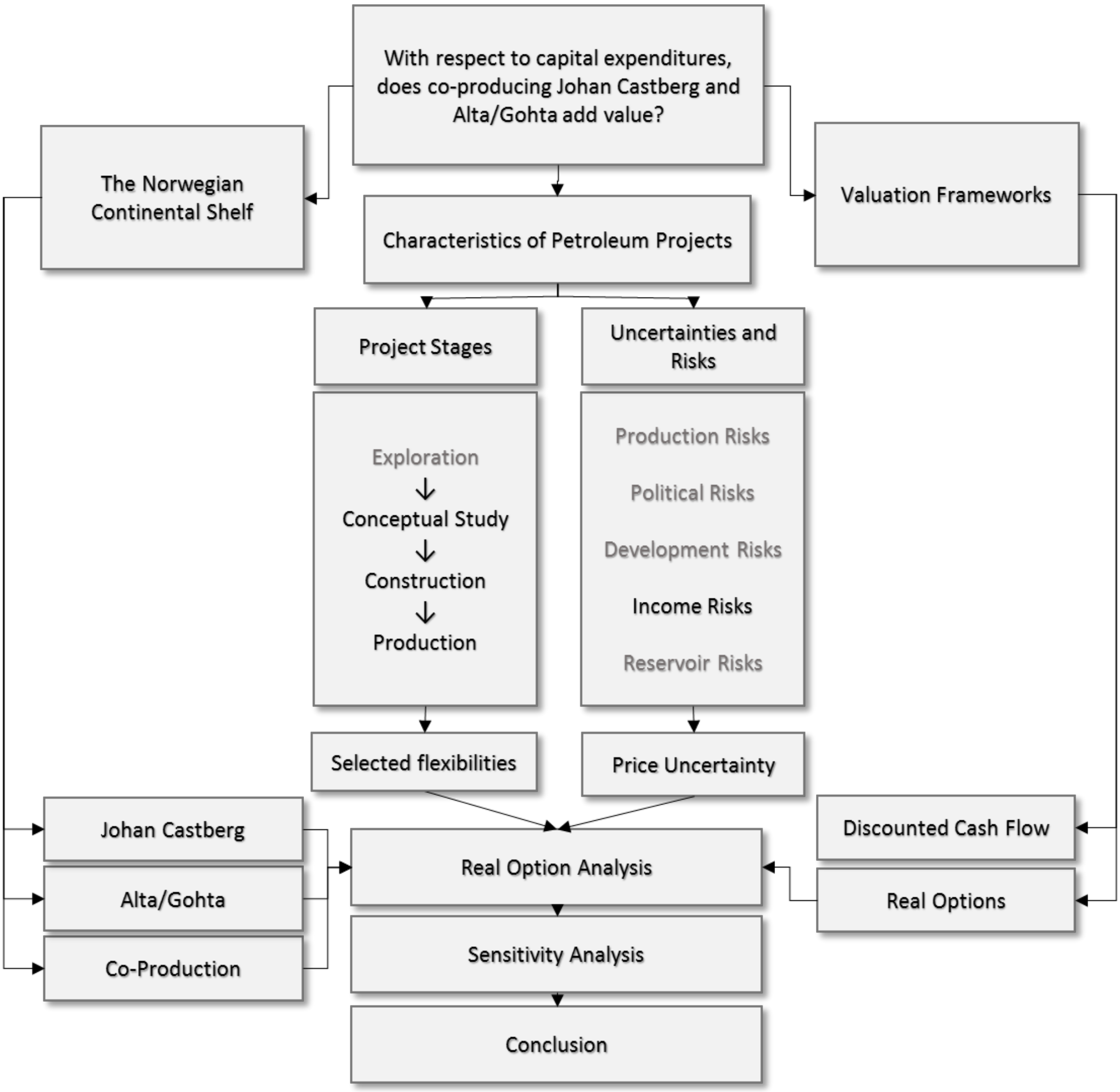
Operating expenses, fixed cost sharing and other value-adding aspects of co-production are omitted when valuing the co-producing fields.

The thesis further only takes financial value into consideration, omitting all aspects of political and inter-organizational behavior.

1.2 Structure

The structure in the thesis is summarized in figure 1.1.

Figure 1 1: Structure of the Thesis



The seven core chapters of this thesis are chapter two to nine. Theoretical framework is presented from chapter two to four, intertwined with relevant methodology.

Chapter two introduces the theme of petroleum in Norway, with central issues set in context of two undeveloped oil fields. Chapter three presents the characteristics of petroleum projects, with relevant flexibilities and uncertainties, and defines the scope of this thesis. Chapter three also contains an in-depth introduction to the main uncertainty factor, oil price. Two income-based valuation frameworks are introduced in chapter four, together with frameworks of estimating value under uncertainty.

Chapter five to eight includes analysis and results. Chapter five includes data analysis, while valuation process and an introduction to the results are presented in chapter six. Chapter seven describes and compares the estimations in detail, and discusses the precision of the two valuation models included. A sensitivity analysis is presented in chapter eight, while chapter nine concludes the thesis, with suggestions for further research.

2.0 The Norwegian Continental Shelf

This chapter introduces central aspects on the Norwegian Continental Shelf, and substantiates the importance of the problem statement.

2.1 The Norwegian Petroleum Economy

The Norwegian petroleum era began with the discovery of Ekofisk, in 1969, and first oil in the area was in 1971 (Norwegian Government, 2016). The still producing field is located in the North Sea, one of the three petroleum ocean areas on the Norwegian Continental Shelf (NCS). The North Sea is still the powerhouse of the Norwegian petroleum production with 65 producing fields at year-end 2015. At the same point, the Norwegian Sea had 16 producing fields. The Barents Sea is the final frontier of offshore petroleum in Norway, and suffers from challenges. Located far from markets and existing infrastructure, together with limited geological knowledge in the basin - cost of being present in the Norwegian High North is unfavorable (INTSOK, 2015). Snøhvit and Goliat, respectively a gas and an oil field, are currently the only fields in production in the Norwegian part of the Barent’s Sea. The two most prominent new oil fields in the area are Johan Castberg, and Alta/Gohta.

The largest and most promising fields on the shelf were produced first, and maturity is reached on many fields. With declining production, peak was in 2000. At that point, Norway was the third largest exporter of both oil and gas (Ryggvik, 2014). Following up to date, the rate of finding new fields at significantly higher, but as new fields are generally smaller – developments are dispersed over a high number of marginal fields with higher break-even price (NPD, 2014). As a result of this, the Norwegian Government increased the focus on cooperation between fields. Gjøa, Vega and Vega Sør is an example of co-production of fields between different licensees (Offshore Technology).

Creation of values, technology and competence in the sector, makes it imperative for the Norwegian welfare. Excluding production of relating goods and services, petroleum sales has contributed with NOK 12.000 to the Norwegian GDP (2016 value, NPD, 2016)). This amounts to production of close to half of the recoverable reserves on the NCS. Recoverable reserves are defined to be the amount of oil that is economically and technologically feasible to extract. Petroleum sales and sales of related goods and services accumulated to 16 percent of the Norwegian GDP in 2015 (21% in 2013), and 39% of total export value (Statistics Norway, 2016; NPD, 2016).

2.2 The Norwegian Licensing System

Production licensees of a field, or area, is awarded to joint ventures through licensing rounds hosted by the government. The license give exclusive rights to explore for and produce petroleum within a specified area. Normally it is awarded for an initial period for exploration, for a maximum of ten years. Within this period, the consortium must meet certain work obligations determined in the license. If the predetermined obligations are met, the production license will normally be prolonged by 30 years (Statoil, 2011).

2.3 The Arctic Oil Fields: Johan Castberg and Alta/Gohta

The following three sections introduce Johan Castberg, Alta/Gohta and the notion of co-developing the fields.

2.3.1 Johan Castberg

Johan Castberg was discovered April 2011, located approximately 240 kilometers offshore from Hammerfest (NPD). It consists of three separate fields: Skrugard, Havis and Drivis which together is estimated to contain recoverable reserves of between 400 to 650 million barrels of oil (Haugstad, 2016; Melberg, 2016). Operator of the field is Statoil (50%), and licensees of the projects include Eni (30%) and Petoro (20%). The licensees will further be noted as the consortium. After two set-backs, first from finding less oil than expected (Lindberg, 2016) in addition to the price drop, the investment decision has been postponed. The intended concept of an investing in an individual oil field, and transporting the oil ashore to Hammerfest does not seem feasible in the current environment, with an estimation of NOK 100 bln in CAPEX. Early in 2016, Statoil announced that they are able to decrease capital expenditures to NOK 50-60 bln, choosing a Floating Production, Storing and Offloading Plus (FPSO Plus) concept over a platform, and separating out the pipeline onshore to Hammerfest as a potential, individual project. The FPSO Plus unit is able to process oil from other licenses. The investment decision on the field is assumed to be in 2017, with the first oil expected in 2022 (Lorentzen, 2016)

2.3.2. Alta/Gohta

Alta/Gohta are located approximately 200 kilometers from Hammerfest, with 20 kilometers from each other (Bjørsvik, 2015). The recoverable size of the joint fields are estimated to be in the range of 216 to 584 million barrels of oil equivalent, with 351 million barrels as a probabilistic mean (Lundin Petroleum, 2016). Gohta and Alta were found in 2013 and 2014 respectively. Operator on the fields are Lundin Norway AS while partners are Idemitsu (30%) and RWE Dea (30%) on Alta, and Det Norske (40%) and Noreco (20%) on Gohta (Petro.no, 2015). Developments on the joint fields are in exploration, and Lundin stated that they are initiating feasibility studies in 2015 (Bjørsvik, 2015).

2.3.3 Co-Production

With regards to locational disadvantages, pursuing exploration and development in harsh climate with high development costs, the developments suffer from challenges (INTSOK, 2015). The operators on the fields have discussed the theme of co-development, without reaching a conclusion (Bjørsvik, 2015). Among other reasons, Johan Castberg is in a more mature position than the relatively newly explored Alta/Gohta area (area, field and license will be used intertwined in the thesis.) For an agreement to be present, resource-contingencies has to be settled (Bjørsvik, 2015).

Figure 1 2: Barents' Sea Fields (Lundin Norway AS, 2016)

To maximize value creation on NCS, co-production is an important theme for the Norwegian Government. Conjunctly producing fields can increase value both in terms of economy of scale, and effective use of existing infrastructure (Norwegian Ministry of Petroleum and Energy (NMP), 2005). The investment cost in developing offshore petroleum fields often passes the USD billion mark, and conjunctly producing nearby fields can include a formidable relief on capital expenditures (Bay et al., 2012).

To promote co-production on the NCS; NPD, a department under the Ministry of Petroleum and Energy, launched the initiative Third-Party Access Agreement (TPA). This agreement has the objective of effective resource management through competitive, transparent and non-discriminatory access to infrastructure. The main principles of the agreement are objective and non-discriminatory terms of using infrastructure with respect to cost and privileges, where NPD can be of assistance in dispute-settlement cases. Profits made from production should further be related to the field, while at the same time the agreement should incentivize the infrastructure owner to invest in extra capacity (NPD, 2014). Central to the question of cost of using existing infrastructure, is that it is a commercial discussion between the companies involved. If the two parties do not agree, where the consequence is an undeveloped field – the Norwegian Government can enforce a decision of coproduction.

According to Dagens Næringsliv (2014), the General Director of the Norwegian Oil Directorate, Bente Nyland, stated a clear desire to establish Johan Castberg as a field center for further discoveries. In the case of co-production in the thesis, Johan Castberg will be the production hub, and Alta/Gohta the tie-in field.

Statoil acquired a 20.1 percent share in Lundin Petroleum, the parent company of Lundin Norway AS, following two rounds of acquisition during the first five months of 2016 (Statoil, 2016). The common interest that the two companies now have, can be an argument of co-production at competitive costs.

3.0 Upstream Petroleum Investments

This chapter has introduces characteristics of upstream petroleum investments, with focus on uncertainty and flexibility embedded within the projects. From a set of uncertainties and flexibilities that is present in a petroleum project, the main value drivers will be introduced in the analysis.

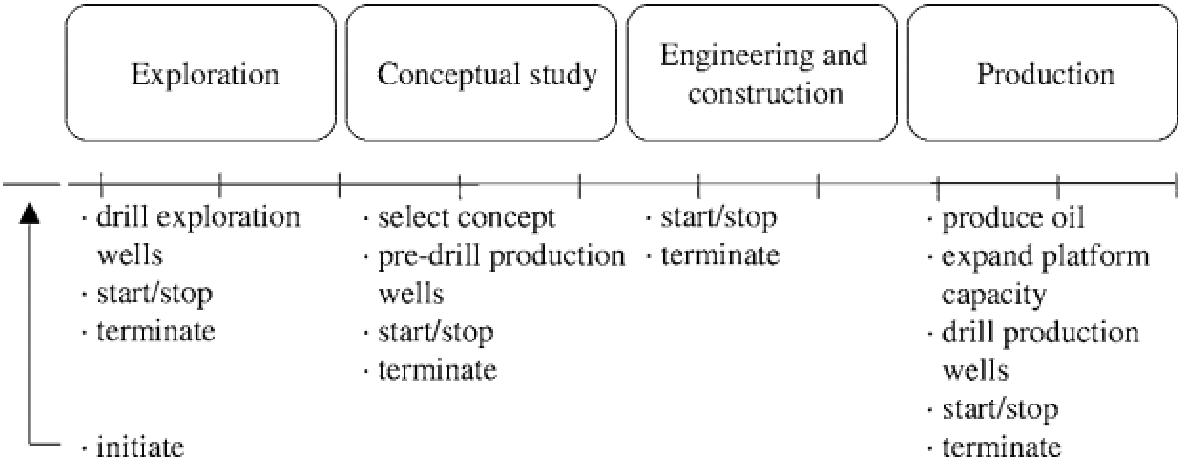
3.1 Characteristics of Upstream Petroleum Investments

The petroleum industry is generally divided into upstream, midstream and downstream. Upstream petroleum, often referred to as Exploration & Production (E&P) is the part of the value chain concerned with exploration and production of crude oil and natural gas. Midstream petroleum mainly concerns storage and transportation of crude products to the refinery. Downstream petroleum refers to the refining and sales of refined petroleum (PSAC).

Upstream petroleum is characterized by long life-time, uncertainty and flexibility. Several decades might pass between the initial exploration phase to end of production.

Lund (1999) divides the stages in four main phases, from before Plan for development and operation of a petroleum deposit (PDO/PUD), to abandonment:

Figure 1 3: Decisions within Petroleum Stages (Lund, 1999)



Uncertainty is present in all stages. Embodied in the projects is potential to make responding decisions. In order to get a better understanding of these flexibilities in petroleum projects, the project life cycle with relevant flexibilities are introduced.

During *exploration*, the owners decide where and how many exploration wells to drill within their license. Seismic operations are performed in order to map the area and the potential field. Some main elements considered are the quality, quantity and mix of oil and gas, reservoir characteristics and pressure in the field. The information mining continues during the lifetime of the project, in order to decrease the uncertainty.

In the *conceptual study*, decisions include to choose if and what type of petroleum producing unit is the most feasible. Statoil stated they will use a FPSO on Johan Castberg, with extra capacity to include production from nearby licenses (Lorentzen, 2016). The nearby license, coupled in to the main field, will further be addressed as a satellite, or tie-in field.

The *engineering and construction* phase describes the step before production, and includes decisions on when and if to invest.

