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A simple but powerful measure of market efficiency

Vu Le Tran*, Thomas Leirvik

Nord University Business School, Universitetsaléen 11, Bodø 8049 Norway

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ABSTRACT

We construct a simple measure to quantify the level of market efficiency. We apply this measure to investigate the level of market efficiency and analyze its variation over time. The main contribution of the new measure is that it makes it easy to compare market efficiency across assets, time, regions, and data frequencies. We find that markets are often efficient, but can be significantly inefficient over longer periods. Our empirical results indicates that in many periods of major economic events, financial markets becomes less efficient. This corroborates earlier results on market efficiency, and simplifies interpretation and comparisons.

1. Introduction

In this paper, we derive a new measure to quantify the level of market efficiency. We use the term *Adjusted Market Inefficiency Magnitude* (*AMIM*). The *AMIM* increases as market efficiency decreases, and decreases as market efficiency increases. The maximum level of *AMIM* is 1, which implies a highly inefficient market. There is no lower boundary, but if the measure produces a negative number, the market is assumed to be efficient. This makes interpretation very simple: a positive *AMIM* indicates an inefficient market, and a negative *AMIM* indicates an efficient market. The *AMIM* is very easy to compute, and is computationally inexpensive. This implies that comparisons over time, assets, asset classes, and geographical regions are carried out with ease. We show that it has several advantages over existing measures of market efficiency, and are able to detect periods of the economy that is known for much uncertainty about prices and values.

The Efficient Market Hypothesis (EMH) is based on the idea that an asset's price should reflect all relevant information and that economic agents, and thus the financial markets, are rational. The EMH was introduced in the seminal paper by Fama (1970). Market efficiency is usually described in three levels: weak, semi-strong, and strong form. There is a vast amount of literature in the field to test if markets are efficient in both weak form and strong form, see for example the papers by Fama (1970), Fama (1991), and Yen and Lee (2008) for more details. The consequence of market efficiency is that future prices, and returns, are random and should not be possible to predict. This randomness can be modeled by a random walk, which is a mathematical description of a stochastic process where each increment is random and independent of earlier increments. That stock returns are not totally random has been shown in several empirical papers, for example (Reinganum, 1983; De Bondt and Thaler, 1985; Jegadeesh and Titman, 1993). In this paper we derive an estimator of the level of market efficiency. The measure, *AMIM*, makes it possible to quantify how efficient the market is, and determine whether it should be classified as inefficient or efficient.

According to Lo (2004), markets are not always rational, nor optimal, but sometimes heuristic, and emotional. Lo (2004, 2017) proposes a concept called *Adaptive Market Hypothesis* (AMH), and suggests that we can use evolutionary models for studying the markets. The assumption is that financial markets are not static objects, but adapt to a changing environment via simple heuristics. If so, market efficiency is also dynamic and can change over time. Tests of the AMH for different markets, assets, and frequencies of

* Corresponding author.
E-mail addresses: vu.l.tran@nord.no (V.L. Tran), thomas.leirvik@nord.no (T. Leirvik).

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observations, has been carried out by numerous authors, see for example (Urquhart and Hudson, 2013; Urquhart and McGroarty, 2014; 2016), and the references therein. These papers largely conclude that markets are adaptive, thus market efficiency varies over time. This variation of market efficiency in time has been studied thoroughly, see, for example the paper by Lim and Brooks (2011) for a survey on the matter. To study the variation of market efficiency, one often applies moving windows that consists of daily, weekly or monthly data. These windows are then applied to investigate whether prices behave according to a random walk process, see for example (Choi, 1999; Kim and Shamsuddin, 2008). The usual focus is on answering the question: *Are markets always efficient?* We enlarge the discussion of market efficiency by addressing the questions: *How large is the inefficiency level, and how does it vary over time?* We find that market efficiency is indeed varying over time. The *AMIM*-measure captures periods with much uncertainty and hence difficulties in determining what information should be incorporated in prices.

The papers by Ito et al. (2014, 2016) and Noda (2016) derive a measure to quantify market efficiency. The authors investigate the variation of the efficiency level by estimating the auto-correlation in stock monthly return through a time variant auto-regressive (TV-AR) model, which is designed to capture a set of auto-correlation coefficients in each observation in time. In particular, Noda (2016)'s measure aim to capture the time-varying degree of market efficiency (*TIME*). From the return auto-correlation coefficients, *TIME* captures the time-varying degree of market efficiency, and hence aims to measure the inefficiency level of the market. In this paper, we extend the results in two main areas: first, our model does not depend on the frequency of the data in the sample, whereas the existing models are more suitable for low frequency data. Second, we do not choose the number of autocorrelation lags in advance. Indeed, Ito et al. (2016, 2014) and Noda (2016) model is challenging to apply to high frequency data when the number of estimations can be up to millions each day.

The *AMIM* is derived using both the autocorrelation coefficients of a time series of stock returns and the confidence intervals of these coefficients. The measure is thus robust against insignificant autocorrelation. Specifically, we start with a measure that we denote market inefficiency magnitude (*MIM*), and use its confidence interval to adjust the *MIM* to produce *AMIM*. Therefore, our measure is also a type of test of market efficiency. *MIM* builds upon (Noda, 2016)'s measure called time-varying degree of market efficiency (*TIME*). The *TIME* measure has many novel contributions, but is relatively difficult to compute. Moreover, a more serious drawback is that the denominator of *TIME* can be close to zero, equal to zero, or even change sign. Thus, there is a discontinuity that is likely to occur, and which will make inference troublesome. *MIM* addresses both drawbacks of *TIME*, and offers a simple solution to make analysis of market efficiency very simple. Our approach also provides a quicker way to find the confidence interval of the inefficiency magnitude. The main reason for this is because our confidence intervals can easily be computed from the sample under investigation. This is a major contribution, as comparable measures, for example the one applied in Noda (2016), relies on simulations and bootstrapping. Finally, *AMIM* helps us to easily compare the inefficiency magnitude between different assets, across different point in time.

