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Commodity Impacts on Time-Varying Market Efficiency

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Abstract

This study builds on the Efficient market Hypothesis (EMH) and the Adaptive Market Hypothesis (AMH) to explore the time-varying characteristics of market efficiency in global equity markets. The research applies Adjusted Market Inefficiency Magnitude (AMIM) and the Hurst exponent to measure efficiency in various markets. This study additionally investigates the impact of commodity futures prices on market efficiency, revealing a heightened sensitivity during periods of market turmoil. The relationship between commodity futures and market efficiency is explored through binomial and multinomial regression in generalized linear models. We find that market efficiency and its dependency on commodities varies substantially over time, with large and abrupt changes during times of global market turmoil. In North America, Far East, and Europe, market efficiency's dependence on corn during covid-19 is consistent across our models. Further discrepancies between models show that there is evidence on long memory under volatile regimes.

Preface

This thesis is the concluding assignment of the Master of Science in Business education at Nord University in Bodø. Through the specialization in finance and investments we have gained knowledge on financial economics, macroeconomic theory, statistical analysis, econometrics, and a host of other topics. All of which have been instrumental in the planning and execution of this research paper. The study is written as a scientific article, and accordingly contains an extended introduction, or theoretical framework.

We would like to praise the contribution of our supervisor Thomas Leirvik, who enhanced the quality of the paper through exceptional guidance and support.

Bodø, May 23rd, 2023

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Sammendrag

Denne studien bygger på hypotesen om effisiente marked (EMH) og hypotesen om adaptive marked (AMH) for å utforske tidsvarierende markedseffisiens i globale aksjemarkeder. Oppgaven benytter to mål på effisiens, Adjusted Market Inefficiency Magnitude (AMIM) og Hurst-eksponenten, for å måle effisiensen i diverse finansmarkeder. Resultatene indikerer varierende markedseffisiens i respektive marked. I studien vurderes det også om prisendring i råvare-futures påvirker effisiensen i markedet, hvor det avdekkes en økende grad av avhengighet under markedskriser. Vi benytter binomial og multinomial regresjon i en generalisert lineær modell for å identifisere forhold mellom råvarefutures og markedseffisiens. Funnene i denne oppgaven antyder at forholdet mellom råvare-futures og markedseffisiens er dynamisk, og varierer betydelig over tid, med brå og plutselige endringer under volatile regimer. Markedseffisiensen i indeksene for Nord-Amerika, Øst-Asia, og Europa, viser i alle modellene våre en gjennomgående sammenheng med futures-prisen for mais under Covid-19. Videre avsløres varierende resultater mellom modellene som benytter Hurst-eksponenten og AMIM, noe som indikerer langsiktig selv-likhet i tidsserier, under ustabile marked.

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Extended Introduction

Our master thesis follows a scientific article format, which comprises two main parts. Firstly, we provide an extended introduction that encompasses a comprehensive literature review, research methodology, and essential concepts necessary for the investigation of our chosen topic. This section serves as a foundation for our subsequent scientific article, where we present our original research findings and analysis. The inclusion of an extended introduction allows us to contextualize our study within the existing body of knowledge and provide a robust framework for our scientific article.

1 Theory

In this chapter we present topics that are relevant for our master thesis, and review literature related to these topics. Our topics can be divided into the sections “Efficient Market Hypothesis”, “Adaptive Market Hypothesis”, “Autoregressive models and time-varying market efficiency”, “Fractal models and dimensions” and “Commodities”. By the end of this chapter, readers will have gained a fundamental understanding of the existing research related to our thesis.