Production has often the longest lifetime of the individual stages, with oil fields that can produce for many decades. Licensees continuously analyze the field and economic climate to enhance understanding of risks and benefits, in order to make value maximizing decisions. Further investments in the reservoir can increase the rate of recovery of oil, and lifetime of the field. At one point the decision to end production is pursued, at a point where expectations on future production is negative earnings on the project. The oil field at this stage is plugged and abandoned.

Upstream petroleum projects are filled with optionality, as the developments have long life-cycles in uncertain environments. These options to react to sequential information adds value if addressed correctly. For flexibilities to add value, uncertainty have to be present. Or else, the optimal choice will be pursued at the initial decision.

3.2 Risks and Uncertainties in Petroleum Projects

This section describes main uncertainties relevant for upstream petroleum investments. The companies involved can evaluate relevant project risks and uncertainties in order to define a value maximizing strategy.

Uncertainty and risk are often used interchangeable. Donald Rumsfeld famously stated that there are “unknown unknowns”, which can allude to the notion of uncertainty. In this aspect, risks are quantifiable, and uncertainties are not. In the thesis, the terms are used interchangeably.

There are relevant and irrelevant risks. Project relevant risks demands risk compensation, often in terms of adjusting the discount rate used. Project irrelevant risk, on the other hand, does not significantly affect the project value. Only project relevant risks are discussed.

Bøhren and Ekern (1985) lists five sources of risks within a petroleum project:

Political risks concerns the stability of the political system and fiscal regime for where the companies operate influences the project value. Changes in taxation policies and regulations revolving one of the steps in the petroleum projects can severely change the value of the prospect, and it can be necessary to change strategy accordingly.

Development Risks includes risks related to the construction of the field. The development process of the field are contingent upon timeliness coupled with expenditures. The energy industry is referred as the globally most capital intensive industry, and it is not uncommon to invest above USD 10 billion in a single mega-project (Goldthau, 2013), and cost-overruns in petroleum projects is normal. A report by NPD (2013) showed that projects presented in the state budget in the same year, had a sum cost overrun, compared to the original budget, of NOK 49 billion. The report also states that EY, in 2013 analyzed the 20 globally largest upstream developments showing an average overrun by 65 percent compared to original budget.

Production Risks relates to uncertainties in the producing fields, a combination of geological factors with the cost of production. The Greater Ekofisk Field was originally intended to produce 17-18 percent of its reserves, with expected abandonment in 2011. 43 years later, in 2011, the new timeline estimated production for a new 40 years (ConocoPhillips, 2013). This is reactions to a combination of production, reservoir, and income uncertainty.

Reservoir risks includes the geological factors, as existence and size of petroleum resources in the reservoir, and the quality of oil and gas. This risk factor is important in early phases of the field cycle, where defining the potential in the field is crucial to choose the appropriate production unit, and if it is profitable to make investments.

Income risks is mainly the uncertainty of future oil price, including interest and exchange rates. The oil price is arguably the main profitability driver of a field, and will be furthered discussed in section 3.3.

Upstream offshore petroleum investments are inherently uncertain and complex. Extreme investment and earnings, combined with long lifetimes and technical challenges within a -complex legal framework creates a convoluted framework for valuation. In the scope of this paper, the flexibilities included are typical options that the consortium faces that are expected to be high-value adding. The price of oil is included as the project uncertainty.

3.3 Oil Price Uncertainty

Oil is the globally most consumed source of energy, playing a central role in the largest industries as transport, construction, petrochemicals and power (British Petroleum, 2016). Changes in oil price affects the global economy on a macroeconomic level, and many state budgets rely on the oil income (IEA, 2014; NPD, 2016). Coupled with a high volatility, oil price uncertainty is important for consumers, producers and in academia. Within upstream petroleum projects the commodity price is one of the main profitability drivers, where solid measures are central in deriving the value of the field. Fluctuations in price are comprised of complex underlying structures, making it difficult to predict. This chapter will try to emphasize the complexity of oil price development, where even today – there is no consensus on the drivers 40 ago. This chapter will be a support to the theoretical framework on the price process, in section 3.4.

In order to simplify the oil traders' role, three benchmark prices for crude oil are commonly used, West Texas Intermediate, Dubai and Oman and Brent Blend. Brent Blend is the benchmark primarily used in Europe, and serves as the benchmark price for oil produced in the North Sea (IEA, 2014).

Historically, Brent and WTI prices have followed each other closely. Brent Crude typically was sold at a slight discount, as higher sulfur content increases refining cost. Following late 2010 to 2014 Brent was sold at a premium. This was mainly the result increased US production and storage in Cushing, weakening the price on WTI crude (Seeking Alpha, 2015). The spread in early to May 2016 revolved around zero (YCharts, 2016)

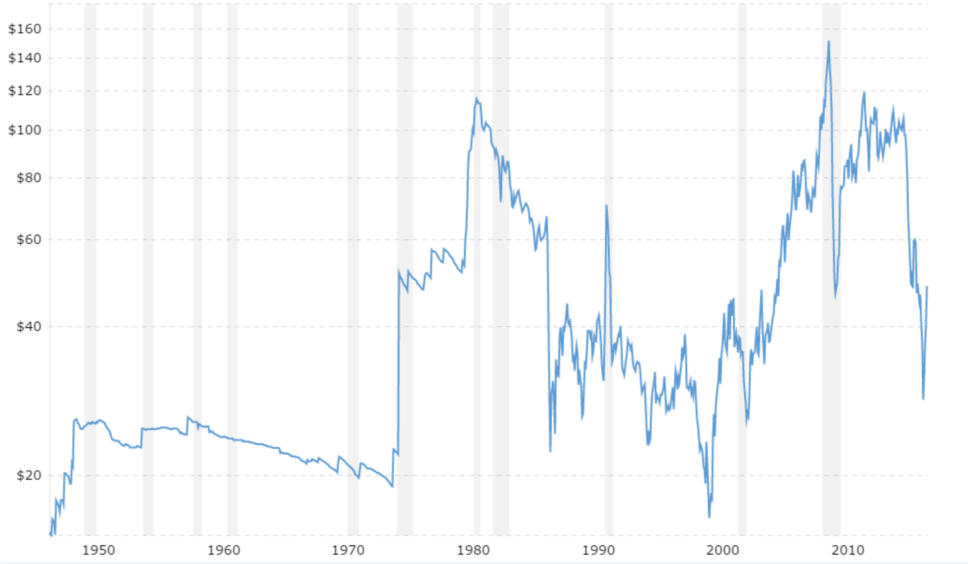
With a close connection between the benchmark in question, Brent crude, and WTI crude – the historic oil price development on WTI will serve as a proxy to describe the complexity in past Brent fluctuations.

A literature survey in 2016 explored causes for major WTI price fluctuations from 1973/1974 until 2016 (Baumeister et al., 2016). The article describes three potentially main price shock determinants. A price shock is here described as a gap between expectations and realized price. *First*, shocks resulting from political events in the oil producing country, and discoveries of new fields and technology. *Second*, demand shocks leading from unexpected changes in the global business cycle. *Third*, demand shocks from above-ground storage, which reflects expectation shifts in supply relative to market demand. These three determinants are used further to describe price shocks further.

It is still not clear what the main drivers have been for major historical fluctuations – and the drivers mentioned still a topic for discussion.

The Yom Kippur war in 1973 broke the trend of a fairly stable oil price. The shock is used as an argument of the competitive power by the Organization of the Petroleum Exporting Countries (OPEC), where a production embargo following American support to Israel tripled the real price. According to the literature review, evidence suggests that this was largely driven by an increase in oil demand, rather than the oil embargo. Regression based on the changes in price on direct measure by the OPEC embargo states that it is difficult to explain more than 25 percent of the price increase based on exogenous OPEC supply shock.

Figure 1 4: WTI Crude, Real Prices (Macrotrends LLC)



A second central price shock occurred in 1979. The shock was a combination of shortage expectations accruing from Iranian Revolution, and expectations of higher future demands spurring from a booming global economy. The remaining two-thirds of the total price increase in 1979 are described as cumulative effects of flow demand shocks triggered by an unexpectedly strong global economy.

Between 1980 and 1990 multiple events led to turbulent fluctuations with a declining trend. One reason for the downward trend was supply increase of new producers or production expansion by incumbents, of mainly United Kingdom, Mexico and Norway. As a response, OPEC, mainly Saudi-Arabia, dampened the decreasing oil price by capping production.

The sharp increase in price in 1990 was a result of the invasion of Kuwait. Subsequently, anticipations of attack on Saudi Arabia, the largest oil producer, magnified the price spike. An explanation for the decrease in late 1990 is that the price decreased when with subsided fear of an invasion in Saudi Arabia.

Through to December 1998, the oil price depreciated further and hit an all-time low in recent history on USD 11. The slide was associated with demand depreciation, possibly caused by the financial crisis in Asia in 1997, and following its expansion to other countries. With a recovery in global economy, and increasing demand for oil, the oil price gradually rose to 2008, with the exception of 2002 and 2003, respectively related to civil unrest in Venezuela and the War in Iraq. The magnitude of these gluts were limited, as the offset production was covered elsewhere, and limited fear that the Iraq war would affect fields in Saudi Arabia.

Mid 2008, the oil price was at its all-time high at USD 145 per barrel (nominal value). This was mainly caused by demand, connected with emerging markets led by South-Eastern Asian countries. The decrease in demand for industrial commodities following the financial crisis in 2008 lead to a major demand and price drop of crude oil. (Baumeister et al., 2016). Following this period, the price rose again over USD 100 per barrel until mid-2014, leading to the current situation.

The drop in mid-2014 and subsequent shocks can among other factors be accredited to the shale oil production. Where OPEC historically have conjunctly adjusted their output to achieve a stabile price, they now pursue their interests in increasing the price in an effort to squeeze competition. The actual market power of OPEC, as previously, is under discussion (Hartmann et al., 2016). Further, lift of sanctions against Iran and a general increase in supply has could have decreased the price further (AT Kearney, 2015).

The price follows a complex pattern, where the supply, demand and events are difficult to anticipate together with their effect on price.

3.4 Literature Review: Oil Price Processes

Several methods exist for forecasting oil price, both qualitative and quantitative. There is no general consensus on the most reliable model, but often time-series models are used, and financial methods are increasingly becoming popular due to their accuracy and flexibility. Financial models estimate the relationship between current spot prices and futures contracts, and whether they are efficient based on the efficient market hypothesis (Behrimi et al., 2013). In the scope of this paper, stochastic processes considered. A stochastic process describes evolution of random values over time.

The simplest stochastic process takes the form of pure random behavior, with no stabilizing element. This is referred to as a random walk, or Geometric Brownian Motion (GBM) in continuous time. Inclusion of a stabilizing element, often reflecting a long-term average, is referred to as Mean-Reverting Models (MRM), often an exponential Ornstein-Uhlenbeck process (EOU). These two models are central in the discussion on the stochastic process of oil price development in literature. Out of the scope of this paper, other models include multiple factors, a combination of GBM and MRM, to also include other processes as Jump-Diffusion to model price shock events (Ozorio et al., 2010; Larsson et al., 2010).

GBM is often assumed to be the appropriate price process when describing stock price behavior (Hull, 2012), and is a central assumption in Black-Scholes option pricing model. In literature on real options in petroleum, GBM is also often assumed as the process for the oil price (Ekern, 1988; Cortazar et al., 2001; Rodrigues et al., 2006). Ozorio, Bastian-Pinto et al. (2013) argue that the main advantages and the reason for preferring GBM over other processes, is the ease of deriving analytical solutions to asset valuations, a small number of parameters to obtain and its mathematically simplicity. The tendency of GBM models, which is based on purely random behavior, to diverge to unrealistic prices in long-run creates artificial scenarios, as shown in figure 1.10.

The opposing price processes has its origins from microeconomics, on the assumption that in the long-run – a commodity price tends to revert the long term marginal cost (Bastian-Pinto et al., 2007). Schwartz (2012) argues that implications of neglecting mean-reversion induces investments too late. Pindyck (1999) argues that both through theory and analyzing 127 years of oil price, nonstructural forecasting models should incorporate mean reversion to a stochastically fluctuating trend line. MRMs can introduce reversion in either fully form, or partially through for example a mean reverting convenience yield. Convenience yield will be discussed in section 3.5.

An adaptive model to create new equilibrium, instead of assuming a static long-term average, is shown in the two-factor model developed by Schwartz and Smith (2000). Where a one-factor model includes one uncertainty to describe a process, two- and multi factor models assumes multiple uncertainties. Bastian-Pinto et al. (2013), state that on economic theory assumptions should be considered when choosing models. A price equilibrium assumption argues for MRM inclusion, and gradual increased marginal costs together with occurrence of rare events argues for mixed models including MRM, GMB and Jump Diffusion Models. Lund (18, 2011) acknowledges this reasoning, and states that the choice of stochastic process in the model is subjected to the analyst's beliefs, economic assumptions and intuition. He further states that both GBM and MRM models are considered acceptable based on empirical evidence.

There is divided support for which single-factor process to assume, between GBM and MRM/EOU. In chapter five, the appropriate price process will determined, with respect to random behavior and mean reverting models. See appendix 1 for characteristics of the GBM and EOU process.

3.5 Net Convenience Yield

Net convenience yield is sum of cash flows deriving from unspecified services over time. This can make the value of having ownership over an asset differ from having the opportunity to invest in the asset. In equity derivative pricing, this is referred to as the (continuous) dividend yield of a stock (Ronn, 2003). The net flow of monetized services accruing to the holder is the occurrence of contango or backwardation markets, where the future prices either respectively higher or lower than the current spot price. Contango implies disadvantageous early ownership, and reversed for backwardation. Hull (2009) shows that the futures price is an unbiased expectation of the future spot price, estimated in chapter five. This assumes that the no-arbitrage argument holds, and that there is no correlation between the asset return on the stock market.

3.6 Closing Remarks on Upstream Petroleum Investments

Upstream petroleum is characterized by long project lifetimes, a set of uncertainties with related flexibilities. The lack of consensus of drivers of oil price shocks over the past 40 years, and difficulty to forecast the oil price has led that one of the prevailing forecasting methods is based on random behavior. Nevertheless, a model is always an approximation of a phenomenon in the real world, it is more a question of how good the assumptions we make are, than modeling the world perfectly. In chapter five the appropriate price process is determined.

The flexibilities included in the models will be the flexibility to prematurely abandon field, and to decide when to invest. These two flexibilities are expected to be significantly value-adding with highly volatile prices. In order to incorporate these, a versatile framework must be present.

4.0 Income Based Valuation Methodologies in E&P

Managers rely on investment assessment tools to determine the course of action and comparing different investment opportunities. This chapter will discuss the main income-based valuation methods' fit in upstream petroleum. The income approach of valuing assets estimates present value of future cash flows accruing from the asset. Petroleum companies commonly use a discounted cash flow (DCF) method to appraise project investments, and real options valuation (ROV/ROA) serves as a compliment to the DCF model (Mun, 2006).

4.1 Discounted Cash Flow Method

The DCF, or Net Present Value (NPV) method is the traditional valuation tool used to determine value of a project (Mun, 2006). The method assumes a fixed line of future cash flows, discounted by a rate covering risk and alternative cost to derive the net present value of future cash flows. The general rule is to accept a project with positive net present value, and for mutually exclusive projects the one with the highest NPV.

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1 + WACC)^n} \tag{1}$$

The net present value is derived by discounting the cash flows from time 0 to n by the Weighted Average Cost of Capital (WACC). The formula can be extended to include perpetual growth for a firm in stable growth, not applicable for petroleum fields with limited lifetime.

