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Effect of Alpha-Type External Input on Annihilation of Self-Sustained Activity in a Two Population Neural Field Model

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ABSTRACT In the present work, we investigate the annihilation of persistent localized activity states (bumps) in a Wilson-Cowan type two-population neural field model in response to α -type spatio-temporal external input. These activity states serve as working memory in the prefrontal cortex. The impact of different parameters involved in the external input on annihilation of these persistent activity states is investigated in detail. The α -type temporal function in the external input is closer to natural phenomenon as observed in Roth *et al.* (*Nature Neuroscience*, vol. 19 (2016), 229–307). Two types of eraser mechanism are used in this work to annihilate the spatially symmetric solutions. Initially, if there is an activity in the network, inhibitory external input with no excitatory part and over excitation with no inhibition in the external input can kill the activity. Our results show that the annihilation of persistent activity states using α -type temporal function in the external input is more robust and more efficient as compared to triangular one as used by Yousaf *et al.* (*Neural networks*, vol. 46 (2013), pp. 75–90). It is also found that the relative inhibition time constant plays a crucial role in annihilation of the activity. Runge-Kutta fourth order method has been employed for numerical simulations of this work.

INDEX TERMS Annihilation of bumps, α -function, integro-differential equations, neural networks, Runge-Kutta fourth order method, symmetric solutions.

I. INTRODUCTION

The human brain is stimulating system contains billions of neurons (nerve cells). The average human brain weighs about 1.2–1.4 kilograms and it is the most complex organ in the human body. The most important thing in brain is neuron which processes information and communicate through electrical and chemical signals, control the activity of body. The localized persistent states of activity in the cortical networks [3] serve as the short-term working memory which is the ability to store stimulus-related information for few seconds and discard it once it is not longer relevant.

Neuron transfer information in the form of electrical pulses, different mathematical models describe this process. Firing-rate and spiking-neuron models are studied by

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Daniel *et al.* [4] in which they focused on models that are much related to the synapses and biophysical mechanisms of neurons than connectionist models. The connectionist models are used for normal behavior and clinical conditions, neuro-modulation, learning and activity profiles in working memory tasks. Working memory models can be broadly classified on how the persistent activity is generated, although these classes are not disjoint. One most popular mechanism is based on the thought that activity is persistent through strong recurrent excitatory connections in a cell assembly [5]. Another mechanism is the activity circulates in form of loops (called synfire chains) [3], which consists of feed-forward connected subgroups with no direct feedback links between succeeding groups of neurons.

In the applied mathematics and in particular mathematical neuroscience, the study of uniqueness, existence and stability of localized persistent activity are important, the generation

of coherent structures for different modeling approaches is investigated in many studies [3], [6], [7], [9]–[14]. They further studied the existence, stability, different shapes and mechanisms that form these bumps in one population neuronal model without recurrent excitations as discussed in Jonathan and William [15].

We investigated the two-population neural field Wilson Cowan model with spatio-temporal external input. Similar studies in firing rate models are done by Folias and Bressloff [16] and Blomquist *et al.* [17]. In Blomquist *et al.* [17], they have studied two-population neural field model with no external input, given as

$$\frac{\partial v_e}{\partial t} = -v_e + \omega_{ee} \otimes F_e(v_e - \phi_e) - \omega_{ie} \otimes F_i(v_i - \phi_i) \quad (1a)$$

$$\tau \frac{\partial v_i}{\partial t} = -v_i + \omega_{ei} \otimes F_e(v_e - \phi_e) - \omega_{ii} \otimes F_i(v_i - \phi_i) \quad (1b)$$

where v_e and v_i are the activity levels of excitatory and inhibitory population, respectively, ϕ_e and ϕ_i represent the corresponding threshold values for firing, τ is the ratio between inhibitory and excitatory time constants called *relative inhibition time*, F_m ($m = e, i$) are the firing rate functions. Neural responses are characterized by firing rates and these firing rate functions converts these neural responses firing rate of the population of neurons. The replacement of the neural response function by the corresponding firing rate is typically justified by the fact that each network neuron has a large number of inputs (see [18] for further details). The functions ω_{mn} , ($m, n = e, i$) are connectivity functions.