1.1 Efficient market hypothesis

The Efficient Market Hypothesis (EMH) was introduced in the seminal paper by Fama (1970), where he describes markets to be of varying degrees of efficiency, categorized as weak, strong, and semi-strong efficiency. Research reviews by Ţiţan (2015) and Yen and Lee (2008) show that market efficiency is a complex topic and exists in all three forms. Fama (1991) argues that stock prices are random at each increment and independent of past values, meaning they display no consistent pattern and would be impossible to predict. Multiple authors have provided evidence of market efficiency moving from one state to another, proving that market efficiency is time-variant as well (Ito and Sugiyama (2009); Kim et al. (2011) Lim et al. (2013); Lim & Brooks, 2011; Urquhart & McGroarty, 2016). For stock markets, Jegadeesh and Titman (1993) shows that predictability exists within a three-year period for most stocks, which is evidence that stock markets typically are not strongly efficient. Furthermore, De Bondt and Thaler (1985) show that markets tend to overreact to dramatic events, which also is not reconcilable with the EMH as it is a direct violation of Bayes’ theorem. Extending on this, Mensi et al. (2022) show how markets for multiple asset classes become less efficient during volatile periods in contrast to benign periods.

Malkiel (2003) show that market anomalies tend to self-destruct upon discovery, which is consistent with EMH as it would mean markets absorb newly available information. In contrast, Banz (1981) shows how the size of the company disproportionately affects its returns but argues that size effect persistence is evidence of model misspecification in the current paradigm rather than market inefficiency. This is supported by Chan and Chen (1991), who show that small firms tend to react differently to the same news as compared to large firms and argues that it is the result of a common model misspecification. Overall, Schwert (2003) argues that persistent market anomalies are likely the result of model misspecification. Further examples are shown by Ariel (1987, 1990), who find that stock returns are higher in the beginning of the month and prior to holidays, which is evident that human income and consumption patterns affect equity valuations.

Declining trend of autocorrelations in stock markets, as shown by Gu and Finnerty (2002), indicates that markets are more efficient today than before, which they partly credit to increasing information availability. As found by Broadstock and Zhang (2019) and L. Jiang et al. (2018), social media has pricing power on stock markets and extend on this by showing that equities with high uncertainty become more efficiently priced when frequently interacted with on online financial forums. This is supported by Chen et al. (2014), who find that efficiency for specific equities increase with frequency of searches on finance websites. In contrast, Drake et al. (2017) found that interactions on social media platforms by non-finance professionals tend to hinder efficiency. This is supported by Polyzos and Wang (2022), who find that only parts of information available on social media platforms regarding clean energy equities is incorporated into its prices, indicating semi-strong efficiency. Examples included show that market efficiency varies across sectors and asset classes (Mensi et al., 2021; Naeem et al., 2022).

In this chapter we showed how research provides varying signals regarding the validity of EMH. In short, the primary concerns about EMH relate to predictability, personal biases, and incomplete incorporation of information. The complexity and time-varying characteristics of market efficiency makes it an interesting topic of analysis, and in this study additional attention will be directed towards the changes that occur during volatile regimes.

1.2 Adaptive market hypothesis

As an alternative to EMH, the Adaptive Market Hypothesis (AMH) is proposed by Lo (2004, 2005), which explains through evolutionary theory how traditional economic financial models can coexist alongside behavioural models. Specifically, Lo argues that through the application of heuristic techniques, individuals will allow behavioural bias to influence their decisions as they adapt to a dynamic environment. Kahneman et al. (1982) explain such deviation from rationale through recency biases and goes on to state that an efficient market is not reconcilable with human nature. This is supported by De Bondt and Thaler (1990), who shows that cognitive biases can be found also among professional security analysts and economic forecasters.

Literature indicates that stock market prices can be predicted through specific market conditions (Urquhart & McGroarty, 2016), which indicates both weak efficiency and adaptivity in markets. In certain cases, there are examples of stock market returns being both predictable and biased as displayed by the existence of calendar anomalies (Reinganum, 1983; Urquhart & McGroarty, 2014). This is extended on in an article by Ito et al. (2015), who find that efficiency in stock markets, in addition to being time-variant, is also long-term cyclical. This is specifically expressed during special events, such as during a crisis or during varying economic conditions. In addition, Gu and Finnerty (2002) show that cognitive biases have affected stock market returns for over a century.