We construct *AMIM* through 4 steps. The first step is to estimate the auto-correlation coefficients in the return series through standard regression methods, and then standardize them. The second steps is to derive a raw measure of the market inefficiency magnitude (*MIM*). The third step derives the confidence interval under the null hypothesis of efficient markets for *MIM*. In the final step, we adjust *MIM* with its confidence interval to derive our measure *AMIM*. The measure is a convenient test score of market inefficiency level; $AMIM > 0$ means that the market is significantly inefficient while $AMIM < 0$ means that we cannot reject the null hypothesis of efficient markets. By design, the inefficiency magnitude will positively correlated with *AMIM*.

Second, our measure can be tested easily on samples that consist of many different assets over time by computing a unique set of confidence intervals. This also decrease the computational burden, especially when analyzing big data. In contrast, even though *TIME* is a very good measure of the inefficiency magnitude, its design implies that it can only be tested sample by sample.

Third, our measure is uniformly continuous, meaning that there are no discontinuities, in all levels of auto-correlation. This is very important when conducting inference and interpreting the results. An inherent challenge with the measure applied in Ito et al. (2016, 2014) is that it is a fraction with sums of the autocorrelation coefficients included. Not only can the denominator be zero, but also the summation can make positive auto-correlation canceling out negative-correlation. In this paper, we address these issues and compute the absolute values of the auto-correlation coefficients before making any summation.

To test the performance of our measure, we estimate the *AMIM* for some US stock market indexes. Concerning robustness test and compatibility with other market efficiency estimators, we apply *AMIM* to the same dataset studied in Noda (2016). We show that *AMIM* can capture similar result in Noda (2016). We also do a simulation to check the power and the size of our test *AMIM* and make some computations to show that *AMIM* is very reasonable in terms of producing estimates that corresponds to financial theory. The results also show that market efficiency varies considerably over time, and reflects major economic events. This is also very important, as, according to the AMH, one can expect market efficiency to change over time. From an economic point of view, the changes should not be completely random, but be linked to economic conditions around the world. Indeed, for the sample under consideration in this paper, we can disentangle major economic events from the movement in the *AMIM*. *AMIM* also provides the main results of Noda (2016) for the Japanese markets.

2. Model and estimation methods

According to Fama (1970) stock prices should, under the Efficient Market Hypothesis (EMH), reflect all relevant information in the market. Therefore, if we are in period t, the return in the next period $t + 1$ should not be predictable. Hence, following the EMH, an auto-regressive process $AR(q)$ of returns (r_t) on its own lags cannot explain the dynamics of returns over time. For example, if EMH holds, then the $AR(q)$ model

$$r_t = \alpha + \beta_1 r_{t-1} + \beta_2 r_{t-2} + \cdots + \beta_q r_{t-q} + \varepsilon_t \tag{1}$$

should have coefficients ($\beta_1, \beta_2, \ldots, \beta_q$) that are all close to zero, or at least insignificantly different to zero. If the EMH does not hold, the β coefficients are (significantly) non-zero. Lo (2004) used the first auto-regressive coefficient to characterize the inefficiency level. If there are more lags with significant coefficients, then there is even more evidence against a strongly efficient market. Our aim is to construct a measure that takes the auto-correlation coefficients into account. The Adjusted Market Inefficiency Magnitude, $AMIM_t$, is constructed following four steps:

2.1. Normalizing the auto-correlation coefficients (Step 1)

Let $\hat{\beta}$ be a column vector which contains the estimated coefficients $(\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_q)'$ from Eq. (1). $\hat{\beta}$ will be asymptotically distributed as follows:

$$\hat{\beta} \sim N(\beta, \Sigma). \tag{2}$$

Here β is the unknown true beta vector, i.e. the vector of auto-correlation coefficients. Σ is the asymptotic co-variance matrix of the estimated $\hat{\beta}$ vector, which can be separated into two triangular matrices by Cholesky decomposition as: $\Sigma = LL'$. The estimated coefficients have different standard errors and can be correlated. Therefore, we standardize the $\hat{\beta}$ vector multiplying it by the inverse of the triangular matrix L. Thus, the standardized beta is given as:

$$\hat{\beta}^{standard} = L^{-1}\hat{\beta}. \tag{3}$$

Under the null hypothesis that market is efficient ($\beta = 0$), then $\hat{\beta}^{standard}$ should be normally distributed as follows:

$$\hat{\beta}^{standard} \sim N(0, I) \tag{4}$$

Where I is an identity matrix. Therefore, the normalizing process in Eq. (3) helps us in two ways. First, by multiplying $L^{-1}\hat{\beta}$, it makes each component in $\hat{\beta}^{standard}$ independent. Second, the standardized coefficients are very convenient for testing any measures constructed from $\hat{\beta}^{standard}$.

2.2. The magnitude of market inefficiency (Step 2)

In this section, we construct the unadjusted, or raw, measure of market inefficiency. To calculate the inefficiency level, we first construct the Magnitude Market Inefficiency, MIM_t, as follows:

$$MIM_t = \frac{\sum_{j=1}^{q} |\hat{\beta}_{j,t}^{standard}|}{1 + \sum_{j=1}^{q} |\hat{\beta}_{j,t}^{standard}|} \tag{5}$$

As we are interested in violations of the assumption that the auto-regressive coefficients are zero, we use the absolute value to eliminate the sign effect. MIM_t is the Market Inefficiency Magnitude at time t whereas $\hat{\beta}_{j,t}^{standard}$ is the j^{th} auto-correlation coefficient in Eq. (1) after standardization. Following the above construction, the auto-correlation $\sum_{j=1}^{q} |\hat{\beta}_{j,t}^{standard}|$ is positively related to the Market Inefficiency Magnitude. The variation of MIM_t is smooth from 0 (very efficient market) to almost 1 (inefficient market). So when comparing two stocks, the one having a higher MIM_t will be more affected by the past than the one having lower MIM_t.