Tools used assist in the decision process and evaluating different investments are the internal rate of return, return on investment and payback period on Investment among others. These are merely mentioned to describe that the net present value is considered together with other estimates.

The DCF model that can lead to distorted values on a petroleum project following two central factors. These factors are related to the discount rate, and issues related to uncertainty and flexibility.

The discount rate is usually referred to as cost of capital, or Weighted Average Cost of Capital (WACC), defined by Damodaran (no date) as "*the opportunity cost of all capital invested in an enterprise*". The rate incorporates some sort of exclusivity, and a risk factor coupled with the investment. The rate is compromised by weighted average of the cost of equity, and cost of debt. The cost of equity is a rate of return expected by the equity investors, while the cost of debt usually is the tax-deducted cost on long term debt financing. The rates are then weighed with the size of financing in order to determine the discount rate for the firm or project. The result, WACC, is then used to discount the future cash flows. Two potential problems arises here. First, a cash flow should be discounted relating to the relevant risk of the cash flow. For market based cash flows, as income, the value should be discounted at a market risk-adjusted rate. For capital expenditures, the cash flows should be discounted at a risk-adjusted rate relevant to relevant contingencies, as the market only will compensate the firm for taking market risks – not private risks. The cash flows deriving from a project is often discounted at the same rate (Mun, 2006). Second, the risk-adjusted rate is usually the most difficult parameter to estimate, as well as one of the most sensitive parameters to project value. In petroleum projects, this is amplified by the length of the investment, where NPVs for projects with a lifetime of 30-40 years are extremely sensitive to the discount rate (Smith et al., 1996). The real option methodology resolves this issue by risk-adjusting the cash-flows, which then can be discounted at a risk-free rate. This will be discussed in 4.2.3.

Further, two issues using the DCF method relates to uncertainty coupled with managerial decisions. The method assumes one fixed development and production time-line, based on decisions made in the beginning. After the project is launched, the project assumed to be passively managed until the end. For an oil field, characterized by being a long term capital investment project (>15 years) subjected to a set of influencing risks, the single-line assessment yields unprecise values. The DCF method can include the effects of changing oil price in a sensitivity analysis, to check robustness, but prevailing oil price, or probabilistic mean will determine the value of the project.

Threats and opportunities rise with uncertainty. For petroleum production, oil price can open for increased production, or investing in satellite fields, and low prices can call for decreasing production or abandoning. Real option methods incorporates uncertainty, and managerial decisions related to sequential information.

4.2 Real and Financial Options

An option is the right, but not obligation to buy or sell an underlying asset, while real options describes the right to undertake business initiatives. Capital budgeting can be seen as possible courses of actions managers are exposed to in a set environment, e.g. a set of real options (Gamba, 2003).

Defining capital budgeting as a string of opportunities and flexibilities, it will be a more precise to allocate values to potential outfalls and reactions during the timeline of a project.

ROV include parts of the NPV method, as a forward-looking income-based valuation method, but extended to be dynamic and capture the value of flexibility. In order to determine if the flexibility is an option – the holder must have exclusive rights to the underlying asset (Damodaran, 2012), which is the case for licensees on an oil field.

Real options methods use financial option theory to value assets, relying on an equivalent martingale measure (EMM), where the cash flows are risk-adjusted and discounted at a risk-free rate. EMM will be described in section 4.2.3.

An option can be priced, or valued, using different methods. The most commonly used financial option pricing model is a closed-form variant; the Black-Scholes model (B&S), and is used as a basis for many other types of models. Merton and Scholes were awarded a Nobel Prize in economics, constructing a pricing model for European Options, under a set of conditions. See appendix 2 for a description of B&S.

A call option gives the holder the opportunity to buy a fixed amount of underlying assets at a pre-determined strike-price. On the other hand, a put option gives the right to sell the underlying asset. A European option is exercisable only at the expiration date, where American options can be exercised at any point. Changing the underlying asset from financial entities to real projects, where the options are decisions, and changing some assumptions leads us to real options. Here, the exercise date tends to be more flexible than assumed in European options, but the structure of the underlying project decides level of flexibility.

Upstream petroleum developments are stage-based and inherently uncertain, as described in chapter three. With sequential information in an uncertain environment, and with the possibility to react, a string of potential outcomes are present. ROV models can capture these uncertainties and assign probabilities on outcomes with attached decisions and values.

The following three paragraphs describe examples of basic options in an upstream development. In any of the examples below, decision makers have the *right,* not the *obligation,* to act - but it is assumed that value-maximizing decisions are pursued.

Timing options gives the holder of the option flexibility in when to exercise an option. Deferring from making the decision until the investment climate yields more secure data, and until estimations yield highest profits is a natural choice. Timing options add value from giving the opportunity to negate value destructive decision, and *striking* when the estimations yield highest profit.

Option to *abandon* can be seen as the right to withdraw from a project before completion. Field abandonment is a certainty, at one point the expectation of future value is negative and exercising this option negates unfavorable cash flows. The option to abandon can be seen as a put option.

Options to *expand* or *contract* can be seen in the view of production rates. An oil company adjusts production according to new information. With a high oil price and demand, production can be elevated – and opposite. Adds value through taking advantage from increasing profitability and decreasing / cutting losses. Another example is the option to invest in a satellite field. As described in section 2.3.1, Statoil ASA has invested in extra capacity in the oil producing unit to have the option to include oil from adjacent licenses. In this case, the capital expenditures related to the extra capacity can be seen as an initial premium related to the option. This can also be described as a *growth* option, but the term expand will be used in the thesis.

Stage-based investments of a petroleum field are *compounded* options. Compounded types can be a long line of subsequent options, where the stages can all be dependent upon each other. An example of a compound option is the option to defer investment in a field for a set period of time, on the option to expand production if prices expected prices are high, and abandon production with expected low prices.

4.2.1 Literature Review: Real Options in Petroleum Projects

In the sense of valuing real assets using financial option methods, the term real options was first coined in 1977 (Myers, 1977). The original option pricing application in natural resource valuation was discussed by Tourinho in 1979 (Tourinho, 1979), performed on a sequential, non-reversible investment opportunity.

In 1985, Brennan and Schwartz introduced a real options valuation model for a natural resource project. Considering oil price uncertainty, the article describes the value of a switching option, where the possibilities span from pausing, increasing and decreasing to abandoning a mine. Cortazar and Schwartz (Cortazar et al., 1998) integrated Monte Carlo simulation to value an undeveloped oil field, following a two-factor model – considering both stochastic oil price and convenience yield. Longstaff and Schwartz (Longstaff et al., 2001) developed a flexible approach to value American options, using simple regression to determine the highest value between exercising and continuation. The model, Least Squares Monte Carlo (LSM) uses backward recursion to solve the optimization problem and the value of continuation. Fleten et al. (2011) valued an expansion of an oil field to a tie-in field using the LSM method, including the option to abandon field. Chen et al. building on production flexibility using the LSM method, studied the optimal operating strategy of a mining company with a fixed production target for a period, under commodity price uncertainty (Chen et al., 2016).

The uncertainty parameter in ROV in petroleum has often surrounded a stochastic oil price, both by single- and multifactor models. Recently, the uncertainty parameter in real options in petroleum have intensified around other stochastic factors such as geology and engineering (Qiu et al., 2015).

Cox, Ross and Rubinstein’s binomial option pricing model is an alternative method of solving real options problems (1979). Ekern (1988), using binomial lattics, values a hypothetical satellite field, unprofitable at current prices. Including a stochastic price process, with sequential information and the opportunity to react – the article describes that the satellite field can have positive option value. Gamba (2003) argues that these models have drawbacks when applied to real option valuation in terms of implementation complexity when the project has many interacting options, and may suffer from the curse of dimensionality. The upside is the intuitive description of the process, and the ease of deriving value with few layers of complexity.

Valuation will be conducted using both the LSM and the binomial option pricing model. This is in order to verify results, and having the option to choose which model to use in further in the analysis. The choice of model will be based on two criteria: precision and ease of modelling. If values from the models deviate, the choice of model will be based on an argumentation on fit of the models. With coherent values, sensitivity and scenario analysis will be performed with the model that simplest can yield new values with changed parameters.

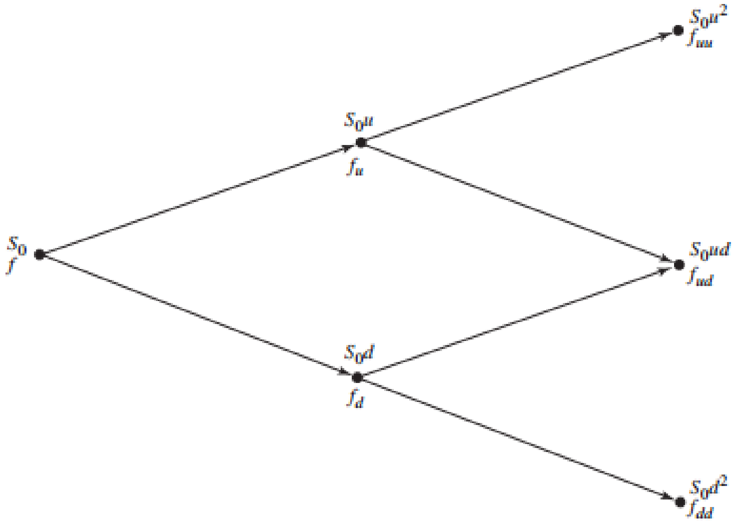
4.2.2 Risk-Neutral Valuation

Real option methodologies rely on risk-neutral valuation (RNV), or Equivalent Martingale Measures (EMM). In the case where the uncertainties in a real option model covers the underlying risk factors of the project, cash flow deriving from the asset can be discounted by a risk-free rate. This estimated risk factor is captured by the risk-adjusted discount rate in the DCF model. EMM is a probability measure where the underlying asset bears the same return as a riskless asset. Existence of risk-neutral valuation assumes the absence of arbitrage, and that all cash flows synthetically can be replicated using market-replicating portfolios (Mun, 2006).

4.2.3 Binomial Lattices

Using lattices, uncertainties are analyzed and allocated probabilities and values, including managerial responses – creating a map of possible outcomes. In order to describe the method used in the thesis, the simplest form of a binomial, recombining lattice with two steps is illustrated below.

Figure 1 5: Binomial Lattices, Price Development



The input parameters used in the example are:

S0 = 50 – initial price

X = 52 – strike price

σ = 30% - volatility of the underlying

t = 2 – number of periods, where one time-step equals one period

rf = 5% - risk free rate.

S0 is current price of the asset, and can either move up or down respectively with the factors *u* and *d*. The two factors are calculated using:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{2}$$

$$d = \frac{1}{u} \tag{3}$$

The calculated factors from equation 2 and 3:

$u = 1.3499$

$d = 0.7408$

S0u and *S0d* describes the two possible prices in step 2, and the method is generalized throughout the tree (Hull, 2009). At step three *S0ud* in the middle reflects that if through two periods, the price moves up in one period and down in the other, *S0ud* equals initial price, and *d* is the reciprocal of *u*. *f* is the option value at the corresponding step. This models assumes the stock price follows a GBM process.

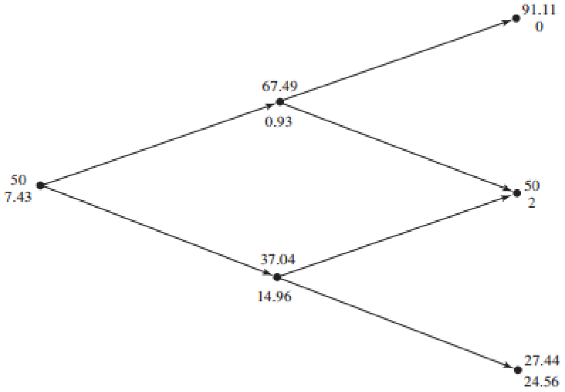
Decision to sell the underlying asset can be made at period 0, 1 and 2. With a put option, the value increases with downward price movement, and where price is below strike price, X, the option is “in the money”. The model can be extended using these parameters, with the three factors derived, the up factor (*u*), the down factor (*d*) and the risk-neutral probability. The risk-neutral probability is defined as:

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{4}$$

$p = 0.5097$.

After building the price-tree, the value of the option is calculated recursively, beginning at the end node, where the holder decides to sell the underlying asset if the option is in the money. In other words, subjected to the decision criteria: MAX(X - S, 0).

Figure 1 6: Binomial Lattices, Option Value



Using the risk-neutral probability, *p*, all option values at step 3 are discounted to step 2 yielding each node’s value at step 2, done through equation (5).

$$((p * value\ in\ state\ u\ +\ (1-p) * value\ in\ state\ d) * exp(r * t) \tag{5}$$

Values in state *u* and *d* refer to values in the subsequent period. By recursively performing the action until the initial period, yields the option value of 7.43.

Further, given the assumptions in this example the holder of the option would be neutral as to exercise the option in time 0 or hold it until time 1. This is equal for step 2 to 3, but in step three the holder would only exercise the in-the-money outcomes.

Increasing the number of time-steps is generally needed for higher accuracy, and between 100-1000 steps are acceptable dependent upon the option (Mun, 2006). When the number of time-steps reaching infinity, making it continuous, the results would converge to the B&S closed form solution for a European option.

This form of deriving the option value is intuitive and easy to create in Microsoft Excel, but introducing some complexity into the process quickly makes the model large and complex. E.g. by introducing different up/down factors (volatility follows a Poisson distribution or changing over time) making the lattice non-recombining, it quickly goes out of hand. Further, with complex types of option, methods based on Monte Carlo simulation is often preferred.

4.2.3 Monte Carlo Simulation

Simulations are performed in order to mimic a real-life system. Monte Carlo Simulation (MCS) generates a line of random variables in order to mimic discreetly sampled paths of a continuous-time stochastic process. The strong law of large numbers (Stanford, no date) states that when the number of simulations reaches infinity, sample mean of the simulated values will converge to the continuous, theoretical value. In finance and capital budgeting, MCS is recommended as it is flexible enough to cope with many real life situations not exposed to the curse of dimensionality (Gamba, 2002).

Usually, the decisions are American type options, and MCS increases the flexibility of the model compared to using lattices. Traditional MCS includes forward looking simulation, but for optimized decision making backwards induction is preferred, as in the binomial model.

4.2.4 Least Squares Monte Carlo

Longstaff and Schwartz presented a regression-based option model to recursively show optimal exercising in an American option pricing model, using MCS (2001*)*. This chapter will introduce the model, and describe its function.

The key insight in the Least Squares Monte Carlo (LSM) is that the conditional expectation of future payoffs is estimated from the cross-sectional information in the simulation using least squares regression. The objective of the algorithm is to find an approximation to optimally exercise an American option. In other words, simulating the underlying risk factor *n* times, the algorithm approximates the period for each simulated path where it is optimal to exercise the value. In the context of this paper, exercising the option refer to either investing in, or abandoning the field. The underlying risk factor, or state variable, is the oil price.

Discrete periods will be included, *t* is defined as period related to different states. $0 < t+1 < t+2 < tn = T$. Decreasing the time between ever step, making *dt* sufficiently small, makes this a continuous American option. In contrast to using binomial lattices, granularity can be introduced both by decreasing the size of the time-steps, and increasing the number of simulations to properly describe the underlying distribution. Using lattices to compute accurate results rely on increasing the number of time-steps.

