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First and second mover advantages and the degree of conflicting interests

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We introduce games consisting of two players where each player's payoff might be differently affected by changes in the decision variable of the rival. The games are classified into three categories: The high-conflict category is characterized by both players having a first mover advantage, the medium-conflict category by (at least) one player having a first mover advantage, and, the low-conflict category by (at least) one player having a second mover advantage. The categories give rise to different equilibria in a prior game where the players are supposed to commit themselves to draw as early or as late as possible.

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1 | INTRODUCTION

When an agent enters a one-shot two-player game, an interesting question is whether the same agent would prefer to become the first or the second mover. A second question, when knowing the preferences of agents as concerning moves, is what the outcome of such a game will be. In this paper, the above questions are discussed in a simple model based on the seminal works by Gal-Or (1985) and Bulow, Geanakoplos, and Klemperer (1985). Both works have introduced concepts that now are well-established in the literature concerned with strategic choices. Gal-Or (1985), considering a model with two identical players (symmetric games) moving sequentially (leader–follower), analyzes the situation where the decision of the follower is positively related to the decision of the leader (positive cross-partial derivative of the payoff functions) and when the decisions are negatively related (negative cross-partial derivatives of the payoff functions). The first case means upward sloping reaction functions while the second implies downward sloping response functions. In Bulow, Geanakoplos, and Klemperer (1985), decisions are denoted as strategic complements for the case of upward sloping reaction functions while they are strategic substitutes for downward sloping reaction functions. The main findings of Gal-Or (1985) is that (*i*) the leader makes a higher payoff than the follower, if the players' reaction functions are upwards sloping (strategic complements); thus, both players have a first mover advantage; and (*ii*) the follower makes a higher payoff than the leader, if the reaction functions are downward sloping (strategic substitutes); thus, both players have a second mover advantage. The work by Gal-Or (1985) has inspired analyses on various types of duopoly markets when firms might be heterogenous. For instance, in a seminal work, Hamilton and Slutsky (1990) discuss the existence of equilibria and properties of the different equilibria when introducing endogenous timing of actions in Cournot–Stackelberg games. Inspired by this work, Amir and Stepanova (2006) are analyzing first and second mover advantages and commitment incentives in a Bertrand duopoly game.[1]

In this work, we generalize the works by Gal-Or (1985), Hamilton and Slutsky (1990), and Amir and Stepanova (2006) by considering the allocation of first and second mover advantages for a wide range of asymmetric sequential games.[2] As proposed by Hamilton and Slutsky (1990), being inspired by Singh and Vives (1984), the introduction of asymmetry introduces a variety of one-shot nonsimultaneous games. Our intention is to provide a complete and systematic overview over such games that might be useful when applying game-theoretic frameworks to analyze possible interactions between two actors. An overview places particular applications in a wider perspective and provides a deeper understanding of mechanisms at play based on the actors' preferences. Furthermore, asymmetric payoff games might be a better fit into data in many real life examples. In undertaking our analysis, we search for possible subgame perfect equilibria, and we assume that the order of moves does not affect payoffs conditional on the strategies (decisions). Besides its theoretical appeal, a systematic overview might make it easier to analyze several situations in models within industrial economics and to arrive at policy-relevant insights. Game theory is also highly relevant for studying possible interactions besides those within industrial organization, for instance for actors meeting in road traffic, for understanding strategical moves in pay-for performance games and in explaining players' position in bilateral lobbying games (see for instance Pedersen, 2003, Bergland & Pedersen, 2019, and Grepperud & Pedersen, 2020).

We find that the introduction of asymmetry allows the sign and significance of the cross-partial derivatives of the payoff functions to differ across players implying reactions functions with different slopes. Using the terminology of Bulow, Geanakoplos, and Klemperer (1985), this means that the decisions can be strategic complements (substitutes) for one player at the same time as they are strategic substitutes (complements) for the rival. Second, the various distributions of the preferred order of moves across players depend on the sign of the cross-partial derivative of the payoffs for each player (the slopes of each reaction function) and the significance and sign of the marginal payoffs with respect to the rival's decision.[3] Third, the combinations of these signs define games that are classified into three different conflict categories each producing a particular allocation of first and second mover advantages (outcomes). Fourth, asymmetry, in contrast to many symmetric games, might produce subgame perfect equilibria where one player has a first mover advantage and the rival a second mover advantage. Fifth, introducing asymmetry among the players opens for various Nash equilibria in an ex ante simultaneous commitment game prior to the choice of actions. Games that we term as "high conflict games," defined by both players having a first mover advantage, give unambiguously simultaneous draws in such a commitment game, while games termed "medium conflict games," characterized by at least one player having a first mover advantage, meaning that the player always having the first mover advantage moves first. Moreover, games termed "low conflict games," defined by situations where at least one player has a second mover advantage, are seen to produce several possible Nash equilibria following from a commitment game prior to the choice of actions.

Section 2 gives a short introduction to the simple model consisting of two players where each of them makes one decison. We use this model to identify the various games that may occur and sort them into three conflict categories. In Section 3, we look into each of the conflict categories, and, inspired by the work of Hamilton and Slutsky (1990) and Amir and Stepanova (2006), we discuss possible consequences on commitment incentives. In particular, we look into asymmetric games by proposing a stylized example of an advertising game. Finally, Section 4 summarizes our findings.

2 | THE MODEL AND THE DISTRIBUTION OF ADVANTAGES

Suppose two players that meet in a one-shot game where each player makes one decision (action) that affects own payoff and the payoff of the rival. The payoff (profit or utility) function of player 1 (also termed she) is given by

$$U = U(x, y) \tag{1}$$

where $x > 0$ is the action made by player 1 and $y > 0$ is the action made by player 2. Analogously, player 2's (also termed he) payoff function is given by;

$$V = V(x, y) \tag{2}$$

The U- and the V-functions are supposed to be strictly concave for all x and y, respectively, that is, $U_{xx} < 0$ and $V_{yy} < 0$. Moreover, player 1's payoff might be increasing or decreasing with the level of the action variable chosen by player 2, that is, $U_y > (<) 0$ (the indirect marginal payoff of player 1). Analogously, player 2's payoff might increase or decrease as player 1 steps up her choice of x, that is, $V_x > (<) 0$ (the indirect marginal payoff of player 2).[4] By assuming that both indirect marginal payoffs are strictly positive or negative, the players affect each other payoffs, and possible interactions exist thus defining a "classic" noncooperative game situation. Moreover, it is assumed that a player's marginal utility is either strictly increasing or decreasing in the other player's action, that is, $U_{xy} \geq (<) 0$ and $V_{xy} \geq (<) 0$.

Suppose players that move simultaneously.[5] Then the Nash equilibrium is defined by

$$U_x\left(x^S, y^S\right) = 0 \quad \text{and} \quad V_y\left(x^S, y^S\right) = 0 \tag{3}$$

where $\left(x^S, y^S\right)$ denotes this type of equilibrium in the simultaneous case. In the case where player 1 is leader, and player 2 the follower, we let $y = y(x)$ denote player 2's reaction function. It follows that $\frac{dy}{dx} = y_x = -\frac{V_{xy}}{V_{yy}} \geq (<) 0$ as $V_{xy} \geq (<) 0$. Hence, the case where player 1 is leader (L) and player 2 follower (F) is given by

$$U_x\left(x^L, y^F\right) + U_y\left(x^L, y^F\right) y_x\left(x^L\right) = 0 \ \text{ and } \ V_y\left(x^L, y^F\right) = 0 \tag{4}$$

where $\left(x^L, y^F\right)$ symbolizes the subgame perfect equilibrium where player 1 is leader and player 2 follower.[6] In the opposite case (player 2 as leader and player 1 as follower), we let $x = x(y)$ define the reaction function of player 1, where $\frac{dx}{dy} = x_y = -\frac{U_{xy}}{U_{xx}} \geq (<) 0$ as $U_{xy} \geq (<) 0$. Furthermore, we get

$$U_x\left(x^F, y^L\right) = 0 \ \text{ and } V_y\left(x^F, y^L\right) + V_x\left(x^F, y^L\right) x_y\left(y^L\right) = 0 \tag{5}$$

where (x^F, y^L) symbolizes the subgame perfect equilibrium where player 2 is leader and player 1 follower.[7] In the discussions below, we identify the payoffs and quantities (x and y) for different order of moves, and we apply the following notation: $U^S = U(x^S, y^S), V^S = V(x^S, y^S),\ U^L = U(x^L, y^F),\ V^F = V(x^L, y^F),\ U^F = U(x^F, y^L)$ and $V^L = V(x^F, y^L)$.