The AMH provides an alternative to the EMH as well as a solution to its shortcomings. As research indicates, there is evidence to suggest that AMH explains behaviours in financial market better than EMH. This thesis builds on both hypotheses of market efficiency, as it will examine the effects of uncertainty and common cognitive biases in volatile regimes.

1.3 Autoregressive Models and Time-Varying Market Efficiency

The application of autoregressive models to assess market efficiency was pioneered by Fama (1970), who utilized autoregressive techniques to test the Efficient Market Hypothesis. Nyblom (1989) asserted a drawback in using the autoregressive approach to gauge market efficiency, highlighting the possibility of drawing false conclusions. Nevertheless, this issue can be seen as a phenomenon not exclusive to autoregression. Building upon this technique, Ito et al. (2015) and Noda (2016) expanded its applicability to time series analysis. Examples showcase how these techniques have been applied to demonstrate contrasting perspectives to existing research (Khuntia & Pattanayak, 2018).

Ito et al. (2015) identified a limitation of time-varying market efficiency measures, specifically their lack of robustness when faced with insignificant autocorrelations. In response, Tran and Leirvik (2019) proposed a solution through the development of the Adjusted Market Inefficiency Magnitude (AMIM), providing an example of its application in multiple equity markets. Additionally, the AMIM model has been employed to measure time-varying market efficiency in cryptocurrency and fiat currency exchange rates (Puertas et al., 2023; Tran & Leirvik, 2020), as well as in commodity markets (Lauter & Prokopczuk, 2022; Okoroafor & Leirvik, 2022).

Parfenov (2022) demonstrated that the AMIM is linearly dependent on the size of the sample window, which has implications for comparability across studies utilizing different window sizes. Furthermore, when applying autoregressive market efficiency models, researchers have the flexibility to employ an AR(q) model where q can either be stationary or varied through optimization, such as using the Akaike Information Criterion (AIC). However, it is important to note that since AIC assumes stationarity in autoregressive models, the error regarding estimation of optimal lag length of an autoregressive model increases under non-stationarity. Granger and Ding (1995, 1996) and Cont (2007) demonstrated the slow decay rate of autocorrelation functions for absolute returns in stock markets, indicating the presence of volatility clustering. This finding is supported by Tseng and Li (2012), who suggested that volatility clustering exhibits time variation and varies across markets, specific stocks, and commodities. Consequently, a drawback of the autoregressive approach is its inability to distinguish between predictable patterns and volatility-induced non-stationarity.

Building upon the assumption of predictability in Fama's efficient market hypothesis, this thesis will employ the AMIM in a rolling window analysis to assess time-varying market efficiency. This test is derived using an AR(q) model and AIC optimization, which means that caution to recency bias and non-stationarity must be considered in interpretation of the results.

1.4 Fractal models and dimensions

Fractional Brownian motion was first introduced by Mandelbrot and Van Ness (1968), arguing models following a fractional Brownian motion is better suited to capture heterogeneity and volatility clustering in financial markets compared to model utilizing ordinary Brownian motion. This perspective is extended by Ilalan (2016) who incorporates Elliot waves to support the theory of fractional Brownian motion in Japanese equity markets. Extensions have been made by introducing a deterministic scaling function to transition into fractional geometric Brownian motion models, which are applicable to equity markets (Angstmann et al., 2019), commodity markets (Ibrahim et al., 2021), and option markets (Azmoodeh et al., 2009).

Rogers (1997) presents an opposing argument, suggesting that as the lags between two increments increase in fractional Brownian motion, the correlation decreases. Hence, it is not true Brownian motion, implying the potential for statistical arbitrage exploitation. In practice however, fractional Brownian motion models has shown to be less effective at exploiting mean-reverting patterns compared to simple correlation pairs trading (Bui & Ślepaczuk, 2022). Mandelbrot and Van Ness (1968) explain the properties of a fractional Brownian motion and extract the fractional Brownian motion exponent, also called the Hurst exponent, which is equal to 0.5 if the process is a standard Brownian motion.