This work is an extension of the work done by Yousaf *et al.* [19] where they investigated the bump solution of two-population neural field model with spatio-temporal external input. They formulated the general necessary condition for emergence of persistent activity states. They further investigated the generation and annihilation of persistent states of activity in the network for triangular type temporal function and different spatial functions.

The idea of α function motivated by Roth *et al.* [1], they investigated the thalamic inputs which provide motor, multiple visual and visumotor signals to L1 of mouse V1. The α -type function is observed in the experimental data in this study. We are using the α -function as the temporal part of external input because the α -type function is smooth and more natural behavior as observed in working memory. The emergence of selective persistent states of activity in the working memory investigated by Afzal *et al.* [20] with spatio-temporal external input. The effect of external input on emergence of bumps for different spatial and smooth α -type temporal functions of external input is investigated and found that certain parameters play a key role in the generation of persistent activity states in the network e.g. relative inhibition time constant, total duration and the amplitude of external input by Afzal *et al.* [20].

We have investigated the phenomenon of annihilation of the activity in network using spatio-temporal external input with α -type temporal part. Initially, there is an activity in

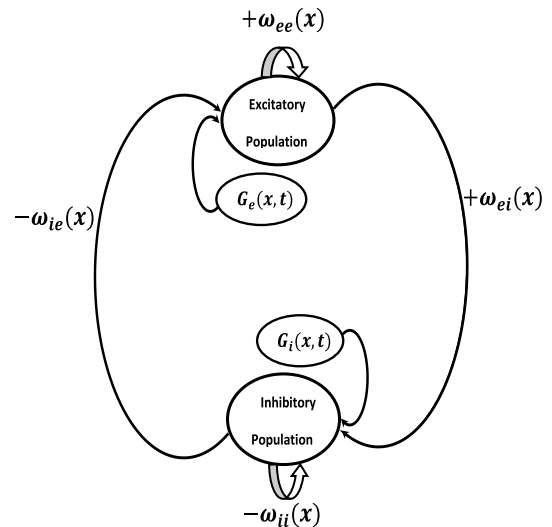


FIGURE 1. Sketch of Two population model (2) with spatio-temporal external input (G_e and G_i).

the network, two types of eraser mechanisms can be used in the network to annihilate the activity, one is inhibitory external input and other is excitatory external input. The results in this work are obtained using MATLAB.

This paper is organized as follows: In Section 2, we describe the two population model (2) with spatio-temporal external input. Section 3 summarizes the general properties of the model (2). In Section 4 we demonstrate numerically the annihilation of persistent activity states for various type of transient external input. The conclusion and discussion of the results shown in Section 5.

II. MODEL

The two-population model with spatio-temporal external input given as

$$v_e = \alpha_e * \omega_{ee} \otimes F_e(v_e - \phi_e) - \alpha_i * \omega_{ie} \otimes F_i(v_i - \phi_i) + G_e \quad (2a)$$

$$v_i = \alpha_e * \omega_{ei} \otimes F_e(v_e - \phi_e) - \alpha_i * \omega_{ii} \otimes F_i(v_i - \phi_i) + G_i \quad (2b)$$

where the functions G_m , for $m = e, i$ are spatio-temporal external inputs, α_m for $m = e, i$ denote temporal kernels which represent the impact of past neural firing on the present activity levels in the network. The parameter τ is ratio between inhibitory and excitatory time constants named *relative inhibition time*, ϕ_e and ϕ_i stand for threshold values for firing, F_m ($m = e, i$) is the firing rate function (these functions constitute one parameter family of non-decreasing and smooth functions which mapped the set of real numbers on to the unit interval $[0, 1]$). The functions F_m ($m = e, i$) are parameterized by a positive steepness parameter β_m . An example of firing rate function is given by

$$F_m(v) = \frac{1}{2}(1 + \tanh(\beta_m v)) \quad (3)$$

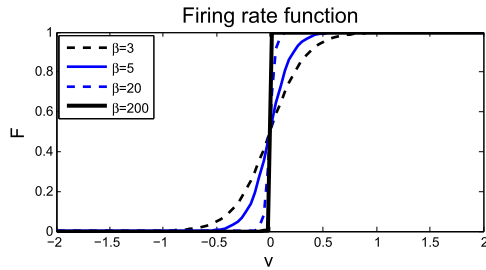


FIGURE 2. The firing rate function for different values of steepness parameter β .