At maturity, the last period with flexibility, the option is exercised for the paths that are profitable, called paths in-the-money. Conditional that the option will not be exercised in a previous period, the payoff at maturity is assumed to be the realized cash flow in the simulated path. Where the option is not exercised, the realized cash flow is zero. Moving backwards to one period before maturity, *T-1*, the underlying risk factor, oil price, is regressed to the discounted payoffs at maturity. The fitted values of the regression is the conditional expectation function, a function describing expected payoff in the next period, when multiplied with the underlying risk factor for each path. The outcome is the value of continuation, the value of not exercising the option in the given period, which is subsequently compared with the payoff of immediately exercising the option. If exercise value is higher than the continuation value, the option is exercised, and the realized cash flow in the simulated path is the exercise value in period *T-1*, conditional on that the option is not exercised at an earlier date. If the optimal exercise period is in period *T*, the payoff value is simply discounted back to the initial period, period zero. If the continuation value exceeds the exercise value at time *T-1*, the realized, discounted cash flow from period *T* is effectively the optimal payoff on period *T*, not the continuation value. Longstaff and Schwartz argue that choosing the discounted continuation value as realized payoff can lead to an upward bias.

Moving back one more period, *T-2*, payoffs leading from all future optimal decisions are used as the basis for the regression. This process is iteratively performed until the initial period, where values from all paths are averaged to find the value of the option. Extensions of this algorithm include several state variables and interactive options (Gamba, 2003).

In order to increase the efficiency of the regression, only the paths that are in the money in each period can be used to estimate the continuation value. A mathematical description of the LSM method is located in appendix 3.

4.3 Closing Remarks on Valuation Frameworks in E&P

Real options methodologies assesses the value of flexibility and uncertainty better than straight-line DCF models. The petroleum project in focus is embedded with both. By taking price uncertainty into consideration, the models used in the valuation is both based on binomial lattices as well as the LSM method. The expectations by including both are that they yield similar values, but the simulation method will be superior in modelling when layers of complexity are introduced.

5.0 Data Analysis

The purpose of this chapter is to find the appropriate price process for Brent Spot between the GBM and EOU processes. Further, the net convenience yield to use in the valuation will be determined.

5.1 Data

5.1.1 Brent Crude Prices

The time series includes daily, nominal data for current month delivery Brent crude - used as a proxy for Brent oil spot prices. A spot market for Brent does not exist. The data included is between 1st of September 1983and 22nd of February 2016. This includes the longest time-series before weekly prices (Thomson Reuters, 2016).

5.1.2 Risk-Free Rate

The futures price serves as a proxy for the spot price, and with limited length on futures contracts on Brent crude led to choosing a five year contract, and deriving the net convenience yield with a five year US Treasury Bond (Bloomberg, 2016; Barchart, 2016). Choosing a shorter interest rate than the length of the asset valued, assumes no reinvestment risk. Further, the risk-free rate assumes no default risk (Damodaran, 2012)

5.2 Analysis of Price Process

Section 3.4 discussed the two main price processes used in the context of real options, and building on divided academic research result - the expectation is that a GBM or MR type model best fits the data. A time-series analysis is used to determine the appropriate price process to use in the valuation. Time-series models predict future values based on historical data, and only including one variable makes it a univariate time-series. These models are often used when data display a systematic pattern. An example of a systematic pattern is that when variables in period *T* increases, values in *T+1* tend to increase. Further, if the number of explanatory variables is large, and that forecasting the dependent variable requires forecasting the explanatory variables as well. As discussed in section 3.3, the development of oil price is determined by a complex set of underlying conditions. All three moments are present for the oil price. (Behrimi et al., 2013)

A commonly used univariate time-series forecasting model is an Autoregressive, Integrated, Moving Averages (ARIMA) models. ARIMA models' function is to describe autocorrelation in the data. Autocorrelation, or lagged correlation, is the presence of a systematic pattern in lagged values of the univariate time-series.

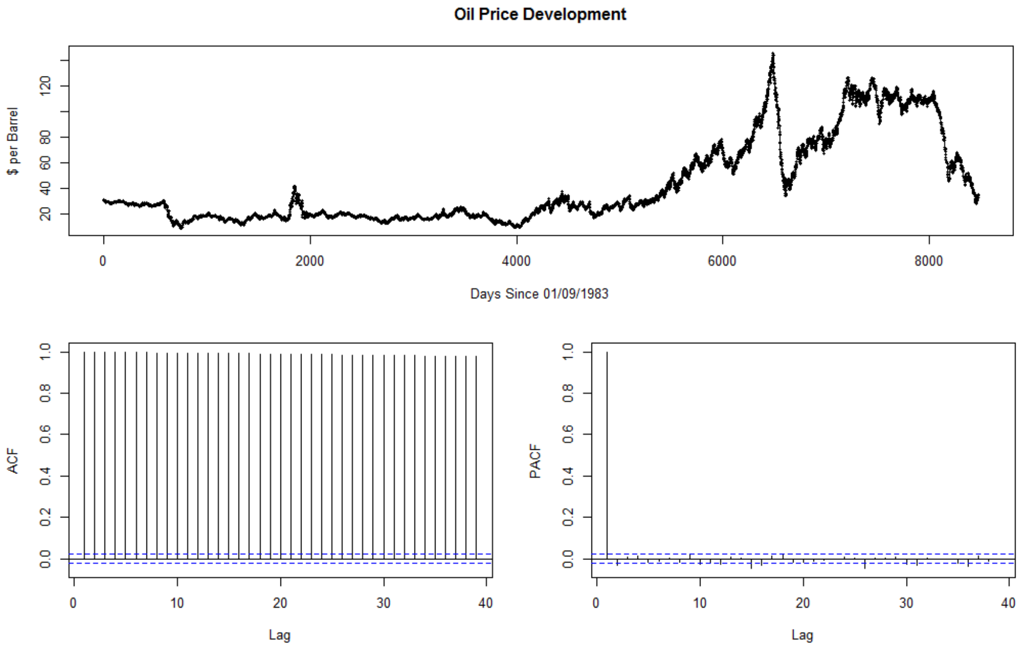
A collection of random variables, or a stochastic process, contains information and noise, and an ARIMA(p,d,q) can be seen as a filter that tries to separate and use the information to forecast the next steps. ARIMA models are considered a general class of forecasting models for stationary time series or time series that can be made (nearly) stationary through transformation. A stationary process is where at any point in the series one will find the same probability distribution. In other words, its statistical properties as mean and variance does not change

The model can consist of three parts, an autoregressive *(p*), an integrated *(d)* and a moving average *(q)*. The autoregressive part refers to serial correlation, where the *p* notation is the number lags to include when forecasting – or where the independent variable is lagged *p* periods. Integrated refers to differencing, i.e. observations' changes, and (*d*) refer to the number of non-seasonal differences is needed to make the series stationary. The moving averages part includes correlation between a variable and the residuals, or forecast errors, from previous periods. The number of lagged periods included is (*q*). The discrete approximation of a GBM model is equivalent to an ARIMA(0,1,0) model, i.e. only differencing is used and the only relevant parameters are volatility (logarithmic standard deviation) and eventual drift. The simplest form of an autoregressive model, AR(1), or an Ornstein-Uhlenbeck process, is equivalent to the simplest MRM model. In other words, an AR(1) model describes a process where all information to define the subsequent step is found by 1 period lag.

According to Hyndman (2013), the following steps are appropriate in order to determine the suitable ARIMA-model, including using an automated algorithm in R:

The time series is plotted, with autocorrelation (ACF) and partial autocorrelation (PACF). In the time-series plot, observations include variance, trend, structural breaks, seasonal and cyclical behavior. Trend includes both single upward and downward movement, or in combination in different periods. Cyclical behavior and seasonality differ in terms of standard length, where seasonality generally have a set period of fluctuation, and cyclicality are not. Cyclicality is often at least 2 years in fluctuation.

Figure 1 7: Development, Autocorrelation and Partial Autocorrelation of Historic Brent Daily



The series plot shows changing variance, upward and downward trends, possible structural breaks or cyclically behavior. Seasonality, structural breaks and cyclicality is difficult to spot from this graph. Structural breaks can be a part of longer cyclically behavior, as with the two large price spikes during 2000-2008 and during 2010-2014, or they can be separate occurrences. For convenience, all data will be included in the model. That the volatility of the variance is unequal in the time-series is a sign of heteroscedasticity. If this is the case, a separate model for the volatility should then be present. This will be discussed further.

A time-series with trend needs to be transformed to become stationary in order to use the model, i.e. differenced until no unit root is observed.

The ACF plot shows strong autocorrelation between the daily spot prices. The plot shows a slow, tapering curve – a second sign of non-stationarity and effectively means that the changes in observations are strongly, positively correlated with the previous'. This could be indicative of an AR(1) model with no differencing, if differencing is performed this is indicative of a random walk (0,1,0). The PACF is included to see the single-lag correlation. The plot shows a sinusoidal movement with non-zero value at point 1 and zero res, meaning that lower order lags are explained by lag-1 autocorrelation.

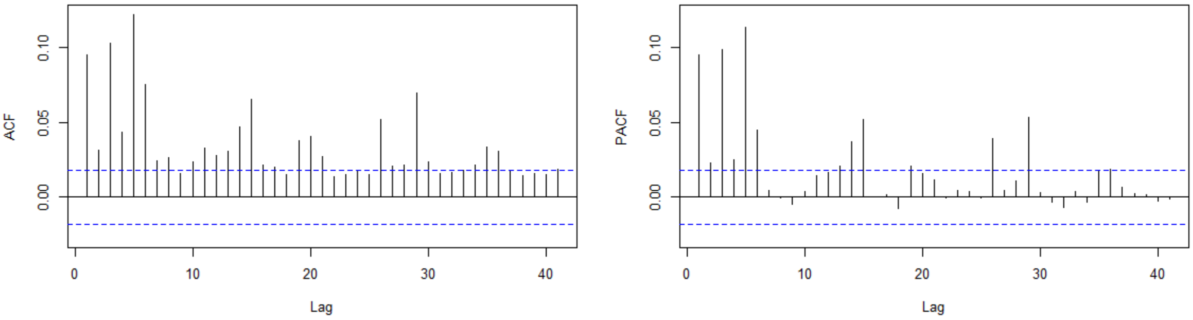
The logarithmic values are further used to reduce variance. In order to determine the model, the auto.arima() function in "forecast" package by Rob Hyndman is used. See appendix 4 for a description of the algorithm.

The output from the automated algorithm can be found in appendix 4. A random walk model with drift, or a discrete GBM with drift, best explains the time-series of Brent oil. The assumption underlying the model is that one-step expected value is the previous value adjusted by a drift coefficient and with an error term.

The error term is assumed to be white noise, i.e. normal variable with finite variance and mean of zero, lacking correlation between the noise-observations. In other words, the error-term should contain no information and follow no systematic pattern.

In order to determine if the error-term contains information, two tests are performed. First, the ACF and PACF of the squared residuals are plotted in order to look for autocorrelation. Further, the portmanteau test Ljung-Box is performed on the residuals to confirm. Ljung-Box test null-hypothesis is independently distributed observations, and that any autocorrelation found is the result of randomness of the distribution. The alternative hypothesis is presence of autocorrelation.

Figure 1 8: Autocorrelation and Partial Autocorrelation of Squared Residuals From ARIMA(0,1,0)



The plot reveals autocorrelation, meaning that volatility seems to be building up in clusters.

The results are checked with the portmanteau test. Tsay argue the number of lags to be observed should be equal to the natural logarithm of the sampling number, 9.04 (Tsay, 2010) . The p-value of <0.001 states that the null-hypothesis cannot be rejected, and heteroscedasticity is present.

For simplicity, a simple ARIMA(0,1,0) model will be included. The parameters used are not the outcome of the auto.arima() model, but will reflect the expected future drift using the risk-free rate and the relationship between the current spot – and futures prices. The volatility in the model is the annualized standard deviation of logarithmic daily returns on the oil price.

5.3 Analysis of Net Convenience Yield

Equation (6) shows an estimation of implicit net convenience yield, which can be derived by inverting the theoretical relationship between the spot price and futures contract price (Hull, 2012).

$$\delta = r - \frac{1}{T} ln(\frac{F_0}{S_0}) \tag{6}$$

δ is net convenience yield, r is risk free rate, T is the futures contract period, *F0* is price of the futures contract price at time *0*, and *S0* is the spot price at time 0.

Net convenience yield estimated to -0.5 percent.

5.4. Closing Remarks on Data Analysis

The assumptions made for the price process is consistent with the assumptions using the B&S model, supported by theory on oil price processes from section 3.4. No mean reversion, seasonality, heteroscedasticity and jumps are included, volatility is identically and independently distributed. The result of this is that the factor driving the price is random behavior and a drift parameter (Ronn, 2003). See appendix 4 specifications of ARIMA(0,1,0) under EMM, and output from auto.arima(). The drift parameter in LSM is calibrated with the negative net convenience yield, which will increase the drift rate (appendix 5), and calibrate the risk-neutral probability using lattices. A negative net convenience yield means that there is a net costs associated with holding the underlying, rather than the option contract.

6.0 Real Option Models

This chapter describes the models in detail, including a short description of the value deriving from introducing flexibility in the projects. Analysis of option values in detail, including model comparison is located in chapter seven. Estimations will both be done using the LSM and binomial lattice model.

Section 6.1 describes the assumptions and input used in the models

Section 6.2.1 and 6.3.1 includes estimations of the individually producing fields without flexibility. This is the base case, serving as a benchmark of the value of the add-on options.

Section 6.2.2 and 6.3.2 includes estimations of the individual fields including the option to abandon.

Section 6.2.3 and 6.3.3 includes a waiting option, and option abandon for the individual fields.

Section 6.2.4 and 6.3.4 includes a simplified option to expand, including both fields' option to defer investment and option to abandon.

The following analysis chapter will include results, comparison of the models and sensitivity analysis.

6.1 Assumptions and Input

Assumptions and inputs used in both models used are:

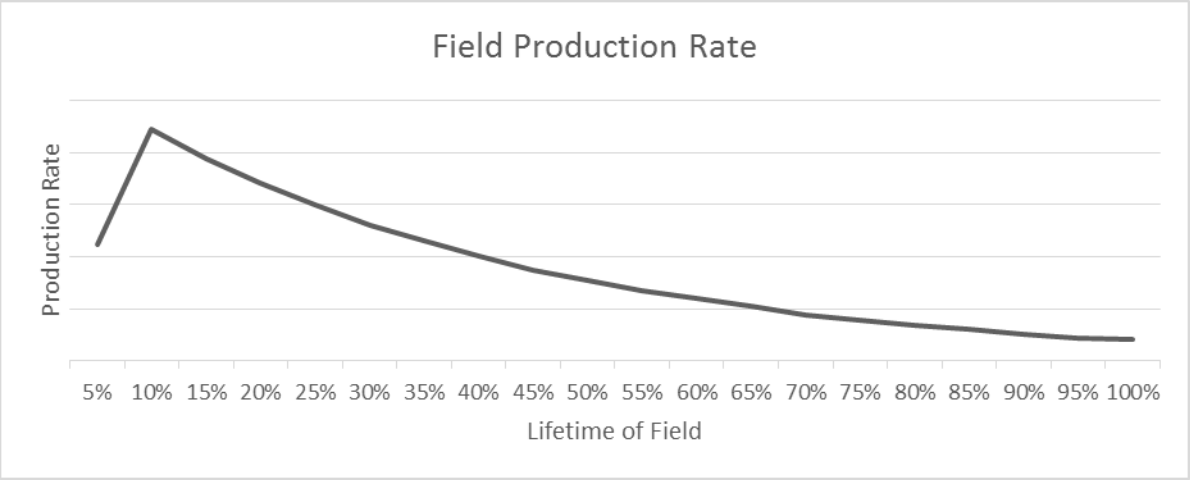
Discount rate:

Discount rate is 2.6 percent, based on a 30 year Treasury Bond (Bloomberg, 2016).