Examples of application of the Hurst exponent in market efficiency analysis exists in both a static approach (Aloui et al., 2018; Gaio et al., 2022; Galluccio et al., 1997; Mensi et al., 2021; Scalas, 1998) and dynamic approach (Engelen et al., 2011; Y. Jiang et al., 2018; Morales et al., 2012; Tzouras et al., 2015). However, critiques identify issues with artificially high Hurst exponents in finite Brownian motions (Couillard & Davison, 2005) and sensitivity to time-dynamic scaling characteristics with exceedingly large sample sizes (Vogl, 2023), indicating that stock markets in reality follow a geometric Brownian motion. In a paper by Ghazani and Ebrahimi (2019) it is solidified that a dynamic Hurst exponent varies with sample size in oil price data.

In the seminal paper on self-similarity in fractal sets, Mandelbrot (1967) shows that fractal geometric shapes depend on scaling of fractal dimensions. The relationship between the fractal dimension (D) and the Hurst exponent (H) is defined as D = 2-H (Mandelbrot, 1985; Voss, 1989). This implies that when the Hurst exponent approaches one, the time series move towards a one-dimensional, predictable flat structure. Conversely, when the Hurst exponent approaches zero, the time series move towards a two-dimensional jagged structure, which is more volatile, but not necessarily unpredictable. Examples demonstrating the linear relationship between the Hurst exponent and fractal dimensions for fractal Gaussian noise processes are presented by Bassingthwaighte and Bever (1991). For specific markets, issues persist regarding the linear relationship between the Hurst exponent and fractal dimensions (Chen et al., 2011; Gneiting &

Schlather, 2004; Li & Lim, 2008). Furthermore, the Hurst exponent and fractal dimensions have been beneficial for exploring the long-term memory of commodity futures markets (Fernandez, 2010; Kristoufek & Vosvrda, 2014).

In this chapter, we showed that fractal techniques can be applied to test for self-similarity, and ultimately, to test for Brownian motions. In this thesis we will employ the Hurst exponent in a rolling window analysis, to act as measure of market efficiency. This is a fractal approach, which can be used to determine if the market follows a Brownian motion. Furthermore, whether it deviates negatively or positively from its benchmark value can be analysed to determine the dimensional structure of the dataset, to specify the characteristics of the inefficiency. While the Hurst exponent tests for the presence of Brownian motion, it is reasonable to assume that stock markets follow a geometric Brownian motion, which is a Brownian motion with drift. Therefore, results can be of inflated, and how much of the stock market drift is captured in our sample must be considered.

1.5 Commodities

Various research indicates that spot prices and future contract prices are positively correlated and that future contract prices lead spot price for multiple asset classes (Asche & Guttormsen, 2002; Bannigidadmath & Narayan, 2022; Choi & Hammoudeh, 2010; Huth & Abergel, 2014; Zhang & Liu, 2018). Expected supply tightening roughly explains divergence of future contract price from spot price (Fama & French, 1988; Gorton et al., 2013; Ng & Pirrong, 1994), which implies that commodity future contract prices increase disproportionate to the spot price when uncertainty regarding its underlying commodity’s scarcity increases.

Commodity futures are also subject to efficiency variation, as many studies show. Bilson (1981) argues that commodity future markets are particularly prone to speculative trading, while Bohl et al. (2021) found a negative relation between speculation and efficiency in commodity markets. Ramírez et al. (2015) found evidence of AMH on the returns of agricultural commodity futures. Okoroafor and Leirvik (2022) study the time-varying properties of Brent and WTI oil prices, finding that even though highly related, the prices does not react equally to various shocks in global financial markets. Berger and Uddin (2016); López (2014); Wadud et al. (2023) and Nguyen et al. (2020) show that correlation between commodity and equity is positive in benign periods, and generally increases during times of crisis.

Intersectoral commodity future contract price co-movement is generally positively correlated and of varying strength (Fan & Qiao, 2023; Wadud et al., 2023). Especially, agricultural commodities stick out with very high intersectoral correlation (Dai et al., 2022). Evidence also exists to suggest that commodities are interconnected to related equity, such as agricultural commodities and the Food & Beverage Index (Billah et al., 2023) and oil commodities and oil corporations (Diaz & de Gracia, 2017). Additionally, there is evidence of correlation between future contracts of similar underlying commodity from the American and Chinese markets (Fan & Qiao, 2023; Li & Lu, 2012), implying interconnectedness through globalization.