Based on 4 and 5, denote the situation where both players have a first mover advantage as a situation where $U^L > U^F$ and $V^L > V^F$. Moreover, a situation where player 1 has a first mover advantage and player 2 a second mover advantage is defined by $U^L > U^F$ and $V^L < V^F$, and the case where player 1 has a second mover advantage and player 2 a first mover advantage is given by $U^L < U^F$ and $V^L > V^F$. Finally, the situation where both have a second mover advantage is defined by $U^L < U^F$ and $V^L < V^F$.[8]

Based on the above assumptions, it is possible to discuss and compare what happens to player 1's (player 2's) choice when becoming the leader and player 2's (player 1's) choice when becoming the follower, compared to the simultaneous case. Such comparisons reveal what happens when player 1 (player 2) moves from being the leader (follower) to becoming the follower (leader). From 3–5, it follows that the answers to our questions rely on the sign and significance of (*i*) the slopes of both players' response functions, that is, y_x and x_y, and (*ii*) what happens to player 1's (player's 2) payoff when player 2 (player 1) increases his (her) action, measured by U_y (V_x). As concerning the slopes of the reaction functions, unlike the symmetric model of Bulow, Geanakoplos, and Klemperer (1985), the response functions, given asymmetry, must be defined contingent upon the identity of each player. Here, we apply the following definition:

When $x_y \geq (<) 0$, the actions are strategic complements (substitutes) for player 1, as she will respond by increasing (decreasing) x as y becomes higher. Moreover, when $y_x \geq (<) 0$, the actions are strategic complements (substitutes) for player 2, as he will respond by increasing (decreasing) y as x becomes higher.

Before presenting the various games under asymmetry, we first describe the games defined by identical actors considered by Gal-Or (1985) within the framework presented in 3–5. Given payoff functions, U and V, being the same, that is, $U = V = \pi$, Gal-Or analyzes the following two situations: (*i*) the actions being strategic complements for both players, that is, $\pi_{xy} = \pi_{yx} > 0$, combined with strictly positive indirect marginal payoffs, that is, $\pi_y = \pi_x > 0$; and (*ii*) the actions being strategic substitutes for both players, that is, $\pi_{xy} = \pi_{yx} < 0$ combined with strictly negative indirect marginal payoffs, that is, $\pi_y = \pi_x < 0$. In addition to the two games described above, there exist two additional ones for identical actors that are not discussed by Gal-Or, presumably because her focus was at two particular duopoly markets being widely discussed in the literature (Stackelberg–Cournot competition and nonsimultaneous Bertrand competition, respectively).[9,10]

Given asymmetry, the number of possible games, defined by the slopes of the reactions functions and the signs of the indirect marginal payoffs for both players, is substantial. To limit the number, we restrict ourselves to consider games where the cross-partial derivatives of the payoff functions differ from zero, that is, $x_y > (<) 0$ and $y_x > (<) 0$, meaning that both players react either positively or negatively to changes in the action of the rival.[11] Furthermore, we rule out the possibility of multiple equilibria, no equilibria and unstable equilibria; that is, all the identified possibilities fulfill the conditions for stationary Nash equilibria. Sufficient conditions are strictly monotonic reaction functions, $y(x)$ and $x(y)$, combined with the slopes, in absolute values, being less than 1, that is, $|y_x| < 1$ and $|x_y| < 1$.

Using 3–5, in combination with the properties of the payoff functions, it is possible to compare the simultaneous case with the cases, where player 1 is leader (follower) and player 2 follower (leader). In Appendix A, we have, based on 3–5, ranked the x and y and the values of U and V that belong to these specific cases (the results are summed up in Table A1 in Appendix A). These rankings, for all combinations of signs with respect to U_y, U_{xy}, V_x, and V_{xy}, define various games as functions of combinations of signs. Table 1 presents the distribution of advantages related to the order of moves (outcomes). Generally, the games in Table 1 produce different distributions of first and second mover advantages across players (outcomes). For game *A*, *FF*, *LF*, and *FL* (see line 1, column 1 in Table 1) represent the three possible outcomes. Given *FF*, both players prefer to act as followers (both have a second mover advantage), given *LF*, player 1 prefers to be leader while player 2 prefers to be follower (player 1 has a first mover advantage and player 2 a second mover advantage), while for *FL*, player 1 prefers to be follower while player 2 prefers to be leader (player 1 has a second mover advantage and player 2 a first mover advantage).

Table 1 is designed in such a way that it makes the various games symmetric around the diagonal. For instance, the type described by line 2 and column 1 (game *C*) has its counterpart in the game described by line 1 and column 2 (game *B*). Following the cases along the diagonal, the assumptions regarding the payoff functions of the

TABLE 1 The distribution of first mover and second mover advantages (outcomes) across games

		The actions are strategic complements for player 2; $V_{xy} > 0$		**The actions are strategic substitutes for player 2; $V_{xy} < 0$**	
		$V_x > 0$	**$V_x < 0$**	**$V_x > 0$**	**$V_x < 0$**
The actions are strategic complements for player 1; $U_{xy} > 0$	$U_y > 0$	(A) FF, LF, and FL	(B) LL	(E) LL and FL	(F) LL and LF
	$U_y < 0$	(C) LL	(D) FF, LF, and FL	(G) LL and LF	(H) LL and FL
The actions are strategic substitutes for player 1; $U_{xy} < 0$	$U_y > 0$	(I) LL and LF	(J) LL and FL	(M) LL	(N) FF, LF, and FL
	$U_y < 0$	(K) LL and FL	(L) LL and LF	(O) FF, LF, and FL	(P) LL

players are symmetric in the sense that the outcomes identified for player 1 (player 2) now are valid for player 2 (player 1). Furthermore, each of the four symmetric games is part of one of the games described along the diagonal starting in cell 1 (games *A*, *D*, *M*, and *P*).[12]

It is observed that quadrants 1 (games *A*–*D*) and 4 (games *M*–*P*) of Table 1 consist of two possible outcomes. First, for games *B*, *C*, *M*, and *P*, both players have unambiguously first mover advantages (*LL*). In the following, these four games are termed as *high-conflict* games since representing a high conflict of interests among the players with respect to *x* and *y*. This is seen by drawing the reaction functions and the indifferent curves in an *xy* diagram where *x* is measured along the horizontal axis while *y* is measured along the vertical axis. Looking into such a diagram makes evident that when one of the players prefers moving to the north-west, the rival prefers moving to the southeast, and when one of the players prefers moving to the northeast, the rival prefers moving to the south-west.[13]

The four remaining games of quadrants 1 and 4 (games *A*, *D*, *N*, and *O*) are characterized by at least one player having a second mover advantage (*FF*, *LF*, and *FL*). These games are in the following denoted as *low-conflict* games. By drawing the reaction functions and the indifferent curves in an *xy* diagram, it follows that the preference direction for the two players in this category is (almost) the same. Game *A* is characterized by a north-east preference direction for both players, game *D* by a south-west preference, game *N* by a north-west preference, while game *O* is characterized by a south-east preference direction.[14]

It is observed from Table 1 that all *high-conflict* and *low-conflict* games are characterized by actions with similar strategic properties (both players consider the actions to be either strategic complements or to be strategic substitutes). For *high-conflict* games, the signs of the indirect marginal payoffs are opposite for strategic complements but similar for strategic substitutes, while for *low-conflict* games, the signs are the same for strategic complements but opposite for strategic substitutes.

For the eight games belonging to quadrants 2 (game *E*–*H*) and 3 (game *I*–*L*), in the following termed *medium-conflict* games, the common factor is that at least one of the players has a first mover advantage (*LL*, *LF*, and *FL*). Games in this category exhibit a moderate conflict of interest as concerning *x* and *y* since their preferences for one, of the two actions, goes in the same direction. For games *I* and *H*, both players prefer to go in a northern direction for one action while their direction preferences differ for the other action. For game *I*, player 1 prefers north-west and player 2 north-east, while for game *H*, player 1 prefers north-east and player 2 north-west. For games *E* and *L*, both players have a common interest in going in a southern direction; for games *F* and *J*, both have an interest in going west; while for games *G* and *K*, both have an interest in going east. All these games are characterized by one player perceiving the actions to be strategic complements (strategic substitutes) while the rival consider them to be strategic substitutes (strategic complements). Additionally, the signs of the indirect marginal payoffs include all possible combinations. Furthermore, it follows that when U_y and V_x have the same sign, the player having an unambiguously first mover advantage perceive the actions as being strategic substitutes, while when U_y and V_x have opposite signs, the player with an unambiguously first mover advantage perceives the actions as being strategic complements.[15] A final conclusion is that symmetric games cannot be part of the *medium-conflict* category.