Here β_m is the steepness parameter. For $\beta_m \rightarrow \infty$ ($m = e, i$), the firing rate function F_m approaches the Heaviside function Θ :

$$\Theta(v) = \begin{cases} 0, & v < 0 \\ 1, & v \geq 0 \end{cases} \quad (4)$$

This phenomenon is shown in Fig. 2. The connectivity function is denoted by ω_{mn} , ($m, n = e, i$) in (2) with some important properties like symmetric, normalized, real valued, bounded and positive. The functions ω_{mn} are parameterized by means of *synaptic footprints* σ_{mn} ($m, n = e, i$), i.e.

$$\omega_{mn}(x) = \frac{1}{\sigma_{mn}} \Psi_{mn}(\xi_{mn}), \quad \xi_{mn} = \frac{x}{\sigma_{mn}} \quad (5)$$

Here Ψ_{mn} is non-dimensional scaling function. For example, the Gaussian connectivity function of the type (5) is given as

$$\Psi_{mn}(\xi_{mn}) = \frac{1}{\sqrt{\pi}} \exp(-\xi_{mn}^2) \quad (6)$$

The operator \otimes in (2) defines the spatial convolution integral, given as

$$\begin{aligned} & [\omega_{mn} \otimes F_m(v_m - \phi_m)](x, t) \\ &= \int_{-\infty}^{\infty} \omega_{mn}(x - x') F_m((v_m(x', t) - \phi_m)) dx' \end{aligned} \quad (7)$$

and the temporal convolution integral $\alpha_{mn} * f$ is given as

$$[\alpha_m * f](x, t) = \int_{-\infty}^t \alpha_m(t - t') f(x, t') dt' \quad (8)$$

where the function α_m for $m = e, i$ represent temporal kernels which represent the impact of past neural firing on the present activity levels in the network. This impact decays exponentially with time. The common choice for the functions α_m are [2], [17], [21], [22]:

$$\alpha_e(t) = \exp(-t), \quad \alpha_i(t) = \frac{1}{\tau} \exp(-t/\tau) \quad (9)$$

The system of Volterra equations (2) with exponentially decaying temporal kernels is transformed into the following integro-differential equations [17], [21], [23], [24]

$$\begin{aligned} \frac{\partial v_e}{\partial t} &= -v_e + \int_{-\infty}^{\infty} \omega_{ee}(x - x') F_e(v_e - \phi_e) dx \\ &- \int_{-\infty}^{\infty} \omega_{ie}(x - x') F_i(v_i - \phi_i) dx' + G_e(x, t) \end{aligned} \quad (10a)$$

$$\begin{aligned} \tau \frac{\partial v_i}{\partial t} &= -v_i + \int_{-\infty}^{\infty} \omega_{ei}(x - x') F_e(v_e - \phi_e) dx' \\ &- \int_{-\infty}^{\infty} \omega_{ii}(x - x') F_i(v_i - \phi_i) dx' + G_i(x, t) \end{aligned} \quad (10b)$$

by mean of linear chain trick [25], [26]. Finally, the functions G_m , $m = e, i$ model the spatio-temporal external inputs. The general form for the external input is:

$$G_m(x, t) = C_m R_m(x) g_m(t) \quad (11)$$

Here the temporal function is denoted as $g_m(t)$, $R_m(x)$ spatial part and C_m represent the amplitude of external input. Schematically, the model (10) is illustrated in Fig.1, which expresses the rate of change the activity levels in excitatory and inhibitory populations.