Production rate:

Production rates are static and yearly. Johan Castberg and Alta/Gohta contain respectively 491 and 351 million barrels of oil. Production rate for both fields follow the distribution in the graph below (Wood Mackenzie, 2015; Lundin, 2016):

Figure 1 9: Production Rates for Johan Castberg and Alta/Gohta



<u>*Production costs:*</u>

It is assumed that all costs in the producing field are variable, with cost per barrel is USD 16.76 (Wood Mackenzie, 2015). This figure represents an estimate on the average production cost on Johan Castberg, based on public information, and is assumed equal for all cases.

<u>*Capital expenditures:*</u>

In the case of individual production, investment costs for Johan Castberg and Alta/Gohta are respectively USD 6.8 bln and USD 6.25 bln (Lorentzen, 2016; Norges Bank, 2016). Cost of developing Alta/Gohta is based on an averaged approximation of development cost and field sizes of Goliat and Alta/Gohta.

In the case of co-production, no public information was available. The investment cost is assumed to USD 8.8 for Johan Castberg, a USD 2 billion increase for the extra capacity. For Alta/Gohta, the investment costs decreases to USD 0.5 bln, to represent the limited investment needs on the field as attached to the hub. This assumption will be changed in section 7.4. For all models, no investment will be made if deficit is expected. This in practice includes optionality into the base case, the option to turn down unfavorable projects, but included to have a realistic benchmark.

The cost of abandoning field is equal for all cases, on USD 0.4 bln (Wood Mackenzie, 2016).

<u>*Tax:*</u>

Tax rate is the marginal Norwegian Petroleum tax rate of 78% (NPD, 216), all taxes and deduction on income and investments occur simultaneously with their related cash flows. Taxes are not mentioned in the description in the model, still included in the model.

Periodicity:

Discrete time periods will be used in the model, where *t* to t_{-1} or *v* to v_{-1} is one year, oil prices are static during the year and known at the beginning of the period. Decisions and cash flows occur almost simultaneously at the beginning of the period. Decisions with their respective investment costs always happen momentarily before potential income.

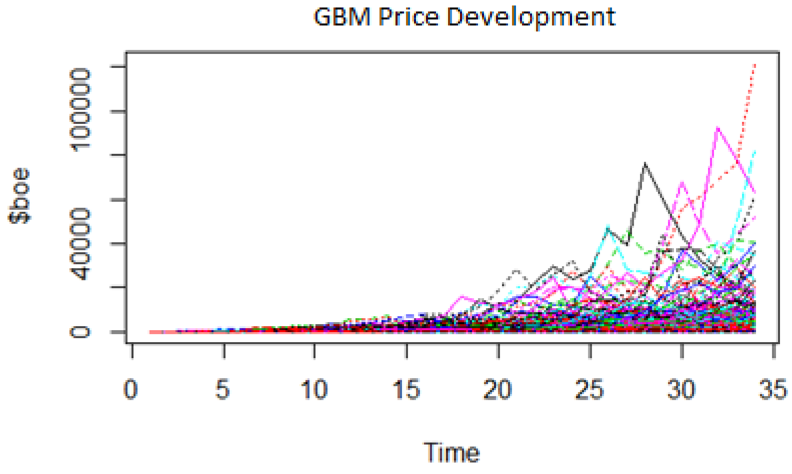
In cases without flexibility to decide when to invest, Johan Castberg and Alta/Gohta initiate production in respectively 2022 and 2023. Prior to this, the fields will have one year construction, and abandonment costs will occur one year after the last year of production. The construction period is only included through discounting cash flows one period. The flexibility to decide when to invest in this case is within the frame of the initial exploration period. This is assumed to be until and included 2022 for Johan Castberg, and 2029 for the relatively less mature Alta/Gohta. Section 2.2 stating that the licensees have to perform obligated tasks in order to get a prolonged license – it is assumed that these obligations are covered until 2029. After initial period, if no investment is made, the license is revoked.

Price development

The initial oil price the first of January 2017 is assumed to be USD 41.6, further exposed to the annualized logarithmic volatility is 37.09% (Thomson Reuters, 2016).

LSM model: The price process is assumed to follow a discrete, Martingale, Geometric Brownian Motion or ARIMA(0,1,0). The price process is simulated 30 years, 70,000 times.in table 1.10.

Figure 1 10: Price Simulation, GBM



It is possible to see the increase in spread, and indications of random behavior. As mentioned above, an argument against using GBM is occurrence of unreasonable values. The counter-argument here is the low probability of it occurring and the weighted effect that price path has on the value of the option. To a certain extent - high values can be argued through the existence of war or instability, where low values can be argued by competing technology, pricing-wars or abundance of oil.

Drift is exposed to the risk-free rate on 2.6 percent subtracted by negative 0.5 percent net convenience yield. See equation 5.2 in appendix 5.

Binomial model: The up, down and risk-neutral probability factors are defined in equation (5.3), (5.4) and (5.9) in appendix 5 .

6.2 Field Value Estimation – Least Squares Monte Carlo

Section 6.2.1 to 6.2.4 describes the valuation processes in detail, using R programming language.

6.2.1 LSM: Base Case

Base case value is estimated in two steps:

First, estimate nominal cash flows from producing field

Simulated oil prices are multiplied with production rate for the given period, v. Abandonment costs are introduced at v=V+1, one year after last year in production.

Second, estimate base case value

Investment costs are introduced at v=0. All paths are discounted back to 2017, where negative net present values are omitted and all paths averaged to estimate base case value. This value is not important in itself, but serves the purpose as a benchmark.

This is done for all simulated paths, with negative values omitted, and averaged to find the based case value. The base-case field values are summarized in table 1.1.

Table 1 1: LSM: Base Case Values (Values in ‘000)

LSM - Base Case	Johan Castberg	Alta/Gohta	Total Value
	$ 2 730 380	$ 1 852 989	$ 4 583 369

6.2.2 LSM: Option to Abandon

The option to abandon describes the consortium’s ability to once discontinue production on the field. At the latest, a one period post production, V+1, the consortium is obligated to plug the field. Before this period, the option to abandon is possible at any periods $0 < v < V + 1$.

The process is recursive, with initial decision point in period V, and explained in two steps:

First, introduce optimal decisions:

Building on the data from the base case, decisions are made in order to maximize expected values for all paths for all periods. In period v=V+1, the consortium bears the cost of decommissioning. At v=V, last year of production, given that the option to abandon is not pursued in earlier periods, the consortium has perfect information on the production value in the current period, and continuation value, the discounted abandonment cost.

Optimal decision in period V:

$$Payoff_V = MAX[CF_V - K * exp(-r), -K] \tag{7}$$

Optimal decision at time V, CF = Cash Flow at V, K = abandonment cost, r = risk free rate.

In period V-1, the consortium has knowledge of the current oil price and production value in the current period, but only expectations of the payoff in the subsequent period. Here, the algorithm is put to use.

All values of the underlying risk factor, oil price, is cross-sectional regressed on the one-period discounted payoffs in the following period. This gives the conditional expectation function, which by multiplying by prices in all paths yields conditionally expected payoff for the following period. The conditionally expected payoff is subsequently summarized with the current year's production value, resulting in the continuation value (CV) in period V-1 for path *w*:

$$Payoff_{V-1} = MAX[CV_{V-1}, -K] \tag{8}$$

Optimal decision at time V-1, CV = Continuation Value.

This is performed until v=1. Note that the conditional future payoff is the discounted result of future optimal decisions, where the actual payoff from continuation is used instead of the continuation value. The actual payoff will include both the production in the current period, and the discounted payoff deriving from later periods. All paths in v=1 yields one net present cash flow deriving from the producing fields including the option to abandon, given that string of simulated oil prices.

Second, estimate value of the field:

The values in period v=1 are discounted one more period, to reflect a one-period construction lag, and subtracted by the investment cost. This gives a vector that includes the value of the field for all simulated paths, which will be used further in section 6.2.3. The field value including option to abandon is estimated through removing negative values, discounting to 2017, and averaging. The field values including option to abandon are summarized in table 1.2.

Table 1 2: LSM: Option to Abandon Values (Values in '000)

LSM - Option to Abandon	Johan Castberg	Alta/Gohta	Total Value
	$ 2 789 977	$ 1 896 612	$ 4 686 589
Added Value to Base Case	$ 59 597	$ 43 623	$ 103 219

The option to abandon only introduces small added-value.

6.2.3 LSM: Individual Options to Defer Investments, on Options to Abandon

The option to defer investments refer to the consortiums ability to invest in all periods before the license is revoked. Here, the consortium will compare payoff of immediately investing in the field, with expected value of waiting.

The underlying asset is the developed oil field, and investment can be made in and in periods *t* to *T*, explained in two steps:

First, estimate all field values:

The input is gathered from section 6.2.2, the vector with value of the field - estimated for all potential investment periods. The vectors containing values of the fields, for all periods and simulated paths, is combined into a matrix. This matrix contains estimated values of the field, for all periods and simulated paths. Cells in the matrix are referred to as simulated field value (SFV).

Second, introduce optimal decision:

The optimal payoff of SFV at time T, last year of the initial license, is be subjected to:

$$Payoff_T = MAX[SFV_T, 0] \tag{9}$$

Optimal value of investment at t=T, SFV = value of developed field in period T.

In period T-1 the algorithm is put to use. This time, in order to increase the efficiency of the regression two conditions are included. First, only positive values are included. Further, only the paths where continuation is estimated to yield value is included. These combined increases the efficiency of the regression, and the paths will be referred to as in-the-money paths (ITM). The ITM paths are cross-sectional regressed over the payoff in the subsequent period, over the same paths. The result of the regression is the conditional expectation function, used to determine the conditional expected payoff in time T for all ITM paths. The exercise function used from t=T-1 until t=1 is as follows:

$$Optimal\ Decision = MAX[CV_{ITM,t}, SFV_{ITM,t}] \quad (10)$$

Optimal exercise decision, ITM,t = in-the-money paths in period t.

This step evaluates the optimal decision between investing in the current period and postponing the decision.

The ITM paths where SFV exceeds CV, the payoff cash flow is investing in the current period. For all other paths, including those not ITM; discounted payoff values from later decisions are the payoffs in the period.

In period t=1, all paths are averaged to find the value of the field with full flexibility. Values, including option to defer and abandon are summarized in table 1.3.

Table 1 3: LSM: Option to Defer, and Abandon Values (Values in '000)

LSM - Option to Defer, and Abandon	Johan Castberg	Alta/Gohta	Total Value
	$ 2 908 274	$ 2 943 588	$ 5 851 862
Added Value to Base Case	$ 177 894	$ 1 090 599	$ 1 268 493

The option to defer investment adds significant value to Alta/Gothta compared to on Johan Castberg, with a longer maturity date on the option.

6.2.4 LSM: Option to Expand

Alta/Gohta is seen as an expansion to Johan Castberg in this case, using the assumptions in section 6.1.

Due to technical limitations in modelling, a simplified method will be pursued and explained.

Investment on Alta/Gohta is contingent upon a positive investment decision on Johan Castberg, and investment can begin on both fields simultaneously. In the paths where Johan Castberg is not developed, Alta/Gohta can be developed at original costs after it is ensured that Johan Castberg will not be developed. This gives the opportunity to treat them as dependent options, but still not compounded as much of the interdependence between the two fields are lacking. Alta/Gohta can produce still if Johan Castberg is abandoned. Still, with equal abandonment and production costs the effect is assumed to be small. A potentially larger problem is that the value of Alta/Gohta does not affect the optimal timing of investment on Johan Castberg, which ultimately leads to suboptimal decisions on both fields.

The process is be described in three steps:

<u>*First, estimate value for Johan Castberg:*</u>

The model follows an equal process as in section 6.2.3 and 6.2.4, including option to abandon and defer investment.

<u>*Second, estimate value for Alta/Gohta:*</u>

For periods and paths with positive investment decision on Johan Castberg, Alta/Gohta can be developed. The model follow the same process as in section 6.2.3 and 6.2.4, with the exception that for unrealized paths on Johan Castberg – a separate model is used to determine Alta/Gohta which can be developed at full cost. Summarizing the estimated values yields the total values of the fields including full flexibility. Value of the co-producing fields, including option to defer and abandon, are summarized in table 1.4.

Table 1 4: LSM: Option to Expand Values (Values in '000)

LSM - Option to Expand	Johan Castberg	Alta/Gohta	Total Value
	$ 2 691 079	$ 3 413 057	$ 6 104 136
Added Value to Base Case	$ -39 302	$ 1 560 068	$ 1 520 766

6.2.5 Closing Remarks on the LSM Method

R programming language the software used to estimate the values, and served as an effective sandbox to create the model and estimate values.

Longstaff and Schwartz (Longstaff et al., 2001) describe that the choice of basis functions can affect the price significantly, which is substantiated in other articles. This model had equal findings, where option values was greatly influenced by the choice. No universal best-fit polynomial basis functions exists, so with limited knowledge in statistics and matrix algebra a preferred method is to follow the examples of basis functions in the original text, and relevant articles (Longstaff et al, 2001; Wu, 2014). In this assignment, a combination of the 4 first Chebyshev polynomials was used.

The number of simulations included was increased until values did not significantly change, with 70,000 simulations.

6.3 Field Value Estimation - Binomial Lattices

Sections below describes the valuation processes, using Microsoft Excel.

6.3.1 Binomial Lattice: Base Case

Using the parameters in section 6.1 and the price development process in section 4.2.4, base case value estimation is described in two steps:

First, create cash-flow tree:

The oil price with volatility is modeled using the up and down factor for all relevant periods. Each node in the spreadsheet reflects one oil price, which will spin out in a tree, or right triangle, of size

$$\frac{Number\ of\ relevant\ timesteps^2}{2}$$

The relevant periods are the steps that has cash flows contingent upon the oil price. Investment cost is introduced at v=0, and tax adjusted decommissioning costs is introduced at v=V+1. All cells between contains oil prices multiplied with the production rate for the given periods.

Second, estimate field value:

All cash-flows are summarized and discounted using *p*, the risk-neutral probability. Summarizing refers to that production value in period *v*, is summarized with the risk-neutral, discounted cash-flows from the following period.

In period v=0, a vector of length t+1 is present, which contains possible field values. t+1 is the number of periods until initial investment, so 5 in the case of Johan Castberg and 6 for Alta/Gohta. This vector is first subjected to an if-statement removing negative values, then discounted using *p* to t=1 which yields the base case value. The results can be checked through multiplying the vector with the risk-neutral probability of arriving in that cell, and discounting the vector to t=1. Values of the fields in the base case are summarized in table 1.5.

Table 1 5: Binomial Lattices: Base Case Values (Values in '000)

Binomial - Base Case	Johan Castberg	Alta/Gohta	Total Value
	$ 2 454 962	$ 1 613 975	$ 4 068 937

6.3.2 Binomial Lattice: Option to Abandon

The option to abandon is included through two steps:

First, estimate cash flow cash-flows:

Introduce the cash-flow tree from section 6.3.1.