3 | A CLOSER LOOK AT CONFLICT CATEGORIES AND COMMITMENT GAMES (ENDOGENOUS TIMING OF ORDER OF MOVES)

In this section, we study in more detail games that belong to each of the conflict categories. This is done by focusing on a selection of games from each category and by discussing an application (stylized example) to provide economic insights. The application can be thought of as a simple advertisement game described by two profit-maximizing firms competing in price regulated markets for substitutable goods. The firms are supposed to decide on advertisement intensities (*x* and *y*), where the advertisement intensity impacts own costs. In order to keep this as simple as possible we have in these examples restricted us to discuss cases where it is asymmetry, i.e. we consider games where a higher advertisement intensity from firm 1 increases the total market demand. Hence, a more intense advertisement from firm 1 will increase the market demand for both firm 1's and the rival's products ($V_x > 0$). The advertisement intensity of firm 2, on the other hand, is assumed to have no market expansion effect (e.g., given market size), thus making a higher advertisement intensity from firm 2 advantageous for firm 2 but disadvantageous for the rival ($U_y < 0$). Furthermore, a higher advertisement intensity is allowed to affect the rival's marginal profit both positively and negatively. This last assumption implies that the advertisement model contains four of the asymmetric games (games *C*, *G*, *K*, and *O*) presented in Table 1 (see Appendix B for a formal setup). These four games only differ with respect to how the two firms perceive the strategic properties of the advertisement intensities.

Most of the applications using the concepts of first and second mover advantages are concerned with analyses of quantity and price competition using either Cournot or Bertrand models (or both). Furthermore, such analyses are typically restricted to symmetry cases. An exception is Hamilton and Slutsky (1990) discussing the case where one of the duopoly firms acts in price setting and the other chooses quantity. Their discussion, being inspired by Singh and Vives (1984), compares the outcomes following from quantity and price competition and shows that the action *variables might become strategic* substitutes for one player while being strategic complements for the rival.

3.1 | High-conflict games

We start out with focusing on the *high-conflict* category. This category contains only outcomes where both players have a first mover advantage (significant preemptive incentives). Game *P*, here chosen to represent *high-conflict* games, assumes actions that are strategic substitutes for both players ($U_{xy} < 0$ and $V_{xy} < 0$) in combination with each player having a decreasing payoff in the other player's action ($U_y < 0$ and $V_x < 0$). The first mover advantages follow from the following rankings of actions: $x^F < x^S < x^L$ and $y^F < y^S < y^L$ (see Appendix A). Based on the two rankings and the assumed properties of the payoff functions for this particular game, we get $U(x^F,y^L) < U(x^F,y^S) < U(x^S,y^S) < U(x^S,y^F) < U(x^L,y^F)$. The first and third inequalities follow from increasing the rival's action, the second from optimizing player 1's action for a given value of the rival's action, and the fourth from optimizing player 1's action given the new value of the rival's action. The same ranking holds for the rival, that is, $V(x^L,y^F) < V(x^S,y^F) < V(x^S,y^S) < V(x^F,y^S) < V(x^F,y^L)$. The intuition behind the rankings is simple. The player, being the first mover, knows that the rival's best response is to react in a defensive manner, and such a response will increase the payoff of the leader. This means that being the leader is preferred to the case of simultaneous draws by both players, which again is preferred to the most unfavorable position, the position as follower.

One of the two games with identical actors considered by Gal-Or (1985), that is, $\pi_{xy} = \pi_{yx} < 0$ and $\pi_y = \pi_x < 0$ (Cournot–Stackelberg competition) fulfills the assumptions that are valid for game *P* (strategic substitutes for both players and strictly negative indirect marginal payoffs); consequently, this case belongs to the *high-conflict* category. This implies that for less restrictive assumptions, relatively to those that define Cournot–Stackelberg competition, will still produce situations for games of type *P* where the only possible outcome is that both players have a first mover advantage. A game similar to case *P* is analyzed by Hamilton and Slutsky (1990).[16]

Similar conclusions, despite different assumptions, are relevant for the three remaining games belonging to the *high-conflict* category (games *M*, *B*, and *C*). For game *M*, the actions are still strategic substitutes for both players, but now the payoffs are increasing with the rivals' action, that is, $U_{xy} < 0$, $V_{xy} < 0$, $U_y > 0$, and $V_x > 0$.[17] Games *B* and *C*, on the other hand, are defined by actions that are strategic complements for both players ($U_{xy} > 0$ and $V_{xy} > 0$), in combination with one player having a decreasing payoff in the other player's action, while the rival has an increasing payoff in the other player's action.[18] Game *C*, since assuming $U_y < 0$ and $V_x > 0$, represents one of the four possible advertisement games (see above) where the advertisement intensities are perceived as being strategic complements by both firms. For this game, we have the following rankings of the advertisement intensities: $x^L < x^S < x^F$ and $y^F < y^S < y^L$ (see Appendix A). For this game, we get $U(x^F,y^L) < U(x^F,y^S) < U(x^S,y^S) < U(x^S,y^F) < U(x^L,y^F)$ and $V(x^L,y^F) < V(x^S,y^F) < V(x^S,y^S) < V(x^F,y^S) < V(x^F,y^L)$. Hence, there is a first mover advantage for both firms. Upward sloping reaction function for firm 2 means that firm 1 (as leader) has an incentive to choose a low advertisement intensity (since $U_y < 0$) to "force" firm 2 to choose a low intensity. Upward sloping reaction function for firm 1 implies that firm 2 (as leader), on the other hand, since $V_x > 0$, has an incentive to choose a relatively high advertisement intensity to "force" firm 1 to choose a relatively high *x*.

To further characterize *high-conflict* games, we discuss the possibility of the players, prior to the game of choosing their actions (*x* and *y*), being able to commit themselves to play Early or Late regarding their actions. Table 2 represents the normal form of such a simultaneous game. In constructing the outcomes of this game, we suppose that if both choose Early or both choose Late, the simultaneous outcome occurs.[19] For all four *high-conflict* games, it follows that the Nash subgame perfect equilibrium unambiguously is simultaneous draw (Early, Early). Furthermore, both players now obtain the payoff level ranked as the second best, and none of the alternative outcomes represent a Pareto improvement relatively to simultaneous draw (Early, Early). For the relevant advertisement game (game *C*), this means that Early draw describes "a race to become the leader" that results in a simultaneous Nash equilibrium.

3.2 | Low-conflict games

Next we focus at games belonging to the *low-conflict* category. These games are characterized by (at least) one player having a second mover advantage which implies that both players have a second mover advantage (*FF*) or that player 1 (player 2) has a second mover advantage and player 2 (player 1) a first mover advantage, *FL* (*LF*). From this category, we discuss game *A*, that is characterized by actions being strategic complements for both players ($U_{xy} > 0$ and $V_{xy} > 0$), in combination with the payoffs of both players being increasing with the opponent's action ($U_y > 0$ and $V_x > 0$). The following three rankings are possible: (*R1*) $x^S < x^F < x^L$ and $y^S < y^F < y^L$, (*R2*) $x^S < x^L < x^F$ and $y^S < y^F < y^L$, and (*R3*) $x^S < x^F < x^L$ and $y^S < y^L < y^F$(see Appendix A). Using the same logic as above, *R1* implies $U(x^S,y^S) < U(x^S,y^F) < U(x^L,y^F) < U(x^L,y^L) < U(x^F,y^L)$ and $V(x^S,y^S) < V(x^F,y^S) < V(x^F,y^L) < V(x^L,y^L) < V(x^L,y^F)$; that is, both players have a second mover advantage (*FF*).

