Using mathematical ideas from carpet and carpet-weavers as a context for designing mathematics tasks

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Abstract

In this study, the mathematics of carpet will be introduced by presenting the lifestyle of two expert carpet-weavers from Kerman, Iran, who work for many years in carpet-weaving activities through an explanation of carpet weavers' culture. This explanation reveals that carpet weavers can do mathematics and solve related real-world problems without academic education in mathematics according to their needs through practical activities. The main purpose of this study is to investigate the mathematical ideas in the art of carpet weavers, and the ethnography approach is used as a methodological framework. Our findings showed that there are many mathematical concepts in the carpet weaving process, such as mirror axes, parallel and diagonal lines, geometric shapes, ratio, and measurement which can be used as context for developing enrich and meaningful mathematical tasks.

Keywords: Carpet-weaver, Ethnography, Ethnomathematics, Math Activities

D'Ambrosio (2006) define ethnomathematics as a research program about the history and philosophy of mathematics and it is also the program of the way in which cultural groups understand, articulate, and use the concepts and practices, which is described as mathematical, whether or not the cultural group has a concept of mathematics. Since the emphasis on the eastern carpet has been little, while it has a long history (Bier, 2001); this study concentrates on the Kerman carpet which is a kind of Persian carpet as one of the unique carpets in the world and tries to introduce mathematical concepts in processes of carpet production with the hand of carpet-weavers. Kerman carpets are natural carpets and are full of patterns. The variety of carpet dimensions is high. So that from small carpets (less than 1 square meres) to large carpets of 47 square meters, it can be seen among Kerman carpets. In the definition of carpet, it should be said that it is called a hand-woven and knotted carpet. Smaller dimensions of carpet are called rugs. The point that can be mentioned is the difference between the kilim and the carpet, which is in the fluff and the designs that are implemented on the carpet. In Figure 1, an example of the Kerman carpet diagram is illustrated.

Figure 1. Kerman carpet diagram

The art of carpet weaving has a long history in Kerman, Iran. Initially, people did Shawl weaving (Edwards, 1953) but later the carpet industry was developed. The industry has grown so much that the Kerman carpets woven in this region have become the typical types of carpet weaving in Iran and the world. For a carpet-weaver, weaving a carpet or a rug is no different, but there is a difference in the type of knitting that is done in two types of Persian and Turkish knots (See Figure 2).
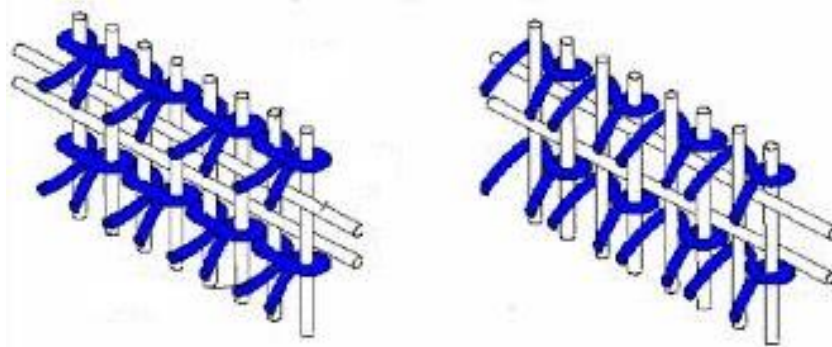

Figure 2. Turkish knots (left hand) versus Persian knots (right hand)

This paper aims to investigate and recognize mathematical concepts and ideas used by carpet-weavers and then integrate these ideas into the school mathematics. In this study, we use real world concrete mathematics ideas for designing mathematical activities.

Today almost all Iranian families have Iranian carpets in their house. So, carpet weaving is a meaningful context for Iranian students as a cultural phenomenon. Bishop (1988) believes that the mathematics curriculum should always consider local culture in both content and method. Indeed, Bishop (1988) distinguishes between Mathematics (with a capital M) and mathematics. He refers to "Mathematics" as western math that omits different cultural backgrounds. In contrast, Bishop (1988) uses "mathematics" for mathematical knowledge that has a cultural and historical basis. In this regard, D'Ambrosio (2006) refers to ethnomathematics as mathematics practiced by every cultural group. Ethnomathematics is the study of the relationship between mathematics and culture. The idea of ethnomathematics came as a broader view of how mathematics relates to the real world. D'Ambrosio (2001) point out that there are different ethnomathematics, each one responding to a different cultural, natural, and social environment. D'Ambrosio (2006) therefore considers ethnomathematics as a combination of three words as follow,

- ETHNO: The natural social-cultural and imaginary environment.
- MATHEMA: of explaining learning knowing and coping with.