Second, introduce optimal decision:

A summarizing and value-maximizing decision criteria is introduced in every cash flow-node, using IF-statements, discounted using *p*:

$$Optimal\ Decision = IF((CFv + (CFu_{v+1} * P + CFd_{v+1} * (1 - P)) * exp(-r) > -K, (CFv + (CFu_{v+1} * P + CFd_{v+1} * (1 - P)) * exp(-r), -K) \tag{10}$$

Optimal decision in period *v* is given by comparing *CFv*, cash-flow at time *v*, summarized with the discounted, risk-neutral outcomes in period $v+1$, CFu_{v+1} and CFd_{v+1}, against abandoning the field in period *v* at cost *–K*.

Introducing this formula to all nodes, the model recursively compares the value of immediately exercising the option, to postponing decision. The spreadsheet at v=0 contains discounted cash flow as a result of optimized decisions subtracted by investment costs. From here, negative values are removed and the nodes are discounted, using *p*, to 2017 values. The single node in t=1, yields the value of the field including the option to abandon. Values of the fields with option to abandon is summarized in table 1.6.

Table 1 6: Binomial Lattices: Option to Abandon Values (Values in '000)

Binomial - Option to Abandon	Johan Castberg	Alta/Gohta	Total Value
	$ 2 485 350	$ 1 622 293	$ 4 107 643
Added Value to Base Case	$ 30 388	$ 8 318	$ 38 706

The option adds marginal value for both fields.

6.3.3 Binomial Lattice: Individual Option to Defer Investment, on Option to Abandon

The process include the steps in 8.2.2, but extended to reflect the exercise dates for investment. Values are estimated in three steps:

First, create price tree

Introducing the option to defer investment extends the price chart by the lifetime of the option to defer investment.

Second, calculate field value including option to abandon:

Field values including the option to abandon is estimated for all periods where the consortium can defer investment, using the process described in section 6.2.2 - with one exception. When the optimal decision reaches v=0, and negative values are removed, the vector containing the values of the field is extracted and included in a new tree. This tree contains field values to reflect all investment periods.

Third, introduce optimal decision to invest:

The *new* tree is subjected to an optimal decision criteria. The decision is based the payoff of immediate investment, compared to the expected value delaying the decision to the subsequent period:

$$Optimal\ Decision = IF((NPVt > (NPV_{t+1} * P + NPV_{t+1} * (1-P)) * exp(-r),$$
$$NPVt, \left(NPV_{t+1} * P + NPV_{t+1} * (1-P)\right) * exp(-r)) \qquad (11)$$

NPVt refers to the present value of the field given investment in the node, and is similar to equation 10. Recursively, this formula will yield the total field value, including the option to abandon and defer investment, in the single node at t=1. Value of the fields including option to defer investment, and abandon is summarized in table 1.7.

Table 1 7: Binomial Lattices: Option to Defer, and Abandon Values (Values in '000)

Binomial - Option to Defer, and Abandon	Johan Castberg	Alta/Gohta	Total Value
	$ 2 485 350	$ 2 293 034	$ 4 778 385
Added Value to Base Case	$ 30 388	$ 679 059	$ 709 448

The option to expand adds significant value to Alta/Gohta. The optimal time to invest is furthest possible into the future – with a drift higher than the risk-free discount rate. For Johan Castberg, the base case assumes the latest investment period in the last period introduced by the option to defer investment. Alta/Gohta can still choose to defer investment, and the option adds additional value.

6.3.4 Binomial Lattice: Option to Expand

Following the limitations in section 6.2.4, an equivalent method is pursued using binomial lattices. With equal input parameters, development on Alta/Gohta is contingent upon development on Johan Castberg. The total value of the fields is derived in two steps:

First, estimate value of Johan Castberg:

The value of Johan Castberg is estimated by increasing capital expenditures in the model described in section 6.3.3.

Second, estimate value of Alta/Gohta:

Alta/Gohta development at a discount is dependent upon an investment in Johan Castberg in earlier, or the same period(s).

The model in section 6.3.3 is updated with lower investment costs. The "new tree", is updated to reflect the risk-neutral probability that non-investment on Johan Castberg inhibits investment on Alta/Gohta. After removing, or adjusting cells according to the risk-neutral probability that non-development on Johan Castberg inhibits development on Alta/Gohta, the field value is estimated using *p*. Including the opportunity to develop Alta/Gohta at full cost, the model from 6.3.3 is included, where the "new tree" only includes the values of Alta/Gohta reflecting the risk-neutral probability that Johan Castberg will not be developed. Summarizing the two values gives the value of Alta/Gohta as an expansion to Johan Castberg. Values of the fields including option to expand, on option to defer investment and abandon are summarized in table 1.8.

Table 1 8: Binomial Lattices: Option to Expand Values (Values in ‘000)

Binomial - Option to Expand	Johan Castberg	Alta/Gohta	Total Value
	$ 2 243 973	$ 2 901 010	$ 5 144 983
Added Value to Base Case	$ -210 989	$ 1 287 035	$ 1 076 046

Compared to the base case, total value added by full flexibility is significant.

6.3.5 Closing Remarks on Binomial Lattices Method

Excel spreadsheet was used to calculate, and especially on Alta/Gohta, including both the abandonment and timing option, it quickly got out of hand. Further, with Alta/Gohta as a tie-in field, the nodes had to manually be adjusted to reflect values. A recommended way to do it would be to either use the Super Lattice Solver included in the book Real Option Analysis (Mun, 2006), or using a programming language.

7.0 Real Option Models: Estimation Analysis

This section describes the estimations from the models, and discusses differences from using the two methods.

7.1 Separate Production

Tables in the section summarize field values, with relevant options using both models:

7.1.1 Base Case

Table 1 9: Comparison LSM and Binomial Lattices: Base Case Values (Values in ‘000)

BaseCase	Johan Castberg	Alta/Gohta	Total
LSM	$ 2 730 380	$ 1 852 989	$ 4 583 369
Binomial Lattice	$ 2 454 962	$ 1 613 975	$ 4 068 937

Both fields are profitable. Larger field size, and longer lifetime on production on Johan Castberg more profitable than Alta/Gohta.

In-between models there is discrepancy of approximately USD 0.5 billion, above ten percent for each case. This will be discussed in section 7.3. As mentioned in section 6.1, model assumptions, the base case effectively included an option to turn down unprofitable investments. The argument was that the consortium will decide at the point of investment whether to investment will happen in the future. This assumption should nevertheless not give significant variation in values.

7.1.2 Value of Option to Abandon

The value of the option to abandon is found by subtracting base case values from the value of the field including the option to abandon.

Table 1 10: Comparison LSM and Binomial Lattices: Option to Abandon Values (Values in '000)

Value of Option to Abandon	Johan Castberg		Alta/Gohta		Total	
	Added Value ($)	(%)	Added Value ($)	(%)	Added Value ($)	(%)
LSM	$ 59 597	2,2 %	$ 43 623	2,4 %	$ 103 219	2,3 %
Binomial Lattice	$ 30 388	1,2 %	$ 8 318	0,5 %	$ 38 706	1,0 %

The value of abandonment is the value of excluding negative production values – traded against the effect of the risk free rate on the decommissioning cost. Relatively small values can be explained by two factors. First, the consortium can turn down bad investments. Second, cash-flow losses are limited to when price reaches zero.

7.1.3 Value of Option to Defer Investment, on Option to Abandon

Value of the options is found by subtracting base case values from the value of the field including the option to defer, on option to abandon.

Table 1 11: Comparison LSM and Binomial Lattices: Option to Defer, on Option to Abandon Values (Values in '000)

Value of Option to Defer	Johan Castberg		Alta/Gohta		Total	
	Added Value ($)	(%)	Added Value ($)	(%)	Added Value ($)	(%)
LSM	$ 177 894	6,5 %	$ 1 090 599	58,9 %	$ 1 268 493	27,7 %
Binomial Lattice	$ 30 388	1,2 %	$ 679 059	42,1 %	$ 709 448	17,4 %

Introducing the option adds significant value to Alta/Gohta, compared to on Johan Castberg. This is the case both for the LSM and binomial method.

Two moments explain drivers of the option value. *First*, the average price increase with each time-step at a rate higher than discount rate. This induces a *relative* optimal investment at later periods, where Alta/Gohta can defer from investment the next thirteen years, 8 years longer than on Johan Castberg. In the binomial method, the drift in price led to optimal decisions at maturity. This is shown comparing the value of the option value on Johan Castberg in this section, only including the option value of abandoning field. Isolated, this means that the value of deferring investment adds no value to Johan Castberg – as the optimal time to invest is the time to invest in the base case. This is not the case for Alta/Gotha, with maturity on the option to defer investment that exceeds the basic investment date.

The *second* moment is taking advantage of discounted investment cost. The post-tax effect of risk-free rate on capital expenditures on Alta/Gohta, equates to an isolated advantage of postponing investment on USD 35, 75 million for a single period. The effect decreases the further capital expenditures are discounted, but the further the sum is discounted, the larger is the effect in sum.

In between models there is a systematic pattern, albeit with a large difference. Added value on Alta/Gotha significantly surpasses the added value on Johan Castberg by introducing the two options.

7.2 Co-production

The value of the co-producing fields are first compared to the base-case, but as they do not in itself make much sense, they are compared to the value of the independently producing fields. The independently producing fields is the total value of both fields including the option to abandon and defer investment.

Only the total values of both the fields are included from here, with respect to the change in capital expenditures.

Table 1 12: Comparison LSM and Binomial Lattices: Option to Expand Values (Values in '000)

Value of Option to Expand	Johan Castberg		Alta/Gohta		Total	
	Added Value ($)	(%)	Added Value ($)	(%)	Added Value ($)	(%)
LSM	$ -39 302	-1,4 %	$ 1 560 068	84 %	$ 1 520 766	33,2 %
Binomial Lattice	$ -210 989	-8,6 %	$ 1 287 035	80 %	$ 1 076 046	26 %

The option values compared to the base-case yields positive figures for both models, and are comparable in-between.

Table 1 13: Comparison LSM and Binomial Lattices: Value of Co-Production (Values in '000)

Value of Co-Production	Johan Castberg		Alta/Gohta		Total	
	Added Value ($)	(%)	Added Value ($)	(%)	Added Value ($)	(%)
LSM	$ -217 196	-7,5 %	$ 469 469	15,9 %	$ 252 273	4,3 %
Binomial Lattices	$ -241 377	-9,7 %	$ 607 975	26,5 %	$ 366 598	7,7 %

Table 1.13 summarizes the benefit of coproduction. The benefit of coproduction is the total value of the fields including the option to expand, subtracted by the total value of the individual producing fields. Between models, there is a systematic pattern again A negative NPV on Johan Castberg leading from an increase in capital expenditures, is covered by the added value on Alta/Gohta from reduced costs. The total added-value of co-production ranges from USD 0.252 to USD 0.367 billion, or an increase from 4.3 to 7.7 percent. Intuitively, the added value is low. Nominal post-tax reduction of capital expenditures equates to USD 0.825 bln, With a low risk-free rate, the maximum effect if discounting, supposing all capital

expenditures happens in year 13, the net effect of lowering capital expenditures equates to USD 0.588 bln. This effect will further be discussed in section 7.4, and can be described by that option models are non-linear. It is difficult to determine if the added value is worth the interdependency. This will be discussed in the following section.

7.3 Model Comparison

Comparing the estimated values show a large, systematic spread between models already in the base case. The LSM method always shows a higher field values compared to lattices. This shows a form of consistency, where the spread increases with most layers of flexibility.

Following the discrepancy, the choice of model to pursue further is based on the model which theoretically shows the correct value of the field. Based on granularity, where Mun suggests using between 100 and 1000 time-steps, the number of time-steps included seems to deflate value. The LSM can counter the problem of granularity through increasing the number of simulations. The LSM method will be used to determine the drivers and added-value from co-production. Further, the notion on waiting is in the context of investment period, as the option to abandon did not significantly affect values.

7.4. Capital-Expenditure Payoff

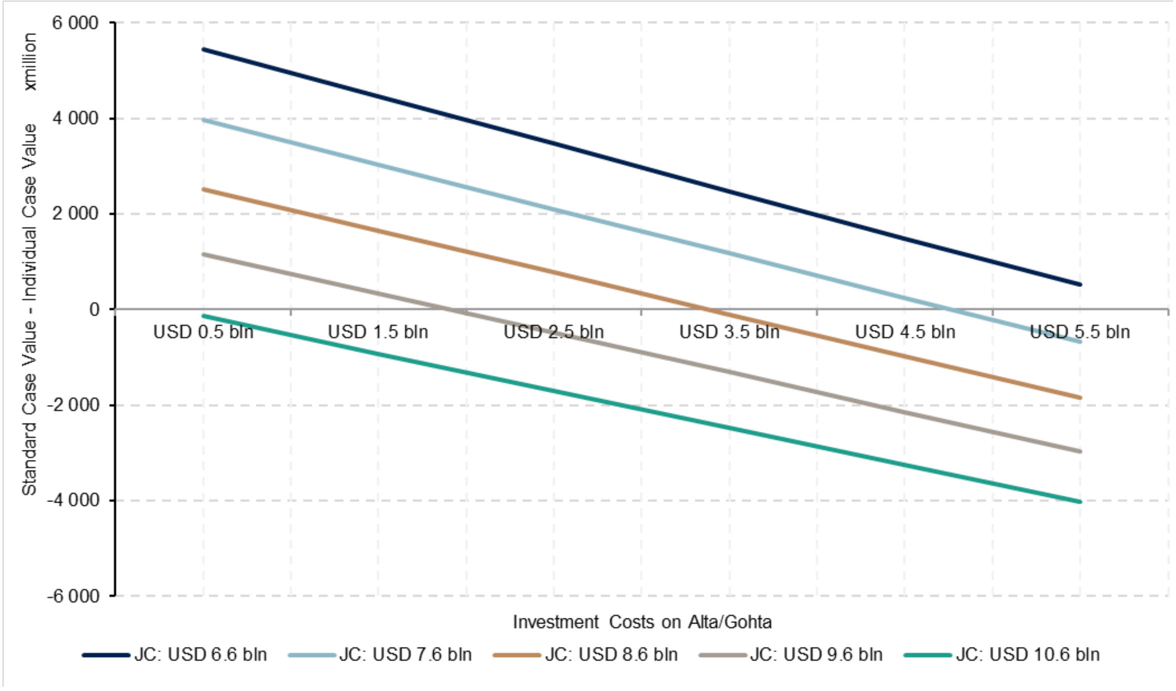
Investment costs used in the case of co-production was a central parameter to the thesis, while difficult get an estimate on. This section will reconcile the assumption by assessing different scenarios of capital expenditures. This section includes a description on how change in capital expenditures on the different fields affects the total value and will serve as the estimator needed to determine if co-production is value-adding. Combinations of investment costs considered are summarized in table 1.14.

Table 1 14: Capital Expenditure Values Considered in Capital Expenditures Payoff (Values for Min/Max in billions)

Values in billion	Minimum	Maximum	Tick
Johan Castberg	$ 6,60	$ 10,60	$ 1,00
Alta/Gohta	$ 0,50	$ 5,50	$ 1,00

At development costs on Johan Castberg at USD 10.6 bln, co-production is not profitable with the minimum cost on Alta/Gohta - compared to individual production. The static case of individual production will be called the individual case. The static case of coproduction from section 7.2 will be called the standard case. The individual fields will be the comparison in all cases. The benefit of coproduction will be the total values of the field, either subtracted or divided on the total value of the individual fields. Figure 1.12 shows the total benefit in value of co-producing the fields. With a tick per USD 1 billion, all values in-between the limits are considered.