TABLE 2 The normal form of the commitment games (simultaneous game of timing)

		Player 2: *Early*	Player 2: *Late*
Player 1	*Early*	U^S, V^S	U^L, V^F
	Late	U^F, V^L	U^S, V^S

The second case analyzed by Gal-Or (1985), nonsimultaneous Bertrand competition for identical actors, belongs to game *A* when *R1* is valid. From this, we can conclude that asymmetric players performing Bertrand competition might lead to *R2* or *R3*, hence deviating from the outcome where the last mover undercuts the price set by the rival (both players have a second mover advantage). For game *A* and *R2*, we get $U(x^S,y^S) < U(x^S,y^F) < U(x^L,y^F) < U(x^L,y^L) < U(x^F,y^L)$ and $V(x^S,y^S) < V(x^L,y^S) < V(x^L,y^F) < V(x^F,y^F) < V(x^F,y^L)$; thus, player 1 has a second mover advantage and player 2 a first mover advantage (*FL*). This situation may arise if the impact on each other's actions differs significantly across players.[20] If this is the case, player 2 (as leader) has the power to "force" player 1 (as follower) to set a value of *x*, higher than player 1 would have done if being the leader. Turning to the example of nonsimultaneous Bertrand competition, if *R2* holds, player 2 (as leader) will set a relatively high price so that player 1 will raise her price, thus ending up in a situation being advantageous for both. This situation might occur if player 2 can increase his price significantly, for a small increase in the price of player 1, without a significant loss of own profits, while player 1 will increase own price marginally, in response to a small increase in player 2's price, without having a significant impact on her profits. Finally, when *R3* is valid, the roles of the two players become exactly opposite of the ones identified for *R2*; thus, player 1 has a first mover advantage while player 2 has a second mover advantage (*LF*). The cases that follow from rankings *R2* and *R3* are discussed in Amir and Stepanova (2006). A game similar to case *A* is analyzed by Hamilton and Slutsky (1990).[21]

From Table 1, we observe that the set of outcomes identified for game *A* is also relevant for the three remaining *low-conflict* games (games *D*, *N*, and *O*). First, for game *D*, being defined by actions being perceived as strategic complements for both players ($U_{xy} > 0$ and $V_{xy} > 0$), in combination with both payoffs being decreasing in the opponent's action ($U_y < 0$ and $V_x < 0$).[22] Second, game *N* is defined by actions being strategic substitutes for both players ($U_{xy} < 0$, $V_{xy} < 0$), in combination with the payoff of player 1 being positively affected by the rivals' action and player 2 being negatively influenced by the rival's action ($U_y > 0$ and $V_x < 0$). The final game, game *O* ($U_y < 0$ and $V_x > 0$), mirrors an advertisement game defined by the advertisement intensities being perceived as strategic substitutes by both firms, that is, $U_{xy} < 0$ and $V_{xy} < 0$. In this game, firm 1 as leader has an incentive to choose a high advertisement intensity in order to "force" firm 2 to choose a low intensity. Firm 2, being the follower, may find this situation favorable, compared with being the leader, since a high advertisement intensity from firm 1 stimulates the demand also in his market. On the other hand, firm 2 as the leader has an incentive to choose a low advertisement intensity to "force" firm 1 to choose a high intensity. Firm 1, being the follower in this situation, may find this position as leader to be advantageous since a relatively low advertisement intensity from firm 2 does not reduce her demand as much as in the situation where she acts as leader. The analysis shows that at least one of the firms prefers being the second mover to being the leader, possibly both.

We now consider *low-conflict* category games when the players take part in a commitment game prior to the game of choosing *x* and *y*. First, from Table 2, it follows that the outcomes (*Early*, *Late*) and (*Late*, *Early*) represent subgame perfect equilibria for all three rankings. Second, both equilibria represent Pareto improvements relative to the other outcomes (*Early*, *Early*) and (*Late*, *Late*). Third, when one of the players has a first mover advantage (defined by ranking *R2* and *R3* above), the subgame perfect equilibrium characterized by the player having the first mover advantage draws *Early*, and the player having the second mover advantage draws *Late*, will Pareto-dominate the other subgame perfect equilibrium.

All games belonging to the *low-conflict* category, where both players have a second mover advantage (*R1*), mirror the "battle of the sexes" in which both players have ambitions of reaching a coordinated solution but disagree about the preferred candidates. For such games, given the existence of two subgame perfect equilibria, both considered as reasonable outcomes, it might be interesting to consider the Nash equilibrium in mixed strategies. Using Table 2, defining *p* as the probability that player 1 chooses Early and *q* is the probability that player 2 chooses *Early*, the equilibrium values of these probabilities are given by the following.

$$p = \frac{V^L - V^S}{V^L - V^S + V^F - V^S} \quad \text{and} \quad q = \frac{U^L - U^S}{U^L - U^S + U^F - U^S} \tag{6}$$

From 6, it is now seen that the lower the increase in the payoff obtained by player 1, from being a leader relatively to drawing simultaneously, that is, the lower $U^L - U^S$, and the higher the increase in the payoff by player 1, obtained from being follower relatively to drawing simultaneously, that is, the higher $U^F - U^S$, the less is the probability *q* for player 2 committing to draw *Early*. A similar reasoning related to player 2's payoff gains for the different outcomes holds for player 1's probability of choosing to commit to draw *Early*, that is, the value of *p*. Generally, as also discussed by Amir and Stepanova (2006), when considering Bertrand competition, the existence of several equilibria where neither Pareto-dominates the others means that it is not obvious what will be the outcome of a commitment game belonging to the *low-conflict* category (both players have a second mover advantage).[23] Hence, surprisingly, low conflict games, defined by players having (almost) the same preference direction, lead to various types of equilibria in a commitment game prior to the choice of actions. Only in the case where one of the players has a first mover advantage within this category, the equilibrium where this player moves first and the other second, this Pareto-dominates the others, and hence, this equilibrium is likely to become the outcome.

3.3 | Medium-conflict games

Finally, we study the games that belong to the *medium-conflict* category. Such games are characterized by (at least) one of the players having a first mover advantage meaning that (*i*) both players have a first mover advantage (*LL*), or (*ii*) player 1 (player 2) has a second mover advantage while player 2 (player 1) has a first mover advantage, *FL* (*LF*). Up to now, all games being discussed are defined by actions that either are strategic substitutes or strategic complements for both players. However, for games belonging to the *medium-conflict* category, they are defined by actions that are perceived as being strategic substitutes by one player while being considered as strategic complements by the rival. Furthermore, for these games, there exist two possible rankings for one of the two actions, while there is only one possible ranking for the other action (for details, see Appendix A). For games *E*, *H*, *J*, and *K*, the two possible rankings are related to the action of player 1 (x), while for games *F*, *G*, *I*, and *L*, the two possible rankings are related to the action of player 2 (y). A game similar to case *F* is analyzed by Hamilton and Slutsky (1990).[24]

In the following, we choose game *L* as the representative of the *medium-conflict* category ($U_{xy} < 0$, $V_{xy} > 0$, $U_y < 0$, and $V_x < 0$). The following two rankings are possible: (*R4*) $x^L < x^F < x^S$ and $y^F < y^S < y^L$ or (*R5*) $x^F < x^L < x^S$ and $y^F < y^S < y^L$ (see Appendix A). Note that the ranking of the actions taken by player 1, y, is the same across *R4* and *R5*. For player 2 (as leader), both for *R4* and *R5*, we arrive at a level of y that is higher than the level of y given by simultaneous moves that again is higher relatively to the situation where player 2 acts as follower. The ranking of x, however, differs across *R4* and *R5*. The highest value of x, in both cases, follows from simultaneous moves, the lowest value of x follows for *R4*, when player 1 acts as leader, while the lowest value of x follows for *R5*, when player 1 is the follower. Hence, for *R4*, we get $U(x^F, y^L) < U(x^F, y^S) < U(x^S, y^S) < U(x^S, y^F) < U(x^L, y^F)$ and $V(x^S, y^S) < V(x^F, y^S) < V(x^F, y^L) < V(x^L, y^L) < V(x^L, y^F)$. This means that *R4* implies a first mover advantage for player 1 and a second mover advantage for player 2 (*FL*). Given ranking *R5*, the ranking of the payoff for player 1 is the same as for *R4*, while the ranking for player 2 changes to $V(x^S, y^S) < V(x^L, y^S) < V(x^L, y^F) < V(x^F, y^F) < V(x^F, y^L)$; that is, both players now have a first mover advantage (*LL*). Player 1, since the actions are strategic substitutes for her, responds to a high y by reducing x, while player 2, since the actions are strategic complements for him, responds to a higher x by increasing y. At the same time, we know that the payoff of player 1 decreases in y, while the payoff of player 2 decreases with x. The remaining games that belong to the *medium-conflict* category (games *E–K*) are similar to game *L*, in the sense that the strategic properties of the actions are considered to be opposite across the two players and because they produce the same distribution of possible outcomes (one player has a first mover advantage and the other a second mover advantage, or both have a first mover advantage).