- TICS: modes, styled, arts and techniques.

Ethno-mathematician believe that mathematics is a cultural product with a long history that can emerge and develop in certain cultural, economic, and social conditions. They believe that teaching critical mathematics can enable students to reflect on and develop the realities in which they live, and to use and empower themselves as a liberating tool in mathematics (Gerdes, 1994). So, in this area we are looking for hidden mathematics that exists in cultural groups and this mathematics should be used in the curriculum in a way that facilitates knowledge, understanding and integration of common methods in the curriculum. Pradhan (2020) try to identify mathematical ideas in mat weaving of people who live in rural part of Nepal and use them for classroom teaching.

Ethno-mathematics examples are not only a resource for teaching formal mathematical concepts in school mathematics but can also be investigated further for their higher mathematical insights commonly pursued in academic mathematics (Matang, 2002). Although many researchers recommend considering ethnomathematics in mathematics textbooks (Katz, 1994; D'Ambrosio, 2006; Orey & Rosa, 2004; Rosa & Orey, 2007; Putra, 2018) and teacher education programs (Presmeg, 1998), there is little emphasis on Iranian mathematics textbooks and teacher education programs in this regard. For example, in the new version of the Iranian mathematics textbook in grade ten, there are only five examples that have historical aspects and all of them are classified as an illumination approach in terms of Jankvist (2009). So, we can say new version of Iranian textbooks have not implemented the strategy of using culturally relevant situations to help students for learning mathematics.

In this study, we use ethno-modeling as a research lens. Because this lens helps us to search contexts for rich mathematical tasks. Rosa and Orey (2010) define ethno-modeling as a process of translation and elaboration of problems taken from systems that are part of the daily life of the members of any given cultural group. Rosa (2000) considers ethnomathematics as the intersection of three research fields as cultural anthropology, mathematics, and mathematical modelling which is used to help us to understand and connect diverse mathematical ideas that found in our communities and academic mathematics.

Verschaffel (2002) explains that there are many different descriptions of the modelling cycle with only slightly different interpretations, and commonalities among them are strong. So, in this study, we use Rafiepour, Stacey, and Gooya's (2012) explanation of the modelling cycle. They believed the modelling process starts with a problem situated in the Extra-Mathematical World (an EMW problem). Then the modelling process continues with formulating the EMW problem in mathematical terms. The problem now formulated mathematically is solved by the application of mathematical concepts and procedures. Finally, the mathematical solution must be interpreted to provide an answer to the EMW problem, and then checked for its adequacy in answering the original question (if a new cycle of modelling process must repeat). The main research question which guides this study was what the mathematical ideas within the context of carpet weaving that are could be used for task design in school mathematics activities.

METHODS

In this study, we use an anthropological approach (Flick, 2009) because we were looking for a method that could work well to extract mathematical ideas in the carpet weaver's activities. This research has close relations to the culture of people who weave carpets, so an anthropological approach was selected for conducting this study.

Participants of this study were two expert people who weaved carpets for many years (more than 60 years) in Kerman. The first one was an old woman (Grandmother) of 75 years who started carpet weaving when she was six years old. She did not read and write, even to the extent of writing her name. In interviews, she mentioned that in rural areas, carpet weaving was important because of living conditions. Carpet weaving was more interesting than going to school. She learned carpet weaving from her parents in childhood. She had seen most of the calculations in the workplace, so she was able to calculate easily without formal instruction. For example, she was able to divide specific amount of money between 15 carpet weavers who work jointly in weaving a carpet. The second participant was a 66-year-old man (Mister M) who had lost his parents when he was a teenager. Thus, he had to work for his subsistence. Mister M had some formal education just in level 2 at primary school. He could write and read and did his arithmetic's calculation in his workplace.

In the current study, we used field notes, interviews, pictures, film, and artifacts (weaving a small carpet) as instruments for data collection. First, we conducted semi-structured interview with two carpet weavers (Grandmother and Mister M) in five 90 minutes' session and we visit their nearby workshop which people were working on a carpet. We take picture and film during our visiting. Carpet weavers use a sort of poem for following map reading during their weaving. The carpet weavers' workshop was located near the city (Zarand) that was close to the author's resident (one-hour driving distance from Kerman). In semi-structured interview we prepare some question in advanced, (e.g., her or his name, when he or she starts to be weaving carpet, what is her or his formal education and schooling level) and then leave them to explain their life experience in carpet weaving. In this case we heard several interesting stories from their life, and we try to find mathematical ideas that emerged during this interview. We try to note every relevant idea as field notes. We discuss about these writhed ideas with Grandmother and Mister M for better understanding the life and culture of carpet weavers. We also use carpet maps (e.g. see Figure 1) as artifact which is most important things in carpet weaving. For data analysis we use qualitative content analysis and try to find any related ideas to mathematics. Then we try to unify this idea and get more evidence from data collected in this regard.