III. GENERAL PROPERTIES OF THE MODEL

In the work of Faye and Faugeras [27], they investigated the asymptotic stability, uniqueness and stability of a solution of non-linear delay integro-differential equations for multi-population model. For properly designed connectivity functions, firing rate function and external input functions, one can prove that the initial value problem of (2) is locally wellposed in the space of bounded continuous functions in a way analogous to Potthast et al. [28] for one-population models, and Faye and Faugeras [27] for multi-population models with axonal and dendritic delay effects incorporated. The boundedness of solutions to Wilson-Cowan type models have been studied in several papers, for details please see: [2], [17], [28]. Here we prove that the solution of initial value problem (10) are uniformly bounded provided both the initial conditions and the external input functions are bounded and continuous.

A. BOUNDEDNESS

In the brief review of boundedness results, the connectivity function ω_{mn} for $m = e, i$ is taken to be normalized and the convolution $0 \leq [\omega_{mn} \otimes F_m(u_m - \phi_m)] \leq 1$ is uniformly bounded for all x and t . The lower and upper bound function for inhibitory and excitatory terms for the present choice of temporal function will be same as in Yousaf et al. [19].

B. SPATIO-TEMPORAL EXTERNAL INPUT

Inorder to investigate the role of each part of the external input on annihilation of ehe persistant activity states, the spatio-temporal external input is splitted into three different parts.

$$G_m(x, t) = C_m R_m(x) g_m(t - t_{m0}), \quad m = e, i \quad (12)$$

where amplitude of external input is C_m , spatial function is denoted by $R_m(x)$. The temporal part is $g_m(t)$ and t_{m0} is for translation axis for $m = e, i$. Moreover, we are using symmetric and continuous functions as spatial part in external input (12).

1) SPATIAL FUNCTIONS OF EXTERNAL INPUT

Spatial part divided into three following types of external input.

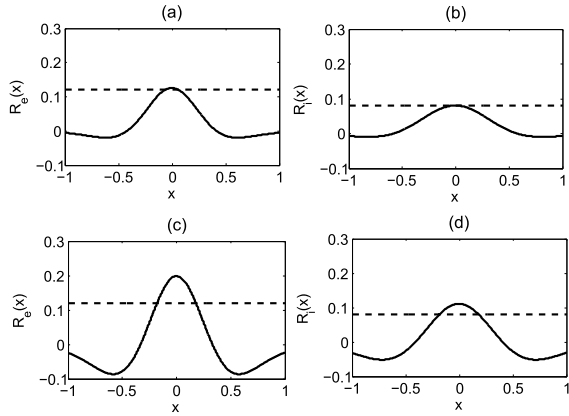


FIGURE 3. Stationary symmetric solution of model 1 with no external input known as bumps [17]. The connectivity function is Gaussian (a) Excitatory narrow bump (b) Inhibitory narrow bump (c) Excitatory broad bump (d) Inhibitory broad bump. The threshold values are $\phi_e = .12$ and $\phi_i = .08$, synaptic footprints are $\sigma_{ee} = .35$, $\sigma_{ei} = .48$, $\sigma_{ie} = .60$, $\sigma_{ii} = .69$.

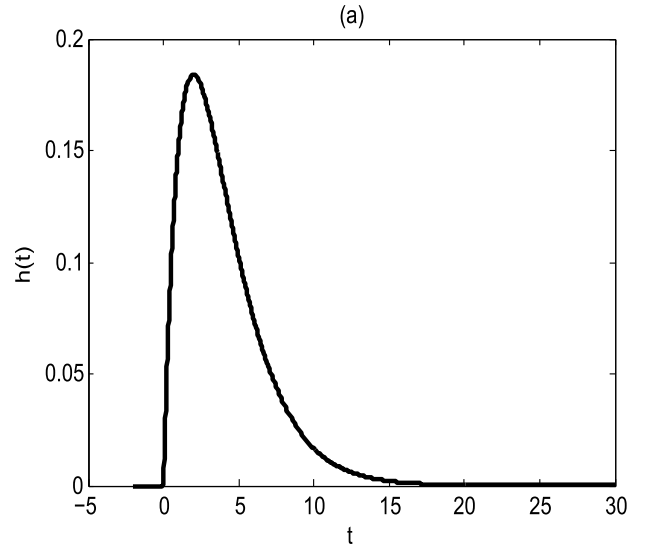


FIGURE 5. Alpha type external input function defined in (15).