Figure 1 11: Net Benefit of Co-producing Johan Castberg and Alta/Gohta



When CAPEX on Johan Castberg and Alta/Gohta respectively reaches USD 6.6 bln, and USD 6.25 bln, the benefit of coproduction reaches zero. At the point where CAPEX on the fields are equal to the individual case, value destruction is .3 percent of total field value. This means that the model does not significantly affect value given these levels of cost, where expectations were that the model inhibits flexibility on Alta/Gohta.

Determinants of capital expenditures payoff will be explained by Alta/Gohta's access to production at reduced cost, and an inelasticity in change in CAPEX. As the compounded option is a fairly complex system, and that options are non-linear, the exact source of added value is out of the context of this paper to describe. Further, an error source is the one-way interdependency introduced – Alta/Gohta is contingent upon investment on Johan Castberg, while development on Johan Castberg does not consider Alta/Gohta.

With higher cost on Johan Castberg, less paths are invested in leading to lower payoff from developing Alta/Gohta at a discount. With USD 6.6 bln in CAPEX on Johan Castberg, 52 percent of the paths includes investments, reduced to 44 percent when investment costs are increased to USD 9.6 bln. Increasing CAPEX on the main field also leads to postponed development to take advantage of the discount effect, together with a limited effect on later periods from discounting over a longer period.

With CAPEX on USD 6.6 and 9.6 bln on Johan Castberg, the average paths invested in at period one and five decreased from respectively 10 to 6 percent and 28 to 26 percent. With lower capital expenditures on Alta/Gohta, early development is induced. The findings is that the net benefit of coproduction, by reduction in costs on Alta/Gohta is highest when Johan Castberg is developed at a low cost.

One tick change in CAPEX on Johan Castberg affects added value in the range of USD 0.1 to 0.15 billion. Change in CAPEX on Alta/Gohta affects added-value in the range of USD 0.08 to 0.1 bln. This skew describes that the average nominal save in capital expenditures on Alta/Gohta needs to surpass the extra cost on Johan Castberg by 44 percent on average. The limited change in total value by change in CAPEX on either fields by one tick, or post-tax nominally USD 0.22 bln, can be explained by non-linearity in the model from a set of interdependent variables, especially the discount rate as CAPEX on Alta/Gohta is discounted over a longer period in average. See appendix 7 for a table describing added value of changing investment costs on one field, given investment costs on the other.

7.4.1 Closing Remarks on Capital Expenditures Payoff

This section described the effects of changes in capital expenditures on two fields with two different maturity dates on the timing option to invest, where the field with the longest maturity date is contingent upon investment on the one with the shortest. The determinants described were access to production on Alta/Gohta and effect of the discount rate. The value of coproduction was at its highest with low capital expenditures on Johan Castberg leading to early investment, in order for Alta/Gohta to be developed at the earliest point. With higher investment cost on Johan Castberg, postponed and lower probability of investment decreased the net effect of added value by co-production.

Alta/Gohta needs to have discount significantly higher than premium added on Johan Castberg, in order to be beneficial compared to the individual case. Within limits chosen, average rebate on Alta/Gohta was estimated to be a formidable 44 percent higher than added premium on Johan Castberg in order to be profitable, compared to individual production.

Mentioned in section 7.2, it is difficult to conclude if the small added value to the standard case is worth the interdependency. As there are two different consortiums of licensees, where Alta/Gohta is dependent upon construction on Johan Castberg – owners of Alta/Gohta can be negatively affected by delayed or development errors on Johan Castberg. As described in section 3.2, there is high uncertainty related to development of upstream mega-projects.

The value of Johan Castberg is reduced, and as described in section 2.3.3, production profits *should* be related to the field. The licensees on Alta/Gohta significantly benefits from the expansion, while value on Johan Castberg is reduced. The contract also has the objective to incentivize increase in capacity on the main field. The incentives could include a higher tariff on processing the oil. There are examples where the licensees on the main field and potential tie-in field do not reach an agreement based on the tariffs for processing oil, also examples where the Government have *forced* the main field to include oil from nearby licenses in order to maximize value creation (Stangeland, 2015). Setting the tariff is arguably easier in the case of Lundin Norway and Statoil AS, with common financial interests (Statoil, 2016).

8.0 Sensitivity Analysis

To determine if other variables influence the value of coproduction, all input parameters included in the model are tested. The parameters are volatility, unit cost, interest rate, initial oil price and field size. Using the LSM method, this chapter describes the isolated effect of altering a parameter, assuming that the parameter is constant throughout the lifetime of both fields. Discussion depends on the effect the parameter have on the value of the fields and added value of coproduction.

8.1 Changes in Low-Affecting Parameters

Volatility increased the value of the fields and options, an expected result. Higher volatility increases the values from higher potential upside in prices, while blocking the unprofitable paths from the decision criteria. Increasing volatility had a marginal, negative correlation with added-value from coproduction. This is explained through the effect volatility has on the paths invested, where increasing volatility increased the value of the paths with investment– but decreases the percentage of paths with investments on Johan Castberg. Higher volatility then leads to less paths to take advantage of the discounted Alta/Gohta, see appendix 6.

The risk-free rate affected values both through oil price drift, and through discounting. Net effect of increasing the risk-free rate is higher value on fields and options. Further, increasing the risk-free rates leads to postponed investment in the fields. This effect introduces two opposing reactions. The discounting effect on total field values pushes for early investments, countered by expectations of higher prices (appendix 6). The risk-free rate marginally reduces the benefit of coproduction.

Increasing unit cost decreases the number of paths invested in. Unit cost therefore is negatively correlated both with the value of the fields, and value of coproduction. Unit cost affects value of the fields in manner as price, with the exception of being static where price increases with the drift rate. See appendix 6.

The remaining parameters; initial oil price, convenience yield and size of the fields will be discussed in more detail in section 8.2 to 8.4.

8.2 The Effect of Initial Oil Price

The assumed initial price of oil is 41.6 dollar per barrel of oil, and changing initial oil price has a significant effect on field values. Changes ± USD 5 dollar is included in the sensitivity analysis, summarized in table 1.15.

Table 1 15: Effect of Initial Oil Price on Field Values (Values in '000)

Total Value of Fields	Effect of Changes in Initial Price		
Initial price	Standard Case Value	Individual Case Value	% Benefit of Coproduction
$ 36,6	$ 5 065 146	$ 4 841 891	4,61 %
$ 41,6	$ 6 104 136	$ 5 851 862	4,31 %
$ 46,6	$ 7 178 055	$ 6 898 587	4,05 %

Increasing the initial oil price by USD 5 from USD 36.6 to 41.6 increases the value of the fields by approximately 20 percent. The percentage growth decays with higher levels of price. Higher initial prices reduces the benefits of coproduction. This finding cannot be explained by the two determinants chosen; access to production on Alta/Gohta and the effect of discount rate. Increasing price increases the benefit of developing Alta/Gohta as an individual field, and while the increased capital expenditures on Johan Castberg induces late investments – this can be the effect of sub-optimal investment decisions on Alta/Gohta. The effect can be seen in context with sizes of the reserves, in section 8.4

8.3 The Effect of Net Convenience Yield

The benefit of coproduction and field values are highly susceptible to changes in net convenience yield. Net convenience yield decreases drift in oil price, and correlates positively with the benefit of coproduction, while negatively with field values. The compounded effect of one percent decrease in net convenience yield leads to an average oil price increase of 39 percent after 34 periods.

Table 1 16: Effect of Net Convenience Yield on Field Values (Values in '000)

Total Value of Fields	Changing Convenience Yield		
Net Convenience Yield	Standard Case Value	Individual Case Value	% Benefit of Coproduction
-1,5 %	$ 7 350 442	$ 7 078 897	3,8 %
-0,5 %	$ 6 104 136	$ 5 851 862	4,3 %
0,5 %	$ 5 080 468	$ 4 847 404	4,8 %

Lower drift together with constant discount rate decreases the value of waiting, and induces early field development (appendix 6). This reduces the net effect of discounting, and with the lifetime of the waiting option on Alta/Gohta compared to Johan Castberg, enhances the benefit of coproduction.

8.4 The Effect of Field Size

The estimated amount of recoverable oil is highly uncertain, together with its effect on the value. The included sizes are the cautions, normal, and ambitious case, which are combinations of all field sizes from section 2.3.1, and 2.3.2. Table 1.17 summarizes the field values for the individual case.

Table 1 17: Effect of Field Size on Field Value in Individual Case (Values in '000)

Individual Production		Johan Castberg		
Total Value of Fields	*Field Size (barrels)*	Cautios (400)	*Normal* (491)	Ambitious (650)
Alta/Gohta	Cautios (216)	$ 3 819 805	$ 4 943 173	$ 5 705 940
	Normal (351)	$ 5 169 022	$ 5 851 862	$ 7 055 158
	Ambitious (584)	$ 7 605 516	$ 8 288 357	$ 9 491 653

Total value is most susceptible to changes in Alta/Gohta, as the waiting option on the field adds high value to the total value due to its lifetime. This effect both is present when using lattices and the LSM method. Using the LSM method, added value was over five times higher than on Johan Castberg, when comparing to the base-case.

Table 1 18: Effect on Field Size on Field Value for Co-Producing Fields (Values in '000)

Co-Producing Fields		Johan Castberg		
Total Value of Fields	*Field Size (barrels)*	Cautios (400)	*Normal* (491)	Ambitious (650)
AG	Cautios (216)	$ 4 056 192	$ 5 208 792	$ 5 983 222
	Normal (351)	$ 5 368 513	$ 6 104 136	$ 7 359 139
	Ambitious (584)	$ 7 693 317	$ 8 451 647	$ 9 743 153

Table 1.19 show that the benefits of coproduction are highest when the amount of recoverable oil on Alta/Gohta is low. In other words, benefits of coproduction is highest when the tie-in field is less valuable. When Alta/Gohta is developed individually, paths with investments decreases with lower amounts of reserves. Reduction in capital expenditures reduces the investment barrier and investment occurs at a higher rate. In the cautious case of Johan Castberg, the effect of co-production is marginal, with the same effect, limiting the benefit of discounted cost on Alta/Gohta.

Table 1 19: Effect of Field Size on Benefit of Co-Producing Fields (Values in ‘000)

Benefit of Co-Production		Johan Castberg		
% Benefit of Co-Production	*Field Size (barrels)*	Cautios (400)	Normal (491)	Ambitious (650)
AG	Cautios (216)	6,19 %	5,37 %	4,86 %
	Normal (351)	3,86 %	4,31 %	4,31 %
	Ambitious (584)	1,15 %	1,97 %	2,65 %

8.5 Closing Remarks on Sensitivity Analysis

The three parameters included in the sensitivity analysis that affected both field values and the value of co-production significantly was field size was net convenience yield, initial price and field size. Added value was largest when the value on the tie-in field was smallest, and the drift rate on price lowest.

9.0 Conclusion

The objective of the thesis was to estimate if co-producing the oil fields Johan Castberg and Alta/Gohta adds value, with regards to capital expenditures. Co-production is an important theme on the Norwegian Continental Shelf in the current climate with a low, volatile oil prices where many new-found fields are small and unprofitable. The two oil fields Johan Castberg and Alta/Gohta are not marginal in size, but challenged by high costs from being located in the undeveloped Barents' Sea.

Co-producing the fields has been described as an option to expand, where Johan Castberg is developed with increased capacity to include oil from Alta/Gohta. Johan Castberg was developed with higher investment cost, while Alta/Gohta at a discount. An underlying theme of the thesis was then to estimate value added by including flexibility on the fields. Two real options methodologies has been used and compared, the binomial option pricing model and Least Squares Monte Carlo. With limited granularity in the binomial model, values were suppressed already in the base case. Granularity can only be introduced by decreasing the time-intervals in the binomial model, where Monte Carlo simulation can address this issue by increasing the number of simulations. Further, the binomial model quickly suffered from complexity when introducing layers of option. Based on this, the results on determining the value of the co-producing fields were estimated using the Least Squares Monte Carlo approach.

The option to defer investment, on option to abandon, added *enormous* value on Alta/Gohta, while limited on Johan Castberg. This is because the option maturity date was thirteen and five years respectively for Alta/Gohta and Johan Castberg. The isolated option to abandon added small value to the fields in all cases.

Co-producing Johan Castberg and Alta/Gohta, two fields that in the model have similar characteristics, added small value with respect to capital expenditures. The determinants discussed was access for Alta/Gohta to initiate production when it can be developed at a discount, and the effect of the discount rate.

Field values were relatively insensitive to change in capital expenditures in the case of co-production, and Johan Castberg was affected at a higher rate than on Alta/Gohta, meaning that the savings on Alta/Gohta must surpass the cost of extra capacity on Johan Castberg. This is due to two factors. First, an effect of higher capital expenditures on Johan Castberg is postponing investment decisions and insensitivity in investments taking place in later periods, to take advantage of the effect of discounting capital expenditures. Savings on investment costs on Alta/Gohta induces early development. Also, higher capital expenditures on the main field leads to lower probability that the field will be developed, affecting the income potential of Alta/Gohta. With development on Alta/Gohta contingent upon positive investment decision on Johan Castberg, this leads to suboptimal decisions. This leads to the second moment, where the model is inadequate with regards to that development on Johan Castberg does not take Alta/Gohta into consideration. The effect of which was not determined. On average, the discount on development costs on Alta/Gohta had to surpass the extra cost of capacity on Johan Castberg by 44 percent.

Benefits of coproduction were only marginally affected by changing other parameters. The exceptions were net convenience yield, initial price and sizes of the oil fields. Net convenience yield reduced oil price drift, and correlated positively with benefit of co-production. Lower drift rate on the oil price induced early investment on the main field, with lower expectations on prices, which opened for developing Johan Castberg in an earlier period on average. This subsequently opened for early investments on Alta/Gohta, taking advantage of development with low capital expenditures. The size of the field also significantly affected the benefit of coproduction. In percent, total benefit was highest when the volume of oil in Alta/Gotha was low. In the cases where the size of Alta/Gohta was high – the benefit of a discounted development cost was largely consumed by the probability that Johan Castberg remained undeveloped and late investments. This reasoning was also made for explaining the effect of initial price.

Coproduction added value under circumstances described - if Alta/Gohta could be developed at a discount higher than 44 percent compared to added cost on Johan Castberg. This is assuming the input parameters in the case of the individually producing fields were precise.

The thesis will close with a theoretical discussion on the subject. Taking into consideration other factors, as uncertainty in development, it is difficult to determine if it is beneficial to co-develop the fields. Based on the estimations, there is a limited upside of co-producing the fields if all value added gains are based on capital expenditures. Including other variables, as sharing variables and fixed costs on the field can yield different results.

An uncertain downside spurs from development uncertainty with different consortium of licensees on the fields. With Alta/Gohta contingent upon development on Johan Castberg, development delays on the main field can have dire effects on the value of the tie-in. As investing in extra capacity *should* be incentivized through the agreement, either through subsidies / beneficial taxation (Norwegian Parliament, 2003), or licensees on Alta/Gohta must assist covering the extra cost. Including the marginal upside estimated in the assignment, the licensees on Alta/Gotha *could*, with respect to taking on extra development uncertainty, be reluctant to increase their cost to the level needed to support licensees on Johan Castberg.