RESULTS AND DISCUSSION

The results show that carpet weavers easily use mathematical concepts in their everyday activities but when they encounter formal and academic mathematical problems, they couldn't use their knowledge to respond to the problems. Because when we ask grandmother to divide two digits, she said I couldn't do that. But when we ask her, please divide 480 Toman (Official Currency in Iran) between 15 carpet weavers, she do this calculation easily. Results of a semi-structured interview with grandmother show that:

- She knows four basics arithmetic operation and uses them in real-world problems with her style (without using any thought). She said that she has gradually learned the relevant calculations experimentally according to her need to calculate the number of yarns and the amount of wool purchased.
- She could recognize different geometrical shapes and calculate the area and perimeter of them.
- She could recognize parallel, diagonal, and Perpendicular lines.
- One of the most important points in carpet-weaving is preparing the wool needed for carpet weaving. Because if this type of calculation is not careful, the amount of wool purchased will be very high and will cost a lot, while the amount of extra wool cannot be used in other rugs. On the

other hand, if the calculations about buying wool are less than the actual amount, it is very difficult and sometimes impossible to prepare this dye. Experienced carpet-weavers like grandmother was able to calculate the appropriate amount of wool that needed for a carpet.

- She was able to recognize Cartesian coordinate system very well.

For example, when we asked the grandmother to divide 480 by 15, she said she couldn't do that. But when we asked her to divide 480 Toman (Iranian currency at this time) between 15 people evenly, grandmother easily calculated the quotient, including a couple of digits after the decimal. Grandmother responded to the division problem showed in Figure 3.

I consider 15 in the form of 10+5. After that I try to find a digit which if I multiply to 10+5, the result becomes closer to 480 Toman (Iranian currency currently). In two steps she finds 30 which if 30 is multiplied by 10+5 we have 450. But there is a difference between 450 and 480. So, she subtracted 450 from 480 and find 30. At this stage she was searching for another digit which if multiplied by 10+5, the result becomes closer to 30. She finds 2 very fast. So, the answer of the division is 32.

$$\begin{cases}10 \times 30 = 300 \\ 5 \times 30 = 150\end{cases} \rightarrow 300 + 150 = 450$$

$$\begin{cases}10 \times 2 = 20 \\ 5 \times 2 = 10\end{cases} \rightarrow 20 + 10 = 30 \longrightarrow \boxed{450 + 30 = 480}$$

Figure 3. Division without remainder by one of carpet-weaver (grandmother)

Carpet-weavers usually sing a poem with rhythm during their works. The whole map of a carpet is formed on very small squares and the main map of the carpet is drawn on these squares (see Figure 4). Carpet weavers sing very beautiful poems when weaving carpets, and these poems are in fact determining the coordinates of the colored dots that the weavers must tie in the specified places.

Figure 4. Map of carpet designed on squares

If the design of the carpet is one weaver, the weaver and the reader are same. But if the pattern of the carpet is double weaving, one of the two people who are weaving the carpet will read the plan of the carpet design and they weave carpet together. Singing while weaving a carpet has a weight and

melody that is in harmony with tying the strings. In such a way that the syllables of the song are adjusted with the speed of the knots. Therefore, the carpet weaver reads more than all the other stages of weaving when tying, and when reading weaving, the reading becomes slower and when the time for tapping the shoulder comes, it becomes silent, because his reading is not commensurate with the sound of tapping the shoulder and is not rhythmic. Expression of these features of using mathematics in a real-world context could encourage and motivate students to learn more mathematical concepts for their future life. Results of a semi-structured interview with Mister M. show that:

- He Understands coordinates and he was able to use it in appropriate way.
- He could calculate.
- He could recognize different geometrical shapes and calculate the area and perimeter of them.
- And he had a good conception of fractions, ratios.

For example, for determining the amount of cotton and wool for weaving a carpet, Mr. M. explains by using proportion concept as below:

- (Interview with Mr. M.): We need 5 kilograms of cotton and wool for 1 square meter (10000 square centimeters) carpet. Accordingly, we need 2/5 kilograms of cotton and wool for 5000 square centimeters carpet, and we must have 1/25 kilograms of cotton and wool for 2500 square centimeters carpet. So, you must buy 1/5 kilograms of cotton and wool for your carpet.
- (Interview with Mr. M.): in responding to a question about how carpet weavers design and determine the margin and border of carpet Mr. M. says this is dependent on customer opinion when they ordered carpet. But, usually, the margin in the length of the carpet is half of one-third of the width of the carpet. In this case, Mr. M. uses the concept of ratio fluently.