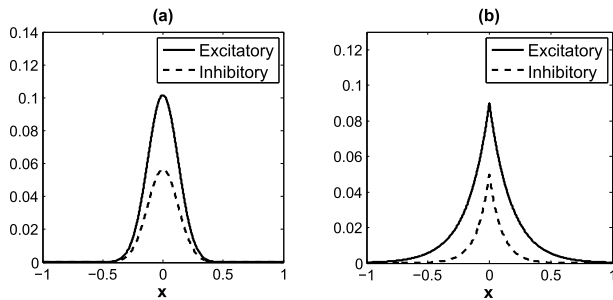


FIGURE 4. Two types of spatial functions, where dashed and solid curves represent excitatory and inhibitory functions, respectively. (a) Gaussian function (13). (b) Exponential function (14).

- 1) The bump pair solution (broad and narrow) determined in Bloomquist et al. [17] and is shown in Fig.3, for more details see [17].
- 2) Gaussian function is defined as:

$$R_m(x) = \frac{1}{\rho_m \sqrt{\pi}} \exp\left(-\left(\frac{x}{\rho_m}\right)^2\right), \quad m = e, i \quad (13)$$

where ρ_m ($m = e, i$) is the width parameter, shown in Fig.4(a)

- 3) Exponential decay function shown in Fig.4(b)

$$R_m(x) = \frac{1}{2} \exp\left(-\left|\frac{x}{\rho_m}\right|\right), \quad m = e, i \quad (14)$$

2) TEMPORAL FUNCTIONS OF EXTERNAL INPUT

In the study of annihilation of the activity in the network the temporal part is assume to be the α -type function is defined as:

$$g(t) = \begin{cases} 0 & t < t_0 \\ d_1 t \exp(-\alpha t) & t \geq t_0 \end{cases} \quad (15)$$

where d_1 is the normalization constant with respect to the area and is given as

$$d_1 = \frac{\alpha^2 \exp(\alpha T)}{\exp(\alpha T) - \alpha T - 1} \quad (16)$$

The α -type function shown in Fig.5. We assume that

$$g(T) = \varepsilon, \Rightarrow \alpha = -\frac{1}{T} \log\left(\frac{\varepsilon}{T}\right) \quad (17)$$

where $\varepsilon = 0.001$ and $T > 0$. This condition (17) imposed due to better comparison between different temporal functions. The α -type function in cortex is observed in Roth *et al.* [1] during some experimental results. Axonal calcium imaging is a major technique used in neuroscience, Roth *et al.* [1] used this technique to measure the information provided visual cortex by the pulvinar equivalent in mice, the lateral posterior nucleus (LP), as well as the dorsolateral geniculate nucleus (dLGN). In [1], they have discussed the thalamic inputs which provide motor, multiple visual and visumotor signals to L1 of mouse visual cortex V1, in this study α -type function is observed in the experimental data in Roth *et al.* [1]. Here V1 is primary visual cortex that receives the sensory inputs from the thalamus and L1 is layer in V1.

IV. RESULTS

In this section, annihilation of persistent activity states is discussed under the influence of spatio-temporal external input (12). Before digging into results its important to mention that we are in the parameter regiem, where two bump pair solutions exist. This is the case for no external input, the most generic case, two stationary and symmetric solutions exist [17], one of which is nammed as narrow and other is broad. The stability of these bumps depend on the parameter relative inhibition time costant τ . The narrow bump is unstable for values of τ , while the broad bump pair is stable for small and moderate values of relative inhibition time $\tau_{cr} = 3.01$.