Noda (2016) has the similar approach of using the auto-regressive coefficients to compute the Market Inefficiency Magnitude, though different formula for market efficiency, $TIME_t$, is applied. $TIME_t$ is given as:

$$TIME_t = \left| \frac{\sum_{j=1}^{q} \hat{\beta}_{j,t}}{1 - \sum_{j=1}^{q} \hat{\beta}_{j,t}} \right| \tag{6}$$

Eq. (6) uses the non-standardized coefficients from Eq. (1). Hence, it will be inconsistent when $\sum_{j=1}^{q} \hat{\beta}_{j,t} \in [0, 1]$. Indeed (Ito et al., 2014)'s ratio will converge to ∞ when $\sum_{j=1}^{q} \hat{\beta}_{j,t}$ is around 1. An interesting implication of this is that sometimes markets are oddly more efficient when the auto-correlation level is high (i.e. $\sum_{j=1}^{q} \hat{\beta}_{j,t} = -2$) than when the auto-correlation level is low (i.e. $\sum_{j=1}^{q} \hat{\beta}_{j,t} = 0.6$). To see this, we get $TIME_t = \left|\frac{-2}{1-(-2)}\right| \approx 0.667$, indicating a level of market efficiency of 0.667. In a more efficient case, for example when the sum of auto-correlation coefficients equals 0.6, the $TIME$ measure is $TIME_t = |\frac{0.6}{1-0.6}| = 1.5$ which indicates a less efficient market even though the sum of autocorrelation coefficients having a very different meaning.

Although a simple example, the consequence is that $TIME_t$ cannot be used for inference in this case. Furthermore, a large-scale analysis using $TIME_t$ will need to be accompanied by an individual check of each case to make sure that the result makes economic sense. Furthermore, Eq. (6) sums all the raw coefficients. This can also make the measure inconsistent. For example, if we have two

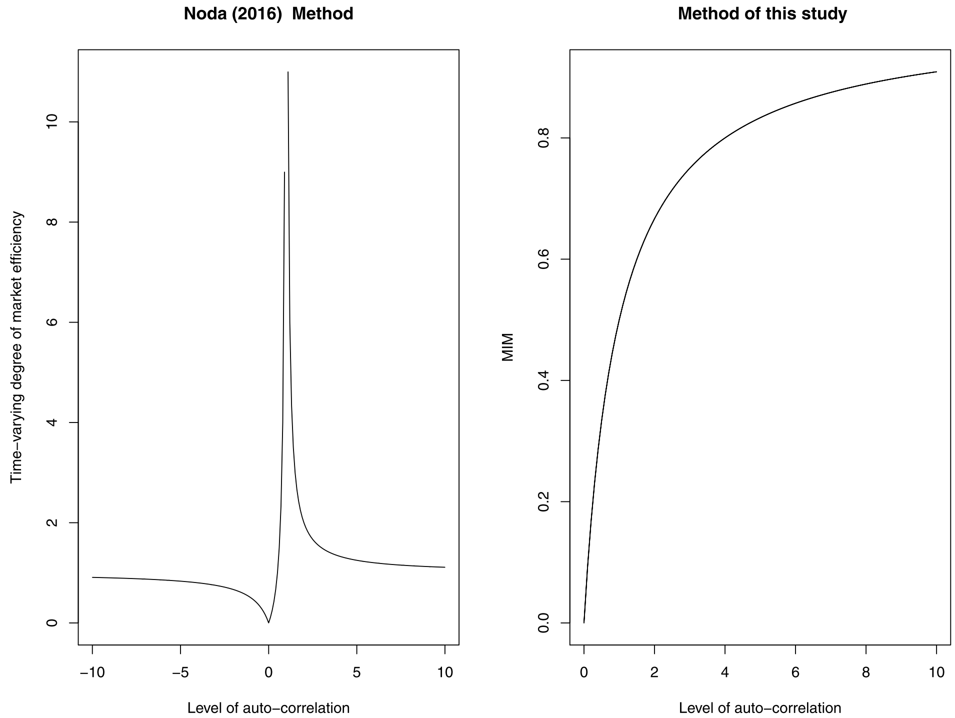


Fig. 1. Market Inefficiency Magnitude MIM_t with auto-correlation level $\sum_{j=1}^{q} \hat{\beta}_{j,t}$ (Noda (2016) methods), and $\sum_{j=1}^{q} |\hat{\beta}_{j,t}^{standard}|$ (our methods).

auto-correlation coefficients, $\beta_1 = -0.5$, $\beta_2 = 0.5$, the Noda (2016)'s measure will equal to zero which indicates an efficient market. Therefore, we take the absolute value of $\hat{\beta}_{j,t}^{standard}$ before sum up all the coefficients. This process will help to avoid the eliminating effect between positive and negative coefficients.

Moreover, we use the standardized $\hat{\beta}$ coefficients before compute MIM_t. This step will be crucial to compute the confidence interval in the following step. By standardizing the auto-correlation coefficients, we can derive a unique set of confidence intervals for *MIM* under the null hypothesis of efficient markets, thus reducing the computational burden.

Fig. 1 illustrates the difference of the two methods of computing the market efficiency level where the level of auto correlation is $\sum_{j=1}^{q} \hat{\beta}_{j,t}$ in Noda (2016) and $\sum_{j=1}^{q} |\hat{\beta}_{j,t}^{standard}|$ in our method.

Second, we use a non-overlapping window method to compute the auto-correlation coefficients of each time interval[1]. Ito et al. (2014, 2016) and Noda (2016) used a *time-varying auto-regressive model (TV-AR)* to compute the auto-correlation coefficients. The latter model will give a set of coefficients for each observation in time. For example, if we have 1 observation/second then Ito et al. (2014, 2016)'s model will have $3600 \cdot q$ coefficients for each hour, where q is a constant number of lags. In brief, the total number of coefficients is equal to the number of observation times q. Thus, this can be computationally intensive when the number of observations increases, in particular using high frequency data.

2.3. Building confidence intervals (step 3)

The Market Inefficiency Magnitude is by construction between 0 and 1. However, the raw value of the MIM_t can give us a false impression of the market efficiency. Due to an absolute process to eliminate the sign effect in step 2, the MIM_t will be, by construction, positively correlated with the number of lags in the Eq. (1). Even for markets that are very efficient, it is likely that MIM_t can be very high. This is undesirable.

To correct for this, we compute the confidence interval of MIM_t. To get the confidence interval under the null hypothesis of efficient markets ($\hat{\beta}_{j,t}^{standard} = 0$), we can either use convergence of random variables, or simulations. The former approach is quite tricky with a function as *MIM* while the latter is more reasonable.