This thesis explores the impact of commodity futures prices on market efficiency during benign and volatile regimes. Overall, research indicates that commodities are weakly dependent and responds similarly as equity markets to crisis, as well as increasing in price when uncertainty regarding its scarcity increases. Therefore, if a crisis impacts resource uncertainty, commodity prices and equity market efficiency must follow suit.

2 Data

This thesis employs the Morgan Stanley Capital International (MSCI) global equity indices as the primary research objects. By utilizing a comprehensive range of statistical testing methods, this study aims to investigate and elucidate the variability in market efficiency inherent within these datasets. In order to identify potential regional disparities, the following MSCI indices have been specifically chosen for analysis: World, North America, Europe, Far East, and Emerging Markets. The broad market index data will be obtained from the official MSCI website. To provide insight into the sources of variation in market efficiency, a diverse array of commodity futures contracts incorporating common industrial input factors will be utilized. These input factors encompass metals, agricultural and food resources, as well as fossil energy sources. To accomplish the research objectives, the following five datasets have been selected:

- Copper Futures – Ticker: HGK3 – Exchange: COMEX
- US Corn Futures – Ticker: ZCK3 – Exchange: CBOT
- Crude Oil WTI Futures – Ticker: CLK3 – Exchange: NYMEX
- Live cattle Futures – Ticker: LCc1 – Exchange: CME
- Natural Gas Futures – Ticker: NGK3 – Exchange: NYMEX

The commodity futures are gathered from Investing.com. We consider both the MSCI webpage and Investing.com as reliable sources when considering price data. The data contains ten timeseries of 6310 daily observations over the period March 1999 through March 2023, formatted in logarithmic returns ln(Pt/Pt-1). Utilizing daily observations is considered the optimal approach for examining high liquidity equity markets through the implementation of responsive autoregressive and fractal models.

3 Methodology

In this chapter we will briefly present which methods we will apply in our thesis. The metrics range between risk measures, as well as statistical testing and a market efficiency test.

3.1 Statistical testing

The Augmented Dickey-Fuller (ADF) test is a statistical technique employed for the identification of unit roots in time series data, which signifies non-stationarity. By assuming the existence of a unit root, the ADF test assesses whether the data displays stationary characteristics. One of the key distinctions between the ADF test and the Dickey-Fuller test is its ability to account for autocorrelation within the time series. The result of the ADF test is contingent upon comparing the test statistic with the critical value.

In addition, we will utilize a Jarque-Bera test, which serves as a goodness-of-fit test to assess the skewness and kurtosis of the data in relation to a normal distribution. For the data to conform to a normal distribution, a Jarque-Bera statistic close to zero is desirable.

3.2 Adjusted Market Inefficiency Magnitude (AMIM)

To investigate the presence of market inefficiency during the test period, our study will utilize the adjusted market inefficiency magnitude (AMIM) test, as extensively described in the paper by Tran and Leirvik (2019). The AMIM model assumes that if the timeseries follows a random walk pattern the lagged values of the AR(q) model seen in Equation (1) in an efficient market will not have any predictive value.

$$r_t = \alpha + \beta_1 r_{t-1} + \cdots + \beta_q r_{q-1} + \varepsilon_t \tag{1}$$

In AMIM an autoregressive model is ran on a given number of lags. We plan to use backwards selection with the Akaike information criterion to determine the number of lags. The significant slope coefficients are stored in a vector, which is used to find a variance-covariance matrix. Since the covariance matrix is equal to Equation (2), then we know that we can use Cholesky decomposition to extract the inverse triangular matrix from the variance-covariance matrix and multiply it by the beta vector to create a vector of standardized betas.