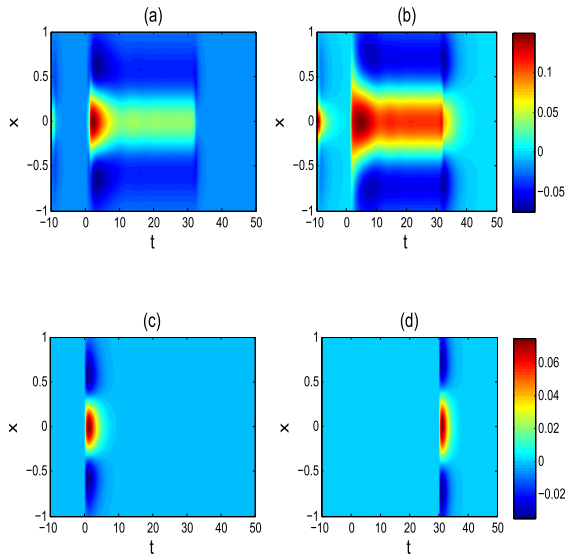


FIGURE 6. Emergence and annihilation of the activity using α -type temporal function (15), the function R is assumed to be broad bump in the external input. The emergence and annihilation of (a) excitatory and (b) inhibitory activity. (c) Excitatory external input. (d) Inhibitory external input. The corresponding parameters are given in table 1. The rest of parameters are same as used in Fig.3.

Once the activity is evoked or emerged in the system, it will converge to a broad bump (attractor state). Now the next challenge is to annihilate this activity.

We are interested to demolish the stable activity states in working memory using α -type external input (15). In Yousaf et al. [19], they have investigated the bump pair formation in presence of external input in two population Wilson Cowan type. They also have investigated the annihilation of activity by using external input which is spatially and temporally dependent, but they used different external input with triangular temporal part motivated by Pinto et. al [29]. We will investigate the annihilation by considering external input with more realistic and natural choice of α -type temporal function (15) in the external input.

A. ANNIHILATION OF THE ACTIVITY

In this section, we are investigating the different aspects of external input to annihilate the activity in the network. We found that the persistent activity in the system is annihilated by using, either inhibitory (i.e. $G_e(x, t) = 0$) or excitatory external input (i.e. $G_i(x, t) = 0$). The activity is also disappeared by a using a combination of both (excitatory and inhibitory) inputs but with stronger inhibitory input as compare to excitatory.

1) INHIBITORY EXTERNAL INPUT EFFECT

Here we focus on the effect of inhibitory external input only (by keeping excitatory input $G_e(x, t) = 0$) to annihilate the activity. Fig.6 which shows the emergence and annihilation of the activity using α -type temporal function ($g_m(t)$) and broad bump $R_m(x)$ as a spatial function.

TABLE 1. Parameters sets used to demonstrate the annihilation of the activity given in Fig.6.

Population	t_0	T	C
Excitatory	0	12	6
Inhibitory	30	40	1

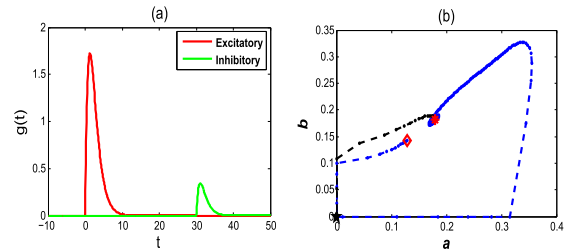


FIGURE 7. Temporal functions for the external input (12) and pulse width coordinate planes corresponding to the activities observed in Fig.6. (a) α -type temporal function $g(t)$ (b) Corresponding pulse width coordinates in the process of emerging and annihilation of activity $g(t)$ and $R(x) =$ broad bump as spatial function. Red and black color diamonds (\diamond) represent the initial and final state, while the symbol asterisk ($*$) stands for the broad bump solution.

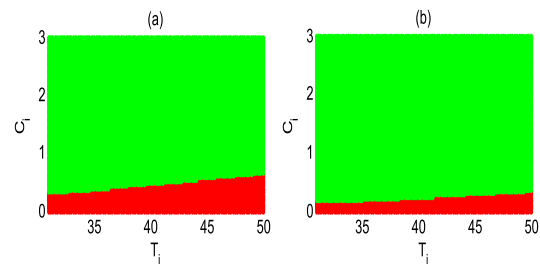


FIGURE 8. T_i versus C_i and the stable activity erase by the presence of inhibitory external input in the system for α -type temporal function (15) for different values of τ (a) $\tau = 1.5$ and (b) $\tau = 2.5$. Successful deactivation and failure of deactivation represented by green and red color respectively. Here $t_{i0} = 30$, so that's way x-axis start at 30.