Let us now consider the two remaining advertisement games both belonging to the *medium-conflict* category (games *G* and *K*). In game *G*, firm 1 considers the advertisement intensities to be strategic complements while firm 2 considers them as strategic substitutes. First, suppose firm 1 is the leader. Firm 1 will now choose a relatively high advertisement intensity in order to "force" firm 2 to choose a relatively low intensity. Firm 2, being the follower, might gain by this choice since the advertisement intensity from firm 1 stimulates the demand for own product as well as the demand for the rival's product. In the opposite situation, firm 2 as leader, will choose a relatively high advertisement intensity in order to stimulate firm 1 to increase her advertisement intensity. This situation is unequivocally worse for firm 1 relatively to her being the leader; however, it might be advantageous for firm 2 compared to the situation where he is the follower. This situation implies that (i) both firms have a first mover advantage (*LL*), or (ii) firm 1 has a first mover advantage while firm 2 has a second mover advantage (*LF*).

In game *K*, firm 2 considers the advertisement intensities to be strategic complements, while firm 1 considers them as strategic substitutes. First, suppose that firm 2 is the leader. Firm 2 will now choose a relatively low advertisement intensity to "force" firm 1 to choose a relatively high intensity level. Firm 2, being the leader, might benefit from this choice since a more intensive advertisement intensity by firm 1 will stimulate the demand for own product as well as the demand for the rival's product. In the opposite situation, firm 1, as leader, will choose a relatively low advertisement intensity in order to stimulate firm 2 to reduce his advertisement intensity. This situation is unequivocally worse for firm 2 relatively to the case where firm 2 acts as leader; however, it might be advantageous for firm 1 relatively to the situation where she acts as follower. This situation implies that (i) both firms have a first mover advantage (*LL*) or (ii) that firm 2 has a first mover advantage and firm 1 a second mover advantage (*FL*).[25]

Next, consider the commitment game for the *medium-conflict* category. It follows from Table 2 that for all eight games that belong to this category, the unambiguous subgame perfect equilibrium is defined by the player having the first mover advantage will choose *Early* while the rival, both when having a first mover advantage and when having a second mover advantage, will choose *Late*. This means that games *F*, *G*, *I*, and *L* produce (*Early*, *Late*) while games *E*, *H*, *J*, and *K* produce (*Late*, *Early*). Hence, all games that are part of the *medium-conflict* category have a unique subgame perfect equilibrium characterized by the player, having an unambiguous first mover advantage, as the first mover, and the rival as the second mover. Additionally, it should be noticed that if we have the situation where the player, possibly having either a first mover advantage or a second mover advantage, prefers to be the follower (as is the case for *R4*), the subgame perfect equilibrium Pareto-dominates the other possible outcomes.

As concerning the two advertisement games belonging to the *medium-conflict* category, the commitment games produce a unique equilibrium being (*Early*, *Late*) for game *G* and (*Late*, *Early*) for game *K*. For game *G*, this means that firm 1 as leader chooses a relatively high advertisement intensity, which implies that the advertisement intensity chosen by firm 2, as follower, will be relatively low. For Game *K*, this means that firm 2 as leader will, given his leading position, choose a relatively low advertisement intensity that, in turn, induces firm 1 to choose a relative high intensity. Furthermore, the assumptions made for game *K* seem to be the most realistic ones compared with the other three advertisement games. Here, the utilities and the marginal utilities of the firms are assumed to be affected in the similar directions (i.e., $U_y < 0$ and $U_{xy} < 0$) at the same time as $V_x > 0$ and $V_{xy} > 0$. This means that firm 1, experiencing a reduction in own demand as firm 2 increases his advertising intensity, also will experience a lesser market effect from own advertising, as firm 2 increases his advertisement intensity. Firm 2, experiencing an increase in own demand as firm 1 increases her advertisement intensity, experiences a higher return from own advertising as firm 1 increases her advertisement intensity. Finally, it should be noted that the two advertisement games belonging to the cases in *G* and *K* differ from the other two advertisement games since for the *high-conflict* advertisement game (game *C*), the subgame perfect equilibrium was defined by simultaneous draws (*Early*, *Early*), while for the *low-conflict* category (game *O*), we ended up with two possible Nash equilibria in pure strategies (*Early*, *Late* and *Late*, *Early*).

4 | CONCLUSIONS

Our study has shown that the distribution of first and second mover advantages primarily depends on the significance and slope of the two reactions functions (whether the decisions are strategic complements or substitutes) and the significance and sign of the marginal payoffs with respect to the rival's decision. Furthermore, allowing the players to differ with respect to the properties of the payoff functions has led to a need for a further clarification of concepts. Now, the strategic properties of the decision variables have to be defined contingent upon each player (player-specific). When a player responds by increasing her value of the decision variable, as the other player reduces (increases) his decision variable, the decisions become strategic substitutes (complements) for this particular player.

The games considered are classified into three categories according to the degree of conflicting preference interests between the actors as concerning the decision variables (conflict categories). For games being characterized by (i) decisions being perceived as being strategic complements by both players combined with each player's payoff function being affected in opposite ways by a change in the decision of the rival and (ii) decisions being perceived as strategic substitutes by both players combined with the payoffs of the players being affected in the same way by a change in the decision of the rival, belong to the *high-conflict* category. Games that belong to this category produce the possibility of a first mover advantage for both. For these *high-conflict* games, a commitment game will result in a subgame perfect equilibrium where both players commit themselves to act as early as possible, that is, (*Early*, *Early*), resulting in simultaneous draws that provide both with the second best payoff of the obtainable ones.

For games that are characterized by either (i) decision variables being strategic substitutes for both players, combined with payoffs being affected in the opposite directions for changes in the rival's decision, and, or (ii) decision variables being strategic complements for both players, combined with payoffs being affected in the same directions for changes in the rival's decision, belong to the *low-conflict* category. Games that belong to this category produce several possible outcomes. One possibility is that both players have a second mover advantage, and another possibility is that one player has a first mover advantage while the rival has a second mover advantage. *Low-conflict* games analyzed as commitment games will result in two subgame perfect equilibria where the players are choosing either to commit as soon as possible or as late as possible. Thus, the equilibria are characterized by leader–follower solutions. In the case where one of them has a first mover advantage, the equilibrium where this player moves first, and the other moves second, will Pareto-dominate the other equilibrium. However, when both players have second mover advantages, there is a coordination problem like in "the battle of the sexes," and the outcome from such situations is not obvious.

Games being part of the *medium-conflict* category are characterized by decisions being perceived as strategic substitutes for one player while being strategic complements for the other. This category of games has not been given the same attention in the literature as the symmetric ones (high-conflict and low-conflict games). These games are characterized by (at least) one of the players having a first mover advantage. For this category, introducing a commitment game, where the players choose whether to commit as early or late as possible to specific actions, results in a unique subgame perfect equilibrium where the player always having a first mover advantage moves *Early*, and the other moves *Late*. In the case where the second player has a second mover advantage, such an outcome Pareto-dominates all other possible outcomes in these games.

As an example of games particularly addressing first and second mover advantages in the cases of asymmetric players, we have discussed possible interactions in advertising behavior between firms supplying substitutes and that compete in price-regulated markets. Generally, firms being asymmetric with respect to how they are influenced by the other firm's advertisement intensity may lead to games of all three conflict categories. However, if the utilities and marginal utilities are affected in the same direction for each firm, and such effects are opposite across firms, the firm with a first mover advantage will choose a relatively low advertisement intensity, while the rival as follower will choose a relatively high advertisement intensity that again stimulates the demands and payoffs for both firms. We believe that this example shows the value of sorting actual game situations into conflict categories when identifying possible outcomes and equilibria.