I suggest further research on five moments:

- Introducing a stochastic convenience yield, as it has high implications for both field values and the value of co-production.
- University papers often introduce yearly decision points and prices when using the binomial model, it would be interesting to see the level of granularity needed to reach the value of a continuous model under similar circumstances.
- Introducing a model for heteroscedasticity in Brent price to assess its impact on option values.
- Assessing other benefits in co-production, as capacity maximization and sharing of fixed/variable costs.
- Constructing a model that incorporates full interdependency between the fields.

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10.0 Appendix

Appendix 1: Characteristics of GBM and Ornstein-Uhlenbeck

Geometric Brownian Motion:

A stochastic process, *St*, follows a Geometric Brownian Motion with drift when it satisfies the differential equation:

$$dS_t = \mu S_t dt + \sigma St * dZ \qquad (1.1)$$

In equation 1, μ is the trend, multiplied with the stock price in a time interval *dt*, representing the deterministic part of the process. The second, the stochastic term, describes a random outcome given the price and its volatility (σ), multiplied by *dZ*, the random behavior. *dZ*= $\varepsilon\sqrt{dt}$ where ε is a random number with mean 0 and variance of 1. With *dz* as an independent draw, the process will only be dependent upon the current state – not on previous outcomes. Changes in time in equation … are infinitely small, and the process is continuous. In order to simulate GBM, a discrete solution is determined. Discretizing the results of applying Itô's Lemma to the exact, discrete analytical solution to the stochastic differential equation (1.2).

$$\Delta St = (\mu - 0.5\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon \qquad (1.2)$$

Δt is later referred to as *dt*, or time-steps. Equation xx is a discrete process, where Δt a period of predetermined size. Absolute changes in *St* are lognormally distributed.

Exponential Ornstein-Uhlenbeck:

A commonly used mean-reverting process is the Ornstein-Uhlenbeck process.
The stochastic differential equation, which only produce positive values, is the exponential Ornstein-Uhlenbeck process is defined in equation (1.3).

$$dSt = \kappa(\mu - ln(St)) * S_t dt + \sigma S_t dZt \qquad (1.3)$$

κ is the speed of the mean reversion, μ is the long-run mean, σ is the volatility and *zt* is a Wiener process. Unlike the GBM, this model dependent indirectly upon past draws. The further *St* is from the long-run mean, the stronger the mean reversion speed constant will *weigh* the values towards the mean. Changes in St are log-normally distributed.
The analytical solution is (Mjell et al., 2010) summarized in equation (1.4)-(1.6).

$$\ln(St) = e^{-k}\ln(S_{t-1}) + \alpha(1 - e^{-k}) + \xi_t \qquad (1.4)$$

$$\xi_t \sim N(0, \eta^2) \qquad (1.5)$$

$$\eta^2 = \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa}) \qquad (1.6)$$

Appendix 2: Black-Scholes Call Option Pricing Model:

Equations (2.1)-(2.3) describes the parameters used in the B&S model, to price call options:

$$C = SN(d_1) - N(d_2)Ke^{-rt} \tag{2.1}$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{s^2}{2}\right)t}{\sigma * \sqrt{t}} \tag{2.2}$$

$$d_2 = d1 - s * \sqrt{t} \tag{2.3}$$

The call option value *C* is determined by the strike *K*, current stock price *S*, risk free rate *r*, time to maturity *t* and logarithmic volatility σ. $N(d_1)$ and $N(d_2)$ is the cumulative standard normal distribution of function of the standard normal distribution.

Appendix 3: Least Squares Monte Carlo – Algorithm Described

The framework assumption is an complete underlying probability space (Ω, $\mathcal{F}$, P) with finite time horizon [0,T]. Ω refers to a set of possible realizations of the stochastic economy between between [0,T]. w refers to a simulated path in the probability space. $\mathcal{F}$ is a sigma field of distinguishable events at time T, and P is a probability measure defined by elements of $\mathcal{F}$. F is a subset of Ω explaining different events at time T, where P is a probability measure for the elements of F. F is defined as {F={ $\mathcal{F}_t$;t ∈ [0,T]}, the augmented filtration by the a price process filter for the relevant securities in the economy, where Ft = $\mathcal{F}$. Further, in compliance with the no-arbitrage theory the existence of equivalent martingale measure (Q) is assumed for this economy.

C(w,s: t, T) denotes the paths of cash flows generated by the option, conditional on that the option is not exercised before time t, and that the holder follows an optimal exercising strategy for all s,t < s ≤ T.

At period T, the holder exercises the option if it is In-The-Money (ITM). At all periods in front, the holder must decide between exercising and continuing to revise the decision at a later point. The option is exercised as soon as the exercise value is equal or greater than the continuation value, efficiently maximizing the option path wise.

The information present to the holder at time t is the value of exercising the option in period t. Continuation value is not known, but no-arbitrage theory states that the value of continuation is the discounted, expected cash flows with respect to risk neutral pricing measure Q. The continuation value (CV) at time tk can be stated as:

$$\mathrm{F}(\mathrm{w};\mathrm{t_k}) = \mathrm{E_Q}[\sum_{\mathrm{j=k+1}}^{\mathrm{K}} \exp(-\int_{\mathrm{t_k}}^{\mathrm{tj}} \mathrm{r}(\mathrm{w},\mathrm{s})\mathrm{ds})\mathrm{C}(\mathrm{w},\mathrm{t_j};\mathrm{t_k},\mathrm{T})|\mathcal{F}_{\mathrm{tk}}] \tag{3.1}$$

$\mathrm{r}(\mathrm{w},\mathrm{s})$ is a static riskless rate, and expectations are taken conditional on information set $\mathcal{F}_{\mathrm{tk}}$ on time $\mathrm{t_k}$

The option can be exercised optimally by maximizing between exercise value and CV.

The algorithm recursively begins in the end period, as C(w,s;t,T) can differ from C(w,s;t+1,T) because it can be optimal to exercise the option at time t+1, thus changing all cash flows along a path w. This argumentation also is used present when using the binomial method. At time tk-1, the model assumes that the unknown functional form of F(w;tk-1) in the equation above can be represented as a linear combination of a countable set of Ftk-1-measurable basis functions (Longstaff and Scwartz, 2001).

Appendix 4: auto.arima():

The steps in the automated algorithm are as follow:

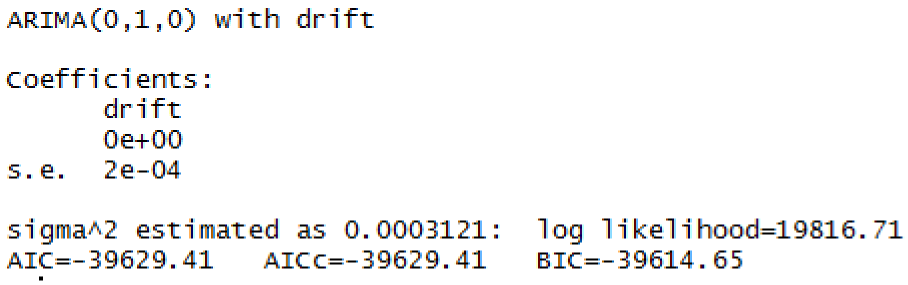
1. A KPSS-test is initially used to determine if the series is trend stationary. If not, differencing is performed together with another KPSS-test. This is repeated until the process is stationary.
2. The autoregressive and moving averages values are determined through minimizing the identification criteria AICc for the following models:

   ARIMA(2,d,2),
   ARIMA(0,d,0),
   ARIMA(1,d,0),
   ARIMA(0,d,1).

   Variations of the models include ± 1 in the current model, which is the chosen model with a constant if d equals to 0. After finding the most appropriate model after variating the values and including/excluding a constant, the step is repeated until the lowest AICc is found.

The AICc measures the relative *quality* of the chosen models, where R will report the log likelihood of the data – i.e. the logarithmic probability that the observed data will come from the model. Thus, the AICc determines the best fitting parameters of the model.

R output from auto.arima():



```
ARIMA(0,1,0) with drift

Coefficients:
      drift
      0e+00
s.e.  2e-04

sigma^2 estimated as 0.0003121:  log likelihood=19816.71
AIC=-39629.41   AICc=-39629.41   BIC=-39614.65
```

Appendix 5: ARIMA(0,1,0) Under Equivalent Martingale Measure

GBM under EMM may be written as:

$$dSt = (r - \delta)Stdt + \sigma Stdz \tag{5.1}$$

δ is net convenience yield. The price drift included in the LSM model is:

$$\Delta St = (r - \delta - 0.5\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\varepsilon \tag{5.2}$$

Binomial lattices, according to Mun (2006), can be solved using either market-replicating portfolios or through risk-neutral valuation. Market replicating portfolios are less used due to the difficulty acquiring them, Mun recommends risk-adjusting the probabilities of specific cash flows. Risk-neutral valuation rests on the assumption of either complete market, or that cash flows are linearly dependent.

The three basic setup for a binomial process includes three equations, an up and down equation, and a risk-neutral probability (p). Deriving the EMM of the underlying uncertainty process is done with the example of Brownian Motion.

$$u = e^{\sigma\sqrt{\Delta t}} \tag{5.3}$$

$$d = \frac{1}{u} \tag{5.4}$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{5.5}$$

These terms are derived from the Exponential Brownian Motion in equation 5.6.

$$\frac{\delta S}{S} = e^{\mu(\delta t)}e^{\sigma\varepsilon\sqrt{\delta t}} \tag{5.6}$$

The deterministic case, the first exponential term, is accounted for in the drift or growth rate of the price. The second exponential term, the stochastic, is transformed. By using a discrete simulation using the binomial approach; $\boldsymbol{\varepsilon}$, usually a simulated variable with mean 0 and finite variance, is accounted for. The stochastic term describes the magnitude of upward movement in the risk-factor after a single time-step – where the reciprocal term equates to $\boldsymbol{e^{-\sigma\sqrt{\delta t}}}$. The up and down factor are used in creating the binomial lattice uncertainty process, where one extra step increases the uncertainty span by one up- and one down-factor simultaneously. The last equation is the risk-neutral probability that does not have any particular meaning other than serving as an intermediate step in a series of calculations. Mun explains it as "the ratio of the exponential function of the difference between risk-free rate and dividend, multiplied by the stepping time less the down factor, to the difference between the up and down factors". The dividend in the case of oil price is net convenience yield.

Using a one-step stock price development, and assuming it can either go up by the factor *u* or down by the factor *d*: $S_u = S_0 * u,\ S_{d1} = S_0 * d$.

The probability of the stock price going up is denoted by *p* in state *u*, and down is (1-*p*) in state *d*. Given *d* < 1+r < *u*, and the probability *p* - the discounted prices are risk-neutral:

$$\frac{1}{(1+\mathrm{r})^0}\mathrm{S}_0 = \mathrm{E}_\mathrm{Q}\left(\frac{\mathrm{S}_1}{(1+\mathrm{r})^1}\right)|\mathcal{F}_0) = \frac{1}{1+\mathrm{r}}(\mathrm{S}_{0\mathrm{up}} + \mathrm{S}_0\mathrm{d}(1-\mathrm{p})) \tag{5.7}$$

Dividing by *S0*, and multiplying with (1+r) on both sides and transforming gives:

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{5.8}$$

Including convenience yield gives:

$$p = \frac{e^{(r-\delta)\Delta t} - d}{u - d} \tag{5.9}$$

Assuming that the underlying risk factors adequately captures the uncertainty in the cash flows, the cash flows can be discounted at a risk-free rate. As cash flows from oil fields are embedded with multiple risks, and the model underneath only takes one into consideration – risk-free discounting does not hold ground. This argument holds both for using LSM and binomial lattices. An alternative of discounting at a risk-free rate is to use a risk-adjusted discount rate, but in the context of this paper it is assumed that oil price solely carries the project uncertainty. In practice, discounting straight line cash flows using a risk-adjusted discount rate should yield the same net present value as discounting cash flows in binomial lattices using a risk-free rate if the underlying volatility is well represented in the model.

Appendix 5: Change in Total Value on Co-Producing Fields Given Change in CAPEX

Coprod-Ind(tot) value increase by CAPEX decrease AG					
AG / JC	JC: USD 6.6 bln	JC: USD 7.6 bln	JC: USD 8.6 bln	JC: USD 9.6 bln	JC: USD 10.6 bln
USD 0.5 bln	99438957,47	93510612,84	87208103,23	82341613,59	77922710,75
USD 1.5 bln	98911332,72	93194361,64	87273291,09	82103909,76	77421689,7
USD 2.5 bln	99105792,46	92871915,13	87258537,88	82136994,06	77530281,1
USD 3.5 bln	98681223,34	93041678,86	87417009,93	82344888,61	77590332,79
USD 4.5 bln	96816019,8	92331043,17	87205541,04	81849947,78	77792506,79
USD 5.5 bln	0	0	0	0	0

Coprod-Ind(tot) value decrease by CAPEX increase JC					
AG / JC	*JC: USD 6.6 bln*	JC: USD 7.6 bln	JC: USD 8.6 bln	JC: USD 9.6 bln	JC: USD 10.6 bln
USD 0.5 bln	0	-147877795,4	-145281354	-136876534,3	-129129889,5
USD 1.5 bln	0	-141949450,7	-138978844,4	-132010044,6	-124710986,6
USD 2.5 bln	0	-136232479,7	-133057773,9	-126840663,3	-120028766,6
USD 3.5 bln	0	-129998602,3	-127444396,6	-121719119,5	-115422053,6
USD 4.5 bln	0	-124359057,8	-121819727,7	-116646998,2	-110667497,8
USD 5.5 bln	0	-119874081,2	-116694225,6	-111291404,9	-106610056,8

Appendix 6: Sensitivity Analysis, Investment Period Johan Castberg:

	Investment Period for Johan Castberg (CAPEX: USD 8.6 bln)					
Volatility	1	2	3	4	5	No Investment
35,09 %	7,24 %	4,95 %	4,10 %	3,97 %	28,11 %	51,63 %
37,09 %	7,16 %	4,89 %	3,99 %	3,87 %	26,47 %	53,62 %
39,09 %	7,21 %	4,77 %	3,88 %	3,78 %	24,89 %	55,47 %
rf	1	2	3	4	5	No Investment
1,60 %	7,55 %	4,64 %	3,71 %	3,68 %	24,25 %	56,17 %
2,60 %	7,16 %	4,89 %	3,99 %	3,87 %	26,47 %	53,62 %
3,60 %	6,84 %	5,05 %	4,22 %	4,09 %	28,66 %	51,14 %
Unit Cost	1	2	3	4	5	No Investment
USD 12	9,34 %	5,40 %	4,33 %	4,26 %	27,42 %	49,25 %
USD 16	7,16 %	4,89 %	3,99 %	3,87 %	26,47 %	53,62 %
USD 20	5,65 %	4,37 %	3,56 %	3,53 %	25,35 %	57,55 %
Initial Price	1	2	3	4	5	No Investment
USD 36,6	5,12 %	4,16 %	3,44 %	3,41 %	25,11 %	58,76 %
USD 41,6	7,16 %	4,89 %	3,99 %	3,87 %	26,47 %	53,62 %
USD 46,6	9,50 %	5,42 %	4,36 %	4,33 %	27,41 %	48,98 %
Net Convenience Yield	1	2	3	4	5	No Investment
-1,50 %	7,13 %	4,93 %	4,23 %	4,04 %	29,91 %	49,76 %
-0,50 %	7,16 %	4,89 %	3,99 %	3,87 %	26,47 %	53,62 %
0,50 %	7,32 %	4,69 %	3,72 %	3,71 %	23,13 %	57,42 %