The parameter-plane T_i versus C_i for the two intermediate values of τ is investigated in Fig.8, where green and red color regions show successful annihilation and unsuccessful annihilation, respectively. The erasure process of persistent activity for the larger values of τ becomes easier as expected, shown in Fig.8. e.g. for the value of $\tau = 2.5$ gives larger green region as compare to $\tau = 1.5$. As we move more closer to $\tau_{cr} = 3.1$, the stable activity will become itself more unstable and a very small input will erase it. The corresponding pulse width coordinates are shown in Fig.7 showing that its relatively easier to annihilate the activity than emergence.

2) EXCITATORY EXTERNAL INPUT EFFECT

Excitatory external input can also be used to erase the activity using alpha type temporal functions $g(t)$ in the external input [30]. This is due to the fact that exciting more excitatory neurons in the network disturbs the balance between excitatory and inhibitory population connections. Since each neuron has its refractory period which makes it impossible for previously excited neurons to fire action potential and in the same time excitation increases inhibitory response.

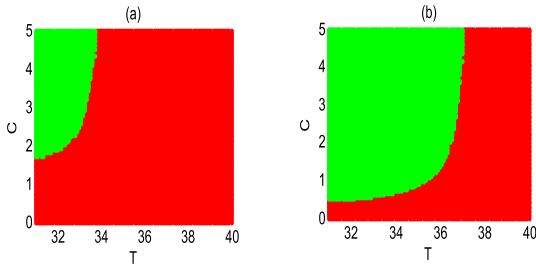


FIGURE 9. Annihilation of the persistent activity states using excitatory external input using α -type function (15) as temporal function and broad bump as a spatial function for different inhibitory time constant τ . The CT -plane for (a) $\tau = 2.2$ (b) $\tau = 2.4$.

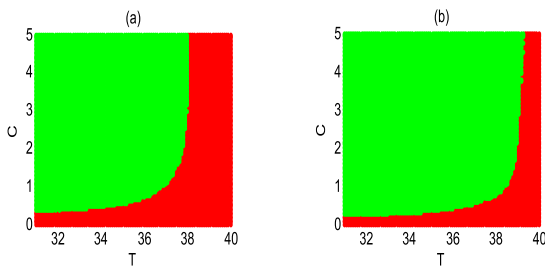


FIGURE 10. Annihilation of the persistent bump state excitatory external input using α -type function (15) as temporal function and broad bump as a spatial function for different inhibitory time constant τ . The CT -plane for (a) $\tau = 2.5$ (b) $\tau = 2.6$.

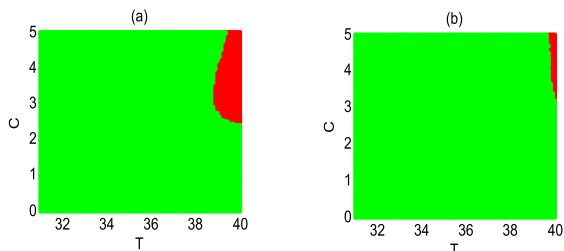


FIGURE 11. Annihilation of the persistent bump state excitatory external input using α -type function (15) as temporal function and broad bump as a spatial function for different inhibitory time constant τ . The CT -plane for (a) $\tau = 2.8$ (b) $\tau = 3$.

This ultimately increases the inhibitory activity which in turn causes the annihilation of activity in the network. This is the basic phenomena about excitatory external input onto excitatory population.