In Hamilton and Slutsky (1990), we find games that belong to each of our three conflict categories. Their game that corresponds to our case *P* is a *high-conflict* game where it generally is impossible to reach Pareto superior outcomes relatively to the simultaneous equilibrium. Moreover, their game corresponding to our case *A* belongs to the category of *low-conflict* games, while their game corresponding to our case *F* belongs to *medium-conflict* games. For both these games, reaching a Pareto superior outcome, relatively to the simultaneous equilibrium, is possible. As follows from our analysis and from Hamilton and Slutsky (1990), games of type *A*, and the other games belonging to the *low-conflict* category, a portion of both reaction functions is located in the Pareto superior area. However, for case *F*, and the other games belonging to the *medium-conflict* category, only the reaction function of the firm that is likely to draw late in the commitment game is located in the Pareto superior area.

Generally, our reasoning has showed that sorting one-shot two player games into *low*, *medium*, and *high-conflict* games makes it easier to answer the question as to whether it will be preferable for a player to draw first or last and what will be the likely outcome in a commitment game prior to the choice of actions. By applying the conflict categories in combination with the characteristics of the players' utility functions, our systematic overview facilitates the characterization of games as well as identification of first and second mover advantages among players. Our analyses are relevant for all situations where each player's payoff is depending on the actions taken by rivals.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

DATA AVAILABILITY STATEMENT

This paper is only based on theory. Hence, no empirical data have been used.

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ENDNOTES

1 Related to these works, many researchers have recently been engaged in analyzing possible first and second mover advantages for various market situations. In particular, different applications of Bertrand and Cournot games (See Yano and Komatsubara, 2018 and Madden and Pezzino, 2019 and the references therein).

2 By symmetry in this paper is meant that the signs of the derivatives of payoffs with regard to the rival's action are the same and that the actions are either strategic substitutes or complements for both; that is, it does not necessarily mean identical players as is the case in Gal-Or (1985).

3 Consequences of having different signs on first and second order derivatives of the payoff functions for actors, with regard to the other player's decision variable, are discussed in entry deterrence games. For example, Fudenberg and Tirole (1984) show that the incumbent might find it advantageous to commit to "overinvest" and "underinvest" in order to deter the potential market intruder, depending on how the payoff functions and the marginal payoff functions are affected by the other player's decision variable. However, unlike our analysis, Fudenberg and Tirole (1984) restrict their discussions on entry deterrence to cases where the action variables are either strategic substitutes or strategic complements for both firms.

4 Amir (1995), using a counterexample, shows that one of conclusions of Hamilton and Slutsky (1990) is invalid if not including that each payoff is strictly monotone in the other player's action. We are indebted to one of the referees for directing our attention to this work.

5 In contrast to Gal-Or (1985), we describe the simultaneous game since it (more easily) enables us to rank the actions and payoffs of the two players for the various type of games that are presented below. The study of commitment games also makes the simultaneous case relevant.

6 The second order condition for the leader's maximization problem is $U_{xx}+2U_{xy}y_x+U_{yy}y_x^2+U_y y_{xx}<0$ that is supposed to be satisfied. It is seen that this means further restrictions on the *U*- and *V*-functions additional to the concave assumptions.

7 The second order condition for the leader's maximization problem in this case is $V_{yy}+2V_{xy}x_y+V_{xx}x_y^2+V_x x_{yy}<0$, that is supposed to be satisfied. It is seen that this means further restrictions on the *U*- and *V*-functions, additional to the concave assumptions and the restriction in footnote 6.

8 Gal-Or (1985) definition of first and second mover advantages are as follows—there are first (second) mover advantages in a sequential-move game if the leader obtains more (less) payoff than the follower. Here, since allowing for heterogeneous players, the definitions of first and second mover advantages must change. We follow the definitions suggested by Amir and Stepanova (2006) where player *i* has a first (second) mover advantage if its equilibrium payoff in the game where acting as leader (follower) is higher than the equilibrium payoff in the game where acting as follower (leader).

9 The two games with identical actors not considered are $\pi_{xy}=\pi_{yx}>0$ in combination with $\pi_y=\pi_x<0$ and $\pi_{xy}=\pi_{yx}<0$ in combination with $\pi_y=\pi_x>0$.

10 Gal-Or (1985), in the general set up, does not explicitly refer to duopoly markets; however, the assumptions made are in line with standard Stackelberg–Cournot competition and nonsimultaneous Bertrand competition. Furthermore, she provides an example with a linear demand and consider two different games: one where the players choose prices as strategies and one where they choose output levels. Other early contributions that compare quantity and price competition in duopoly markets include Singh and Vives (1984) and Cheng (1985).

11 It is easily seen from 4 that when $y_x=0$, $x^L=x^S$, and $y^F=y^S$, and from 5, it follows that when $x_y=0$, $x^F=x^S$ and $y^L=y^S$. Moreover, when $y_x=0$, $y^L>(<)y^S$ as $V_x x_y>(<)0$, and $x^F>x^S$ when $x_y>0$ and $y^L>y^S$ and when $x_y<0$ and $y^L<y^S$, and $x^F<x^S$ when $x_y<0$ and $y^L>y^S$ and when $x_y>0$ and $y^L<y^S$. In the opposite case, when $x_y=0$, $x^L>(<)x^S$ as $U_y y_x>(<)0$, and $y^F>y^S$ when $y_x>0$ and $x^L>x^S$ and when $y_x<0$ and $x^L<x^S$, and $y^F<y^S$ when $y_x<0$ and $x^L>x^S$ and when $y_x>0$ and $x^L<x^S$. However, we do not compare cases where x_y and/or y_x are equal to zero.

12 From Table 1, we observe that in total, 16 cases are identified; however, the table could be simplified by ignoring those cases that are redundant due to symmetry, for example, by blanking the upper off-diagonal part of the table.

13 More accurately, game *B* is characterized by preference orders where player 1's utility is increasing as one moves north/north-east, while player 2's utility is increasing as one moves west/south-west. For game *C*, the preference order for player 1 is south/south-west and east/north-east for player 2; for game *M*, the preference order is north/north-west for player 1 and east/south-east for player 2, and finally for game *P*, the preference order is south/south-east for player 1 and west/north-west for player 2.

14 However, for all these games, the preference orders for the players will be slightly different. For instance, in game *B*, the utility for player 1 is increasing when moving along her reaction function in the direction of north/north-east, while player's utility is increasing when moving along his reaction function in the direction of east/north-east. The same type of nuances applies for games *D*, *N*, and *O*.

15 This finding is similar to a result by Amir and Stepanova (2006). They assume that U_y and V_x always are positive and find that the player with a downward-sloping reaction function always has a first mover advantage.

16 See Fig. 5a at page 40 (Hamilton & Slutsky, 1990).

17 Such a game is discussed in a model analyzing traffic behavior (see Pedersen, 2003).

18 Bergland and Pedersen (2019) present a game concerned with traffic safety having such properties.

19 A similar assumption is made by Amir and Stepanova (2006).

20 The symmetric case assumes that the impact on each other's actions is the same, thus producing ranking *R1*.

21 See Fig. 5b at page 40 (Hamilton & Slutsky, 1990).

[22] Bergland and Pedersen (2019) present a game concerned with traffic actions having such properties.

[23] Games within the *low-conflict category* are defined by having a (almost) common preference order related to the decision variables x and y. It may seem surprising that Bertrand duopoly, normally discussed and characterized as "fierce competition," is belonging to this category. However, this stems from the fact the leader–follower solutions in price competition make it possible for higher prices compared to the situation of simultaneous moves, surely gaining both firms. It should be noticed, however, that even when the players agree on which direction one should move in the xy diagram, they could end up disagreeing on which of the two possible equilibria they will prefer. Hence, in the case of price competition, there is no disagreement that both prices should be high, even though the firms may prefer different equilibria.

[24] See Fig. 5c at page 40 (Hamilton & Slutsky, 1990).

[25] Grepperud and Pedersen (2020) present a game on lobbying and campaigning games having the same properties as case K.

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APPENDIX A.