Because all $\hat{\beta}_{j,t}^{standard}$ are standard normal, knowing the number of lags in Eq. (1), we can identify the confidence interval of MIM_t under the null hypothesis through simulation. We first simulate 100 000 observations for each $\hat{\beta}_{j,t}^{standard}$ following a standard normal

[1] In the empirical part, we also apply the rolling window methods and having the same results qualitatively.

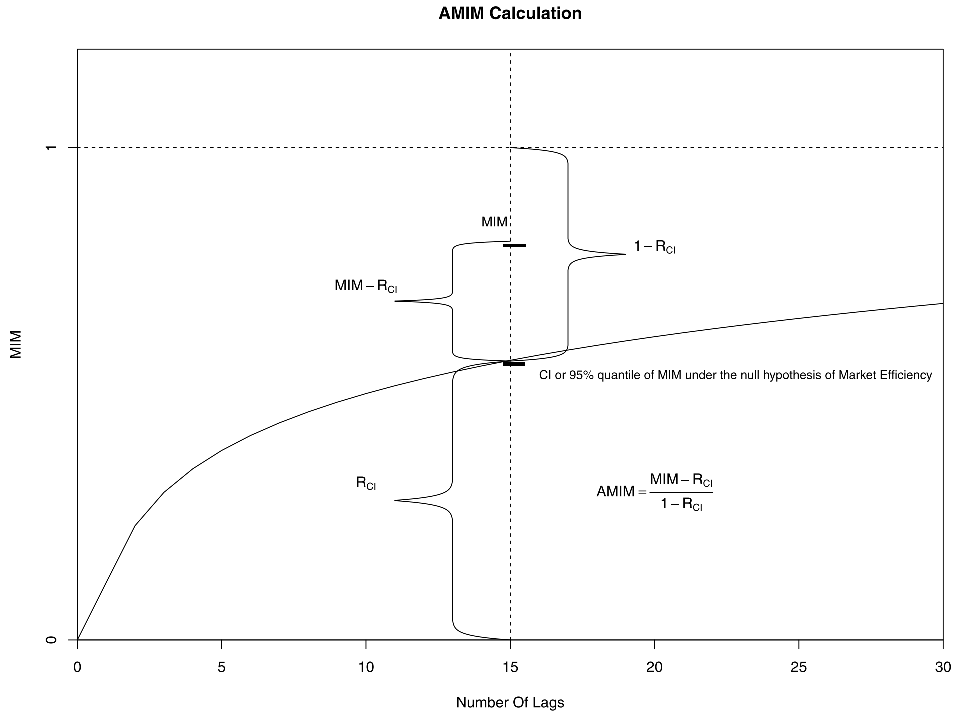


Fig. 2. Illustration of Adjusted Market Inefficiency Magnitude Calculation ($AMIM = \frac{MIM - R_{CI}}{1 - R_{CI}}$). The curvature line is the upper bound of 95% confidence interval of MIM under the null hypothesis of efficient markets.

distribution. Based on $\hat{\beta}_{j,t}^{standard}$, we compute *MIM*. For each number of lags we have 100 000 observations of *MIM* under the null hypothesis of market efficiency. After that, we find the 95^{th} percentile of *MIM*. Because *MIM* is only varying in [0, 1), the interval between this 95^{th} percentile and 0 is the 95% confidence interval of *MIM* under the null hypothesis. This confidence interval is thus unique for each number of lags. This again gives us a table of confidence intervals (CI) which can be used in a different context. See Table 3 in the appendix for details of the computation of the confidence intervals.

2.4. The adjusted market inefficiency magnitude (step 4)

In this section, we derive the adjusted market inefficiency magnitude, *AMIM*. From the previous step, we know the 95% confidence interval of MIM_t under the null hypothesis of efficient markets. First, we compute the range of the confidence interval, basically the distance between zero and the 95% quantile of *MIM* under the null hypothesis of market efficiency. We then adjust the *MIM* by first subtracting the range of the CI from the *MIM*; $MIM - R_{CI}$, then we divide this distance between MIM_t and R_{CI} with the distance between the theoretical maximum value of *MIM*, which is one, and R_{CI}. Mathematically, this is given as:

$$AMIM_t = \frac{MIM_t - R_{CI}}{1 - R_{CI}}. \tag{7}$$

Because $MIM_t < 1$, the estimates of $AMIM_t$ and R_{CI} are always less than one as well. MIM_t is also always greater or equal to zero, which implies that *AMIM* can be negative. In fact, whenever $AMIM > 0$ the market is inefficient, whereas when $AMIM < 0$ the market is efficient. Fig. 2 gives an illustration of *AMIM* formula. Loosely speaking, *AMIM* only stresses on the inefficient part of *MIM*, which passes the null hypothesis CI. The $AMIM_t$ is thus more reliable than MIM_t because it penalizes the mechanical variation of MIM_t due to high number of lags in $\hat{\beta}_{j,t}^{standard}$. We divide $(MIM - R_{CI})$ by the difference between one and R_{CI} to give a common ground for comparison between stocks. Indeed, different stocks will have different *MIM* values with again different R_{CI}'s at different point in time. Adjusting for R_{CI} gives us the same comparison criteria for all assets. By this construction when $AMIM_t < 0$, we cannot reject the null hypothesis that markets are efficient. If $AMIM_t > 0$ we can say, markets are significantly inefficient. Markets are more inefficient when $AMIM_t$ increases.

3. The size and power of *AMIM*

To investigate the size and power of *AMIM*, we carry out a Monte Carlo simulation. We simulate an AR(1) model for returns where ρ is the auto-correlation coefficient. We set the return innovation as normally distributed with mean 0.03 % a day and a daily standard deviation of 1 %. This is the typical long-run mean and standard deviation for the S&P 500 index. We set ρ to be (0, ± 0.3,

Table 1
Simulation of AMIM. We simulate an AR(1) model for returns where ρ is the auto-correlation coefficient. We set the return innovation as normal distributed with mean 0.03 % a day and a daily standard deviation of 1 %. For each ρ value, we simulate 100 000 batches. Each batch consists 200 observations. Each batch gives 1 value of *AMIM*. N is number of observations of *AMIM*. Q is the quantile of the *AMIM* distribution.