The annihilation of persistent bump state under the influence of α -type temporal part in the external input, broad bump is used as a spatial part. The dependence of τ can be observed from figures 9,10,11, eraser of persistent state of activity is easier for larger τ . The eraser mechanism is due to the fact that the value of τ is much smaller then τ_{cr} , e.g., $\tau = 2, 2.2$, no such annihilation was seen in Fig. 9. For the moderate values of τ e.g., $\tau = 2.5, 2.6$, the remarkable difference is shown in Fig.10, but if we choose the value of τ closer to τ_{cr} e.g., $\tau = 2.8, 3$ for which the persistent state of activity is no longer stable shown in Fig.11. It is obvious from the results shown in Fig.8 and Fig.10, the minimum value required to

TABLE 2. Minimum Values of Amplitude in the external input to annihilate the activity. C_z and C_y stands for minimum aplitude in present work and Yousaf et. al. [19], respectively. The minimum time duration and amplitude required to annihilate the persistent activity states.

Input	τ	Min. C_y	Min. C_z
Inhibitory	1.5	0.451	0.303
Inhibitory	2.5	0.224	0.1515
Excitatory	2.5	0.614	0.303

annihilate the activity for both the cases i.e. in inhibitory and excitatory annihilators are smaller than Yousaf et. al. [19]. The comparizon is given in Tab. 2. Thus the present choice of temporal function (Alpha function) is a better choices as compare to triangular one. Present work can be beneficial for those working on brain modeling. This work may provide useful information for designing functions which best fits for external inputs in brain.

V. CONCLUSION AND DISCUSSIONS

The present study is a continuation of an interested and one of best studied area in neuro-science, how to Annihilate the working memory (Bumps solutions) in the prefrontal cortex by using a transient spatio-temporal external input in two population neural field model. Before going into further detail its important to mention that model with no input corresponds atmost two stationary and symmetric solutions called bumps (self sustained activity). One is named as narrow and the other as broad. The stability of these two bump pairs depend on relative inhibition time τ . In this work we have used broad bump as a spatial part of external input. It is stable for small and moderate values of $\tau < \tau_{cr} = 3.01$ [17]. We are in the parameter regime of two bump pair solutions. So if activity is stable then it will converge to broad bump as shown in Fig.7. The idea of external input was introduced in Yousaf et al. [19], where they investigated the annihilation of the activity under the effect of external input with triangular function as the temporal part [19], [29].

In the present study, We have investigated the effect of α -type external input on annihilation of self sustained activity states instead of triangular one. This choce of external input is more natural as observed by Roth et al. [1]. The role of different parameters of the external inputs have been investigated to annihilate the stable activity (broad bump/ self sustained activity) in the system. The most important fact in our observation is α -type function which gives much better results instead of triangular function e.g. the green region is appeared more earlier as compared in case of triangular function as shown in Fig.8 to Fig.11.

it observed that, for larger values of relative inhibition time constant τ , activity state is annihilated quickly as compare to the case $\tau \leq \tau_{cr} = 3.01$ for both cases (Inhibitory or excitatory input eraser). This is due to the fact that the activity near the critical time contant is already about to die and only a very small external input is enough to annihilate it, while for comparitively smaller values of $\tau < \tau_{cr}$, strong external input is required to annihilate the activity Fig.8 to Fig.11.

It is obvious from results (Fig.8) For the choice of Inhibitory eraser ($G_e = 0, G_i > 0$), since excitatory and inhibitory activities are already in the system and there is a balance between them. This balance is resulted in the form of stationary solutions (bumps). Therefore, a small inhibitory external input is sufficient disturb this ballance which results the annihilation of activity. If this process is repeated for larger values of τ , its even more easier to annihilate the activity. For the case of excitatory external input as an eraser its not that much easier to annihilate the activity as compare to inhibitory eraser. This is because of that each neuron has its refractory period which makes it impossible for previously excited neurons to fire action potential and in the same time excitation increases inhibitory response. This ultimately increases the inhibitory activity which in turn causes the annihilation of activity in the networkm (Fig.11).

All above discussion shows that the smooth choice of temporal part in the external input is a better chaoice as compare to temporal one. It is obvious from the results in (Fig.8–Fig.10), the minimum value of the amplited of external input required to annihilate the activity for both the cases (inhbitory and excitatory annihilators) is smaller as compare to Yousaf et. al. The corresponding comparizon is given in Tab.2.

In future, the effect of external input on generation and annihilation of self sustained activity states can further be investigated for some other choices of temporal functions which are enriched with more importnat parameters.

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