Based on 3–5, and the properties of the payoff functions, it is possible to compare the quantities arrived at in the simultaneous game with the games where player 1 is leader (follower) and player 2 follower (leader). First, comparing (x^S, y^S) with (x^L, y^F), it follows that:

$$\begin{aligned} &\text{When } U_y > 0 \text{ and } V_{xy} > 0 \text{ it follows that } x^L > x^S \text{ and } y^F > y^S \\ &\text{When } U_y > 0 \text{ and } V_{xy} < 0 \text{ it follows that } x^L < x^S \text{ and } y^F > y^S \\ &\text{When } U_y < 0 \text{ and } V_{xy} > 0 \text{ it follows that } x^L < x^S \text{ and } y^F < y^S \\ &\text{When } U_y < 0 \text{ and } V_{xy} < 0 \text{ it follows that } x^L > x^S \text{ and } y^F < y^S \end{aligned} \tag{A1}$$

Secondly, by comparing (x^S, y^S) with (x^F, y^L), it follows that:

$$\begin{aligned} &\text{When } V_x > 0 \text{ and } U_{xy} > 0 \text{ it follows that } y^L > y^S \text{ and } x^F > x^S \\ &\text{When } V_x > 0 \text{ and } U_{xy} < 0 \text{ it follows that } y^L < y^S \text{ and } x^F > x^S \\ &\text{When } V_x < 0 \text{ and } U_{xy} > 0 \text{ it follows that } y^L < y^S \text{ and } x^F < x^S \\ &\text{When } V_x < 0 \text{ and } U_{xy} < 0 \text{ it follows that } y^L > y^S \text{ and } x^F < x^S \end{aligned} \tag{A2}$$

From the above comparisons, it is seen that $U_y < 0$, $U_{xy} < 0$, $V_x < 0$, and $V_{xy} < 0$ gives the following unambiguous rankings: $x^F < x^S < x^L$ and $y^F < y^S < y^L$ (see *P* in Table A1). It also follows that for $U_y > 0$, $U_{xy} < 0$, $V_x > 0$, and $V_{xy} < 0$ (see *M* in Table A1), we get the following unambiguous rankings: $x^L < x^S < x^F$ and $y^L < y^S < y^F$ while for $U_y < 0$, $U_{xy} > 0$, $V_x > 0$, and $V_{xy} > 0$ (see *C* in Table A1), we get the following unambiguous rankings: $x^L < x^S < x^F$ and $y^F < y^S < y^L$. Moreover, for $U_y > 0$, $U_{xy} > 0$, $V_x < 0$, and $V_{xy} > 0$ (see *B* in Table A1), we get the following unambiguous rankings: $x^F < x^S < x^L$ and $y^L < y^S < y^F$. Finally, for games belonging to the *high-conflict* category, there exists an unambiguously ranking of payoffs (see *B*, *C*, *M*, and *P* in Table A1), which is defining a first mover advantage for both players, that is,

$$U^F < U^S < U^L \text{ and } V^F < V^S < V^L \tag{A3}$$

(for the reasoning behind A3, see the main text.) For the remaining 12 combinations of payoff function properties, there exist no unique rankings of payoffs and *x* and *y* for the two leader–follower solutions.

Now, by going systematically through all remaining combinations of signs for U_y, U_{xy}, V_x, and V_{xy}, using the information already obtained in A1 and A2, it is seen that for 8 games (*E*–*H* and *I*–*L* in Table A1), two possible quantity rankings appear. For games *E*, *H*, *J*, and *K*, the two possible rankings are related to the values of *x*, while for *F*, *G*, *I*, and *L*, the two possible rankings are related to the values of *y*. Moreover, when using the payoff functions, it is seen that the two possible rankings of *x* lead to analogous possible ranking of the payoffs for player 2, while the two possible rankings of *y* lead to analogous rankings of the payoffs for player 1 (see Table A1). Consequently, at least one player will have a first mover advantage in these cases, possibly both; see the reasoning in Section 3 for further details.

An analogous procedure, based on A1 and A2 for games *A*, *D*, *N*, and *O*, leads to four possible outcomes. However, one of these, the one that expresses a first mover advantages for both players, is not consistent with our prior assumptions. An alternative approach for studying the possible rankings of the two leader–follower solutions for games *A*, *D*, *N*, and *O* is to consider the consequences from moving from a situation, where player 1 is follower and player 2 leader, to the opposite situation. Differentiating Equations 4 and 5, defining $\Delta x = x^L - x^F$, $\Delta y = y^F - y^L$, $\theta = -U_y \frac{V_{xy}}{V_{yy}}$, and $\mu = -V_x \frac{U_{xy}}{U_{xx}}$, considering θ and μ as sufficiently small differences in 4 and 5, we get

$$\Delta x = \frac{-V_{yy}\theta - U_{xy}\mu}{\sigma} \text{ and } \Delta y = \frac{U_{xx}\mu + V_{xy}\theta}{\sigma} \tag{A4}$$

where $\sigma = U_{xx}V_{yy} - U_{xy}V_{xy} > 0$ (fulfilled by prior assumptions). From A4, it is seen that the signs from changes in *x* and *y*, as player 1 moves from being the follower to becoming the leader and player 2 moves from being the leader to becoming the follower, are determined by the signs of the numerators in A4. Using the definitions of θ and μ in these numerators gives the following:

$$n_1 = -V_{yy}\theta - U_{xy}\mu = U_yV_{xy} + \frac{V_x(U_{xy})^2}{U_{xx}} \text{ determining the sign of } \Delta x = x^L - x^F \tag{A5}$$

$$n_2 = U_{xx}\mu + V_{xy}\theta = -V_xU_{xy} - \frac{U_y(V_{xy})^2}{V_{yy}} \text{ determining the sign of } \Delta y = y^F - y^L \tag{A6}$$

As seen from A5 and A6, the quantitative changes in *x* and *y*, when player 1 moves from being the follower to becoming the leader, and player 2 moves from being the leader to becoming the follower, consist of two terms. The first term in A5, U_yV_{xy}, represents a direct effect on player 1 as she becomes the leader instead of being the follower, while the second term in (A5), $\frac{V_x(U_{xy})^2}{U_{xx}}$, represents an indirect effect, capturing that the follower no longer acts as a leader. The same interpretation is relevant for the two terms in A6. For games *A*, *D*, *N*, and *O*, the two terms in both A5 and A6 have opposite signs. Now, by going systematically through the combinations of signs for U_y, U_{xy}, V_x, and V_{xy}, we get for games *A*, *D*, *N*, and *O* that the direct effects lead to $x^L > x^F$ and $y^L > y^F$ for *A*, $x^L < x^F$ and $y^L < y^F$ for *D*, $x^L < x^F$ and $y^L > y^F$ for *N*, and $x^L > x^F$ and $y^L < y^F$ for *O*. Given identical players (symmetric), the direct effects, measured by the first term in both A5 and A6, will always dominate the indirect ones measured by the second term in both A5 and A6, that is, when $U_y = V_x$ and $U_{xy} = V_{xy}$, $x^L - x^F = U_yU_{xy}(1 - x_y)/\sigma$, and $y^L - y^F = V_xV_{xy}(1 - y_x)/\sigma$. This is the result arrived at by Gal-Or (1985), leading to second mover advantages for both players (see *A*). This conclusion also holds for game *D*. Returning to the general case of heterogeneous players related to case *A* and *D*, that also fits the asymmetric cases *N* and *O*; suppose now that the indirect effect dominates the direct effect for one of the players (player 1). Using game *A* as an example, this means that $U_yV_{xy} + \frac{V_x(U_{xy})^2}{U_{xx}} < 0$. Multiplying by the negative value $\frac{U_{xx}}{U_{xy}}$ gives