ρ	N	Q0.01	Q0.05	Q0.1	Q0.25	Q0.5	Q0.75	Q0.9	Q0.95	Q0.99
0	100 000	−0.22	−0.143	−0.061	0	0	0	0.052	0.127	0.235
0.3	100 000	0.104	0.206	0.251	0.316	0.392	0.467	0.525	0.555	0.603
0.5	100 000	0.296	0.359	0.391	0.445	0.506	0.561	0.604	0.627	0.667
0.7	100 000	0.345	0.402	0.432	0.481	0.534	0.582	0.622	0.644	0.681
0.9	100 000	0.322	0.38	0.41	0.458	0.51	0.558	0.599	0.621	0.66
−0.3	100 000	0.125	0.217	0.26	0.324	0.399	0.474	0.534	0.563	0.609
−0.5	100 000	0.297	0.359	0.391	0.446	0.506	0.562	0.605	0.629	0.671
−0.7	100 000	0.341	0.399	0.429	0.48	0.533	0.582	0.622	0.644	0.683
−0.9	100 000	0.319	0.377	0.407	0.457	0.509	0.558	0.599	0.622	0.661

± 0.5, ± 0.7, ± 0.9) respectively. With ρ = 0, we have the efficient market case. For this case, we expect *AMIM* to be smaller or equal 0. For other cases ρ ≠ *0*, we expect *AMIM* to be greater than 0. For each ρ value, we simulate 100 000 batches. Each batch consists 200 observations. Each batch gives one value of *AMIM*. We end up with 100 000 *AMIM* values for each ρ value. Table 1 gives the estimates of the simulation. Here, *N* is the number of observations of *AMIM*, and *Q* is the quantile of the *AMIM* distribution.

In the Efficient Market case $\rho = 0$, hence *AMIM* is supposed to be smaller than, or equal to zero. 90% of our simulated *AMIM* is smaller than 0.052. Subsequently, 95% *AMIM* is smaller than 0.127. These results show that in the efficient market case, we do not make a huge mistake with *AMIM*. Even in case we make a mistake, the error is not big because *AMIM* is wrongly positive, but it is small and very close to zero. Therefore, if we wrongly conclude that markets are inefficient instead of efficient, the wrong inefficient level is also small which makes less harm.

In the inefficient market case, for example $\rho = 0.3$, *AMIM* is supposed to be greater than zero. 99% of our simulated *AMIM* is greater than 0. The logic goes so on with different ρ values. These results show that when market is not efficient, the *AMIM* measure makes very little to no mistake of discovering it. In summary our *AMIM* measure performs quite well in simulation to detect the size and power of the test. Of course, our drawback is not considering all the available alternative hypotheses. Such a full analysis is out of scope of this research and is worth investigating in future research.

4. Data

To have a better comparison with Noda (2016)'s measure, we use the same dataset as they applied. The dataset is price levels for the Tokyo Stock Price Index (TOPIX) and the Tokyo Stock Exchange Second Section Stock Price Index (TSE2). Stocks in TOPIX and TSE2 indexes are different. The TOPIX index has a much higher market capitalization and trading volume than TSE2. The data source is Bloomberg. We compute the log return r_t from the daily prices, p_t, thus $r_t = \log(p_t/p_{t-1})$.

We also investigate the efficiency level of US stock markets for both small stocks and large stocks. We use S&P 500 index as a proxy for large stocks. For US small stocks, we first sort stocks belonging to AMEX, NYSE, NASDAQ exchanges from CRSP database into decile portfolios based on market capitalization, then taking the value weighted return of the first portfolio as an index portfolio for small stocks. We call this portfolio as CAP1.[2]

We use these datasets to compute *AMIM*. The frequency of the data is daily, and cover the period from 1962 through 2017. We compute the auto-correlation coefficients ($\hat{\beta}$) for each index in each year using all daily return. To identify the number of lags of each time interval, we use Akaike information criterion (AIC)[3] We required that each year having at least 200 observations to run the regression. For each year, the model estimates one Market Inefficiency Magnitude, MIM_t, and one Adjusted Market Inefficiency Magnitude, $AMIM_t$.

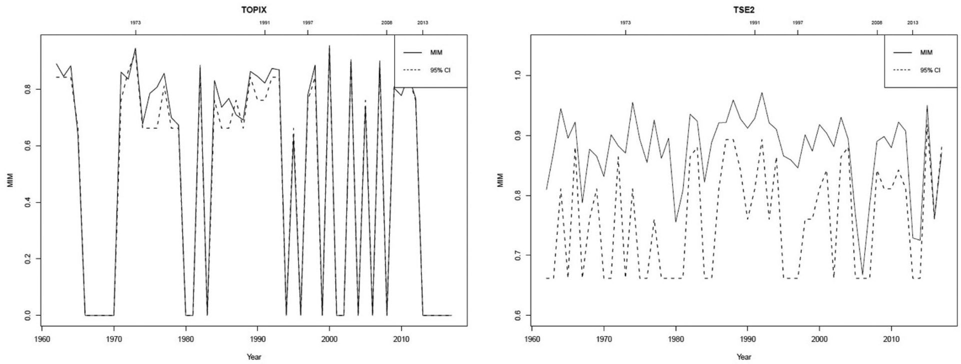
We also estimate *AMIM* using rolling-window data. In detail, we estimate *AMIM* daily using one year rolling-window data. Then every day we will have one value of *AMIM*.

5. Empirical results

Fig. 3 shows the value of *MIM* over year using non-overlapping window of TOPIX and TSE2. We can spot a clear fluctuation of *MIM* over time. However, it is hard to say that TOPIX is more efficient than TSE2 or the other way around. It is also hard to say which time the markets are more efficient than the other is. As discussed above, at each point in time with each asset we have a different confidence interval value of *MIM*. Therefore, we do not have a same base for comparison. Indeed, we can have a high *MIM* value but

[2] The sorting procedure is done through Wharton Research Data Service called "CRSP Stock File Indexes - Daily Index Built on Market Capitalization" at: https://wrds-web.wharton.upenn.edu/wrds/ds/crsp/indexes_a/mktindex/cap_d.cfm?navId=124

[3] *AMIM* is very flexible in choosing the number of lags in contrast with a fixed number of lags setting in TV-AR methods. *AMIM* does not depend only on AIC or any information criteria. With our construction, it is possible to apply other criteria to select the number of lags in the first step. In our paper, we only use AIC as a decision criterion, because the focus is more on introducing our measure *AMIM*, how to use it, and its feature to reflect major economic events.