The very important point that careful calculation of the amount of wool needed for a carpet is necessary to make sure the weavers don't run out of a particular batch of dyed wool before the carpet is finished. One sees carpets where one shade of blue or red suddenly changes at one end of the carpet. That is the result of improper calculation, and it lowers the value of the carpet. Usually, the coloring of the carpet yarns is done manually and the type of wool, the amount of paint, and even the temperature of the dyeing pot can affect the color of yarns and carpet. So, if we want to buy a little bit more yarn with the same color, it's so difficult. In some cases, it is impossible to find that yarn. Hence, accurate calculation is very important in this context. As an example, related to this context, we designed two activities.

- We need five kilograms of cotton and wool for a carpet with a one-meter square area. How much cotton and wool we need for weaving a carpet with 30cm *30cm dimensions.
- Calculate the proportion of borders (see Figure 5) of a 3 meter by 4-meter carpet with respect to whole areas of 3 meter by 4-meter carpet?

Figure 5. borders of a carpet

An expert carpet-weaver can estimate the end of her/his carpet-weaving project because she/he must exact plan for the cost and benefit of the project. In this context, the below activity could be designed.

- Zahra weaves 5 rows (lines) of a carpet per day. How many days long for weaving a 2 meter by 3-meter carpet? (There are 1100 rows/lines for a 3 meter by 4-meter carpet)

Carpets have different symmetry, so the student can find these symmetries through engaging in real-world activities related to finding symmetries of carpets. In this way, the map of the right quarter of the carpet is prepared and one person sits on the right side of the carpet and one person on the left side of the carpet. One of the carpet weavers starts reading the carpet map. While the carpet weaver wants the codes in the form of a poem, the weaving path is determined. By carpet weaver, two people can weave the carpet symmetrically between the right and left sides in the lower half. For the second half of the carpet, each of the carpet patterns rotates 180 degrees at the top, and as before, one person reads the map and two people weave it.

- Draw the mirror axes in carpets shown in Figure 6. How many mirror axes are there on this carpet?

Figure 6. A beautiful carpet and mirror axes

CONCLUSION

This study reported results of an ethnography approach for studying carpet weavers in Kerman. The findings of this study show there are many mathematical concepts and ideas in the carpet weaving process, such as mirror axes, parallel and diagonal lines, geometric shapes, measurement, ratio, and

decimals. Some examples of these mathematical ideas were reported in the previous section. Finding of current study is consistent with some other studies' such as (Jurdak & Shahin, 1999; Millroy, 1992; Pradhan, 2020).

We think that mathematical ideas which appear in cultural context of students' daily life could be and should be used in the process of teaching mathematics at school. Although we agree with D'Ambrosio (2006, p. 31) that "it is a big mistake to think that ethnomathematics can substitute academic mathematics, which is essential for an individual to be an active being in the modern world". But we believe that using ethnomathematics approaches in mathematics classrooms and textbooks could help students to make sense of mathematical concepts. Although some researchers like Rowlands and Carson (2002; 2004) made epistemological and educational critiques on ethnomathematics. They highlight academic mathematics as one of the biggest achievements of mankind. But several researchers recount the advantages of using ethnomathematics activities in the process of teaching and learning mathematics (Millroy, 1992; Katz, 1994; D'Ambrosio, 2006; Rosa & Orey, 2007; Putra, 2018). As a more specific example, Matang (2002) integrates ethnomathematics into the formal mathematics curriculum as one way to address students learning difficulties in Papua New Guinea. Teaching mathematics via the ethnomathematics approach will also formalize the students' ethnomathematical knowledge gained through practical experiences in which students also develop a sense of ownership of their knowledge.

Also, it seems that the carpet-weaving workshop along with the problem-solving and modeling workshop could help students to realize abstract mathematical concepts in a real-world context and deepen their understanding of mathematics. In this regard, Bier (2000) proposed to use of square grids in drawing patterns in hand-woven carpets based on the application of addition, subtraction, multiplication and division, and other activities such as squares and square roots. This study showed the potential of using ethnomathematics ideas in school mathematics curriculum. We recommend further research to reveals more ethnomathematics ideas in other real-life activities which is related to students' daily life. Finally, we expect that studies like these contribute to the enrichment of mathematics textbooks such that students become motivated (Matang, 2002) creativity fostered (D'Ambrosio, 2006) to facilitate constructive and reflective citizens (Rafiepour, 2012).

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