TABLE A1 Possible payoff and quantity (x and y) rankings across games

		$V_{xy} > 0$		$V_{xy} < 0$	
		$V_x > 0$	$V_x < 0$	$V_x > 0$	$V_x < 0$
$U_{xy} > 0$	$U_y > 0$	(A) $x^S < x^F < x^L$ $y^S < y^F < y^L$ $U^S < U^L < U^F$ $V^S < V^L < V^F$ or $x^S < x^F < x^L$ $y^S < y^L < y^F$ $U^S < U^F < U^L$ $V^S < V^L < V^F$ or $x^S < x^L < x^F$ $y^S < y^F < y^L$ $U^S < U^L < U^F$ $V^S < V^F < V^L$	(B) $x^F < x^S < x^L$ $y^L < y^S < y^F$ $U^F < U^S < U^L$ $V^F < V^S < V^L$	(E) $x^L < x^S < x^F$ $y^S < y^F < y^L$ $U^S < U^L < U^F$ $V^F < V^S < V^L$ or $x^L < x^S < x^F$ $y^S < y^L < y^F$ $U^S < U^F < U^L$ $V^F < V^S < V^L$	(F) $x^L < x^F < x^S$ $y^L < y^S < y^F$ $U^F < U^S < U^L$ $V^S < V^L < V^F$ or $x^F < x^L < x^S$ $y^L < y^S < y^F$ $U^F < U^S < U^L$ $V^S < V^F < V^L$
	$U_y < 0$	(C) $x^L < x^S < x^F$ $y^F < y^S < y^L$ $U^F < U^S < U^L$ $V^F < V^S < V^L$	(D) $x^L < x^F < x^S$ $y^L < y^F < y^S$ $U^S < U^L < U^F$ $V^S < V^L < V^F$ or $x^L < x^F < x^S$ $y^F < y^L < y^S$ $U^S < U^F < U^L$ $V^S < V^L < V^F$ or $x^F < x^L < x^S$ $y^L < y^F < y^S$ $U^S < U^L < U^F$ $V^S < V^F < V^L$	(G) $x^S < x^F < x^L$ $y^F < y^S < y^L$ $U^F < U^S < U^L$ $V^S < V^L < V^F$ or $x^S < x^L < x^F$ $y^F < y^S < y^L$ $U^F < U^S < U^L$ $V^S < V^F < V^L$	(H) $x^F < x^S < x^L$ $y^L < y^F < y^S$ $U^S < U^L < U^F$ $V^F < V^S < V^L$ or $x^F < x^S < x^L$ $y^F < y^L < y^S$ $U^S < U^F < U^L$ $V^F < V^S < V^L$
$U_{xy} < 0$	$U_y > 0$	(I) $x^S < x^F < x^L$ $y^L < y^S < y^F$ $U^F < U^S < U^L$ $V^S < V^L < V^F$ or $x^S < x^L < x^F$ $y^L < y^S < y^F$ $U^F < U^S < U^L$ $V^S < V^F < V^L$	(J) $x^F < x^S < x^L$ $y^S < y^F < y^L$ $U^S < U^L < U^F$ $V^F < V^S < V^L$ or $x^F < x^S < x^L$ $y^S < y^L < y^F$ $U^S < U^F < U^L$ $V^F < V^S < V^L$	(M) $x^L < x^S < x^F$ $y^L < y^S < y^F$ $U^F < U^S < U^L$ $V^F < V^S < V^L$	(N) $x^L < x^F < x^S$ $y^S < y^F < y^L$ $U^S < U^L < U^F$ $V^S < V^L < V^F$ or $x^F < x^L < x^S$ $y^S < y^F < y^L$ $U^S < U^L < U^F$ $V^S < V^F < V^L$ or $x^L < x^F < x^S$ $y^S < y^L < y^F$ $U^S < U^F < U^L$ $V^S < V^L < V^F$
	$U_y < 0$	(K) $x^L < x^S < x^F$ $y^L < y^F < y^S$ $U^S < U^L < U^F$ $V^F < V^S < V^L$ or $x^L < x^S < x^F$ $y^F < y^L < y^S$ $U^S < U^F < U^L$ $V^F < V^S < V^L$	(L) $x^F < x^L < x^S$ $y^F < y^S < y^L$ $U^F < U^S < U^L$ $V^S < V^L < V^F$ or $x^L < x^F < x^S$ $y^F < y^S < y^L$ $U^F < U^S < U^L$ $V^S < V^F < V^L$	(O) $x^S < x^F < x^L$ $y^L < y^F < y^S$ $U^S < U^L < U^F$ $V^S < V^L < V^F$ or $x^S < x^F < x^L$ $y^F < y^L < y^S$ $U^S < U^F < U^L$ $V^S < V^L < V^F$ or $x^S < x^L < x^F$ $y^L < y^F < y^S$ $U^S < U^L < U^F$ $V^S < V^F < V^L$	(P) $x^F < x^S < x^L$ $y^F < y^S < y^L$ $U^F < U^S < U^L$ $V^F < V^S < V^L$

$$V_x U_{xy} + \frac{U_y (V_{xy})^2}{V_{yy}} \frac{U_{xx} V_{yy}}{U_{xy} V_{xy}} > 0 \quad \text{(A7)}$$

Now, if the indirect effect for player 2 dominates the direct effect, it follows from A6 that:

$$V_x U_{xy} + \frac{U_y (V_{xy})^2}{V_{yy}} < 0 \quad \text{(A8)}$$

Now, we use the assumption that $\sigma = U_{xx}V_{yy} - U_{xy}V_{xy} > 0$, that is, $\frac{U_{xx}V_{yy}}{U_{xy}V_{xy}} > 1$, which with necessity leads to the conclusion that conditions A7 and A8 cannot be satisfied; that is, both indirect effects could be dominating at the same time. The same conclusion follows for games *D*, *N*, and *O*. Hence, as identified in Table A1, there are three possible rankings of *x* and *y* for each game. For game *A*, these three rankings are (*a*) $x^S < x^F < x^L$ and $y^S < y^F < y^L$, (*b*) $x^S < x^L < x^F$ and $y^S < y^F < y^L$, or (*c*) $x^S < x^F < x^L$ and $y^S < y^L < y^F$. Moreover, for game *D*, the three possible rankings are (d) $x^L < x^F < x^S$ and $y^L < y^F < y^S$, (e) $x^L < x^F < x^S$ and $y^F < y^L < y^S$, or (f) $x^F < x^L < x^S$ and $y^L < y^F < y^S$. The rankings of games *N* and *O* are determined in the same way (see Table A1). The common feature for games *A*, *D*, *N*, and *O* is that they all lead to three possible distributions of first and second mover advantages (outcomes).

APPENDIX B.

The advertisement example presented in Section 3 is defined by the following payoff functions (profit functions)

$$U(x,y) = R(x,y) - C(x) \text{ and } V(x,y) = Q(x,y) - D(y) \quad \text{(B1)}$$

where *R* is the revenue of firm 1, *C* is advertising costs for firm 1, *Q* is the revenue of firm 2, and *D* is the advertising costs of firm 2. Due to the example given in the text, it is supposed that $R_x > 0, C_x > 0, Q_y > 0, D_y > 0, R_y < 0, Q_x > 0, R_{xx} < 0, Q_{yy} < 0, C_{xx} \geq 0$ and $D_{yy} \geq 0$. Before proceeding, it should be remarked that $R_{xy} = U_{xy}$ and $Q_{xy} = V_{xy}$. Then it follows that:

For game *C*, the following assumptions hold in addition to the assumptions made related to B1:

$$R_{xy} > 0 \text{ and } Q_{xy} > 0 \quad \text{(B2)}$$

In this case, firm 2's marginal revenue with regard to its own advertising becomes higher as *x* is increased ($Q_{xy} > 0$), and the higher the advertising originally is for firm 1, the less is the reduction in its demand and revenue following from an increased advertising from firm 2 ($R_{xy} > 0$).

For game *G*, the following assumptions hold in addition to the assumptions made related to B1:

$$R_{xy} > 0 \text{ and } Q_{xy} < 0 \quad \text{(B3)}$$

In this case, firm 2's marginal revenue with regard to its own advertising becomes lower as *x* is increased ($Q_{xy} < 0$), and the higher the advertising is originally for firm 1, the less is the reduction in its demand and revenue following from an increased advertising from firm 2 ($R_{xy} > 0$).

For game *K*, the following assumptions hold in addition to the assumptions made related to B1:

$$R_{xy} < 0 \text{ and } Q_{xy} > 0 \quad \text{(B4)}$$

In this case, firm 2's marginal revenue with regard to its own advertising becomes higher as *x* is increased ($Q_{xy} > 0$), and the higher the advertising is originally for firm 1, the more significant is the reduction in its demand and revenue following from an increased advertising from firm 2 ($R_{xy} < 0$).

For game *O*, the following assumptions hold in addition to the assumptions made related to B1:

$$R_{xy} < 0 \text{ and } Q_{xy} < 0 \quad \text{(B5)}$$

In this case, firm 2's marginal revenue with regard to its own advertising becomes lower as *x* is increased ($Q_{xy} > 0$), and the higher the advertising is originally for firm 1, the more significant is the reduction in its demand and revenue following from an increased advertising from firm 2 ($R_{xy} < 0$).