(a) TOPIX MIM (b) TSE2 MIM

Fig. 3. MIM of TOPIX and TSE2 indexes. MIM is estimated with non-overlapping window. The solid line is MIM value while the dotted line is the 95% confidence interval under the null hypothesis of market efficiency.

being not significant and vice versa a low *MIM* value but showing a significant inefficiency level.

AMIM solve this issue by adjusting *MIM* with the confidence interval. Now it is enough to compare with the same base line of zero; remember that $AMIM > 0$ implies a significant inefficiency level, and $AMIM_t < 0$ implies an efficient market. Fig. 4 shows the evolution of *AMIM* for TOPIX, TSE2, S&P 500, and CAP1. For the Japan market, we can also confirm the major empirical findings of Noda (2016) with our measure. These are: i) first, market efficiency changes over time with both TOPIX and TSE2; ii) TOPIX has a lower and less volatile inefficiency level (mean (μ) $=0.08$, standard deviation (σ) $= 0.15$) than TSE2 ($\mu = 0.47$, $\sigma = 0.20$); iii) both TOPIX and TSE2 inefficiency level significantly decreases after 2010. Hence, both markets becomes more efficient after 2010. Table 2 gives the summary statistics of *AMIM* of the indexes.

For the US market, we find a similar difference in *AMIM* between S&P 500 and CAP1. The small stock index has a higher mean ($\mu = 0.38$), a higher standard deviation ($\sigma = 0.22$) than the ones of S&P 500 ($\mu = 0.09$, $\sigma = 0.17$). So for both US and Japan markets, large stocks indexes (TOPIX, S&P 500) are more likely efficient. Both TOPIX and S&P 500 have a low median near zero. This means that 50% of time, these indexes are efficient.

In addition, our measure also offers an additional feature. *AMIM* reflects very well important economic events, for example, it increases in periods of economic turbulence or crisis, then decreases after such periods. For Japan market, we can see this pattern with both TOPIX and TSE2 through the Oil-Crisis (1973–74), the bursting of Japanese asset bubble (1991–92), the Asian financial crisis (1997–99), and the financial crisis (2008). *AMIM* also decreases in 2013, which reflects the period of quantitative easing.

For US market, we catch the similar pattern. *AMIM* of both S&P 500 and CAP1 raises in the Oil-Crisis (1973–74) then decreases. *AMIM* raises again in the 1987 crisis and in the 2001 dot-com bubble burst. In these two crises (1987, 2001), *AMIM* of small stocks (CAP1) raises more than *AMIM* of large stock (S&P 500). These two crises could hit small stocks harder than large stocks. However in the financial crisis 2008, *AMIM* of large stocks (S&P 500) experienced a sharp increase and being almost as high as *AMIM* of CAP1. This can be due to the fact that a lot of large stocks (especially financial industry stocks) was smashed very hard during that crisis.

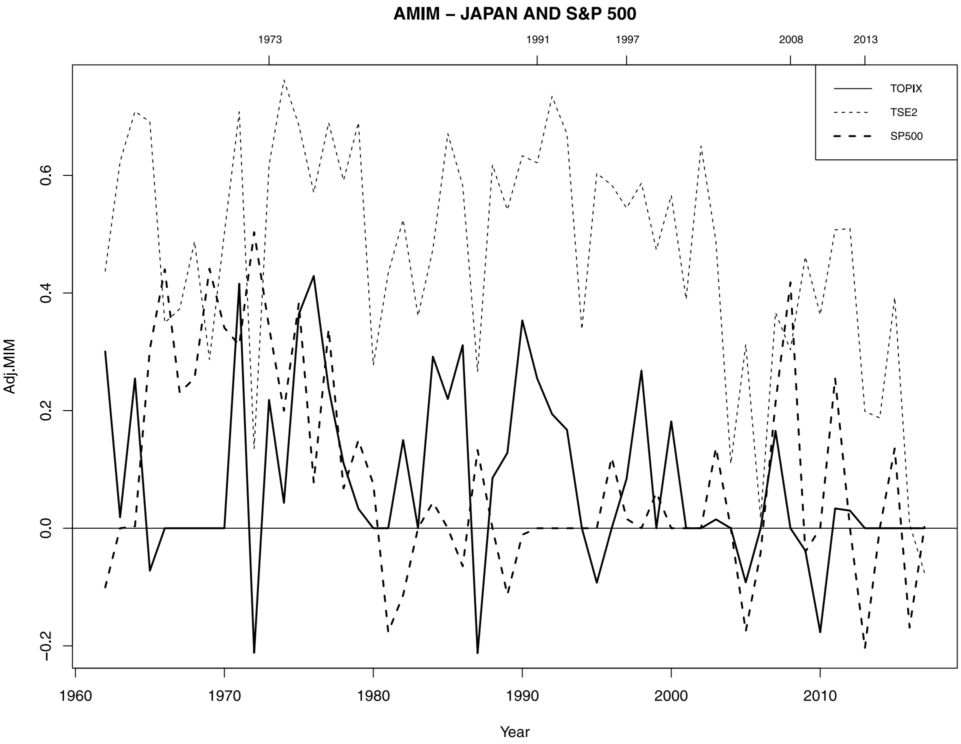
For a robustness check, we also estimate *AMIM* using rolling window. In detail, we estimate *AMIM* daily using a one year rolling window of data. So we will have one *AMIM* value per day. The results of the overlapping window estimates corroborates our earlier results. The summary statistics of *AMIM* using rolling window is in Table 4 in the appendix. For a clearer illustration of the trend of *AMIM*, we calculate a 100-day Moving Average (MA) of *AMIM* and plot it below in Fig. 5. As the figures illustrates, the S&P 500-index indicates an efficient market most of the time from 1980 through 2018, but with some periods of inefficiency. For example, during the oil-crisis in the early 70-ies, and during the more recent financial crisis of 2008-09, the market is inefficient. Smaller stocks are, not surprisingly, less efficient than large stocks, only being significantly efficient over small periods of time.

The fact that AMIM increases in times of crisis, and then decreases afterwards, confirms the results of Lo (2004, 2017) on the hypothesis of adaptive markets; financial markets are not always efficient, nor always inefficient but changing overtime. These results also indicate that our measure is a valid measure of the level of market efficiency.

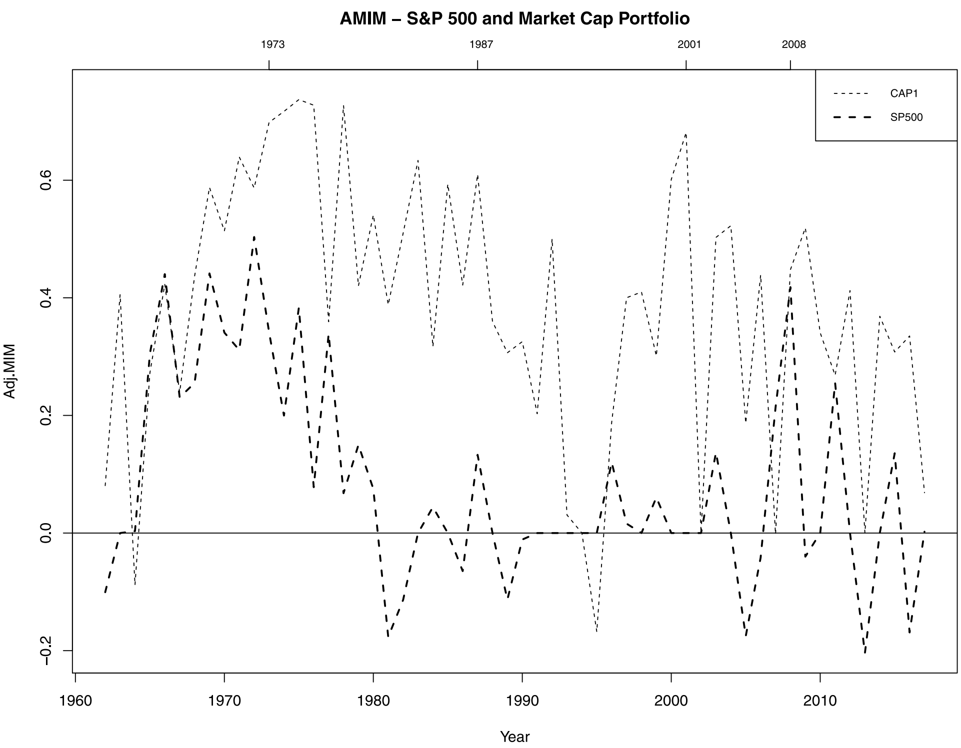
One caveat of our study is that we do not establish the causal effect between different factors (i.e. inflation, interest rates, unemployment rates, etc.) to market efficiency. Another caveat is that we do not have a full horse race between all efficiency measures in all markets. We recognize them as interesting subjects for further research. We herein focus more on developing *AMIM*, explaining how to use it, and showing its important features. Hence, we consider *AMIM* as a good alternative measure of efficiency that is easy to use, light in computation, alongside with other measures such as the variance ratio, *TIME*, etc.

6. Conclusion

This study derives a measure for the level of market efficiency, named Adjusted Market Inefficiency Magnitude *AMIM*. The measure is easy to be applied and computed via four steps. *AMIM* improves two challenges of the measures derived in Noda (2016). First, *AMIM* demands less computational effort and can easily be interpreted. Second, *AMIM* also provides a better foundation for

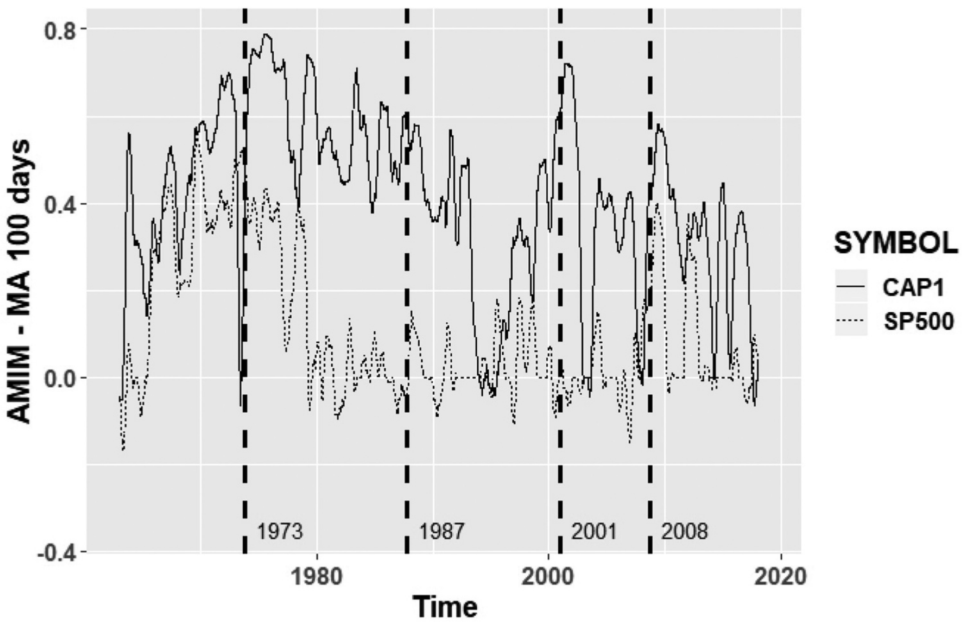


(a) AMIM of TOPIX, TSE2, S&P 500

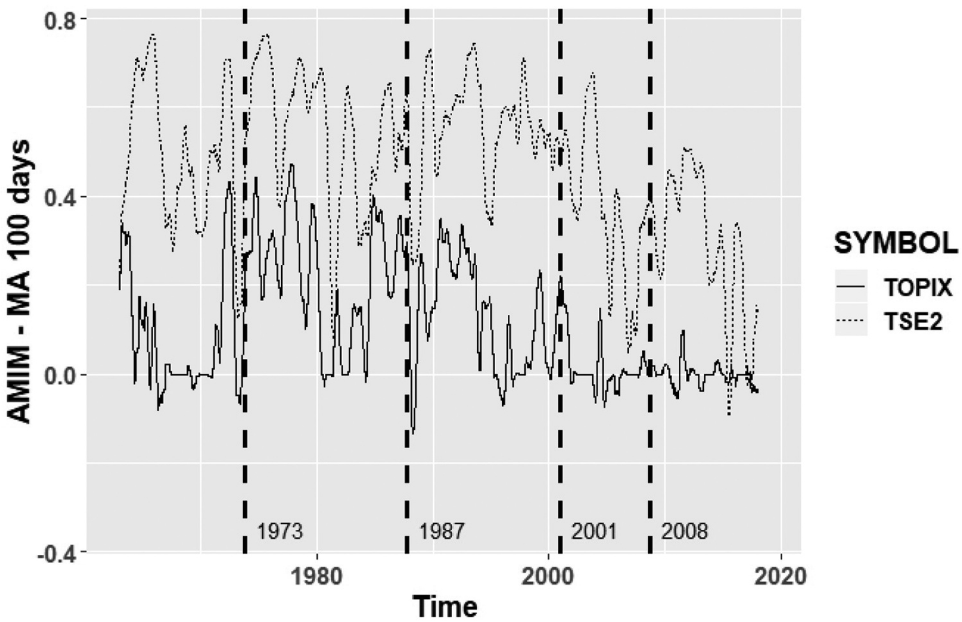


(b) AMIM of S&P 500 and CAP 1

Fig. 4. Adjusted Market Inefficiency Magnitude *AMIM*, using non-overlapping window, of TOPIX, TSE2, S&P 500, and CAP1. CAP1 is the portfolio containing 10% of small stocks on NYSE, AMEX, and NASDAQ exchanges. The data range is from 1962 to 2017.



(a) Moving Average 100-day of AMIM of CAP1, and S&P 500



(b) Moving Average 100-day AMIM of TOPIX and TSE2

Fig. 5. Moving average (MA) 100-day of *AMIM* of TOPIX and TSE2, S&P 500, and CAP1. *AMIM* is estimated daily using a 1 year rolling window data. CAP1 is the portfolio containing 10% of small stocks on NYSE, AMEX, and NASDAQ exchanges. The data range is from 1962 to 2017.

comparison the inefficiency level between different assets in different time. Applying our measure to the same dataset as in Noda (2016), we can confirm the major findings of Noda (2016)'s work. In addition, our measure also reflects very well major economic events in the US and Japanese economies. These empirical results shows that market efficiency is not constant over time, assets, or regions, which corroborates the Adaptive Market Hypothesis of Lo (2004, 2017).

Table 2
Summary statistic of AMIM measure using non-overlapping window for TOPIX, TSE2, S&P500, and CAP1 where CAP1 is the portfolio containing 10% of small stocks on NYSE, AMEX, and NASDAQ exchanges. The data range is from 1962 to 2017.

Index	n	mean	sd	median	min	max	skew	kurtosis	Q0.25	Q0.5	Q0.75
TOPIX	56	0.08	0.15	0.01	−0.21	0.43	0.52	−0.24	0.00	0.01	0.18
TSE2	56	0.47	0.20	0.50	−0.08	0.76	−0.81	0.07	0.36	0.50	0.62
CAP1	56	0.38	0.22	0.41	−0.17	0.74	−0.48	−0.38	0.27	0.41	0.53
SP500	56	0.09	0.17	0.00	−0.20	0.50	0.69	−0.33	0.00	0.00	0.20

Appendix A. Confidence Interval of *MIM* under the null hypothesis of Market Efficiency.

Table 3
The upper bound 95% Confidence Intervals (CI) under the null hypothesis of efficiency market for *MIM*, where: $MIM_t = \frac{\sum_{j=1}^{q} |\hat{\beta}_{j,t}^{standard}|}{1+\sum_{j=1}^{q} |\hat{\beta}_{j,t}^{standard}|}$ with different number of lags in the Eq. (1). The lower bound of this interval is 0. To compute CI, we first simulate 100 000 observations for each $\hat{\beta}_{j,t}^{standard}$ following a standard normal distribution. Then based on these $\hat{\beta}_{j,t}^{standard}$, we compute *MIM*. For each number of lags we have 100 000 observations *MIM* under the null hypothesis of market efficiency. After that, we find the 95^{th}% quantile of *MIM*. So the interval between this 95^{th}% quantile and 0 is the 95% confidence interval of MIM under the null hypothesis.

Number of Lags	95% CI	Number of Lags	95% CI	Number of Lags	95% CI
1	0.6618747	16	0.9441565	31	0.9682057
2	0.7604725	17	0.9468745	32	0.9690909
3	0.81105	18	0.9493466	33	0.9699065
4	0.8423915	19	0.9516287	34	0.9706732
5	0.864342	20	0.9536607	35	0.9714273
6	0.8806096	21	0.9555671	36	0.9721273
7	0.8932211	22	0.9572666	37	0.9727706
8	0.903343	23	0.9588263	38	0.9734095
9	0.9115645	24	0.9603012	39	0.9740274
10	0.9184596	25	0.9616615	40	0.9745969
11	0.9243942	26	0.9629152	41	0.9751548
12	0.9293885	27	0.9641263	42	0.9756867
13	0.9338437	28	0.9652404	43	0.9761773
14	0.9376448	29	0.9662761	44	0.976653
15	0.9411291	30	0.9672589	45	0.9771318

Appendix B. Summary statistics of AMIM using rolling window

Table 4
Summary statistic of *AMIM* measure for TOPIX, TSE2, S&P500, and CAP1 where CAP1 is the portfolio containing 10% of small stocks on NYSE, AMEX, and NASDAQ exchanges. The data range is from 1962 to 2017. *AMIM* is estimated daily using a 1-year rolling window.

Index	N	mean	sd	median	min	max	skew	kurtosis	Q0.25	Q0.5	Q0.75
CAP1	13,898	0.411	0.218	0.432	−0.227	0.796	−0.579	−0.118	0.296	0.432	0.570
SP500	13,898	0.106	0.180	0.000	−0.366	0.610	0.738	−0.306	0.000	0.000	0.223
TOPIX	13,632	0.104	0.154	0.005	−0.395	0.550	0.542	−0.471	0.000	0.005	0.222
TSE2	13,619	0.468	0.194	0.495	−0.223	0.798	−0.683	0.158	0.339	0.495	0.620

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