1. The slope coefficients are stored in a vector $(\widehat{\beta_1}, \widehat{\beta_2}, \widehat{\beta_3} \ldots \widehat{\beta_q})$, which can be separated into two triangular matrices through application of Cholesky decomposition as illustrated.

$$\sum = \mathcal{L}\mathcal{L}' \tag{2}$$

2. The standardized beta vector $\widetilde{\beta_\iota}$ is extracted through multiplication of the inverse triangular matrix.

$$\tilde{\beta} = \mathcal{L}^{-1}\hat{\beta} \tag{3}$$

3. The absolute values of the beta vector are used to create the market inefficiency magnitude (MIM).

$$MIM = \frac{\sum\left|\widetilde{\beta_\iota}\right|}{1 + \sum\left|\widetilde{\beta_\iota}\right|} \tag{4}$$

4. The AMIM is derived by subtracting the confidence interval from the MIM, and then divided by the distance between the maximum value of the MIM, which is 1, and the confidence interval.

$$AMIM = \frac{MIM - CI}{1 + CI} \tag{5}$$

The equation for AMIM will have a maximum value of 1, and no set minimum value. AMIM-estimates between 0 and 1 indicate inefficient markets, while estimates lower than 0 indicate efficient markets. Because of the AR(q) models' affinity towards recent observations, some recency bias must be considered in the interpretation of the results. The AMIM model is robust against insignificant autocorrelations. This process is applied in a one year rolling window analysis.

3.3 R/S Hurst exponent

The Hurst exponent, also known as the Hurst coefficient or Hurst index, is a measure used to quantify the long-term memory or persistence in a time series data set. The Hurst exponent is based on the concept of self-similarity or fractal behaviour. In time series, self-similarity refers to the property that patterns observed at one scale are similar to patterns observed at different scales. The Hurst exponent helps to quantify the degree of self-similarity in the data. The value of the Hurst exponent, denoted as H, ranges between 0 and 1.

The Hurst exponent is computed by partitioning a time series, denoted as x(i), divided into s subsets of length n. For each subset, $\alpha = 1, 2, \ldots, s$, $x_{k,\alpha}$ signifies the element in subset α where $k = 1, 2, \ldots, n$. The rescaled range is calculated as follows:

1. Calculate the cumulative deviate series using a mean adjusted series:

$$y_{k,\alpha} = \sum_{i=1}^{k}(x_{i,\alpha} - \frac{1}{n}\sum_{k=1}^{n} x_{k,\alpha}) \tag{6}$$

2. Calculate the range R_α and standard deviation S_α of the subsets:

$$R_\alpha = \max(y_{k,\alpha}) - \min\,(y_{k,\alpha}) \tag{7}$$

$$S_\alpha = (\frac{1}{n}\sum_{k=1}^{n} x_{k,\alpha} - E_\alpha)^2)^{0.5}, \alpha = 1,2,\ldots,s \tag{8}$$

3. Calculate the mean of the rescaled range (R_α/S_α) for every subset of length n:

$$\left(\frac{R}{S}\right)_n = \frac{1}{s}\sum_{\alpha=1}^{s}(R_\alpha/S_\alpha) \tag{9}$$

Mandelbrot (1969) and Mandelbrot and Wallis (1969) show that the rescaled range asymptotically follows the power-law relation $(R/S)_n \propto cn^H$, where c is a constant and H denotes the fractional Brownian motion exponent, also called the Hurst exponent. The Hurst exponent is found as the slope coefficient of the log linear regression:

$$\log\left(\left(\frac{R}{S}\right)_n\right) = \log c + H \log n \tag{10}$$

Mandelbrot and Van Ness (1968) show that according to the model of fractional Brownian motion a Hurst exponent of 0.5 indicates a Brownian motion. If H > 0.5, then the time series is considered persistent and containing long range dependencies, or a directional trend. If H < 0.5, then it is considered anti-persistent, signalling short term dependencies, and indicating mean reversion. According to the process of fractional Brownian motion, the scaling properties shift inversely between fractal dimensions as H moves between 0 and 1 (Mandelbrot, 1985). This means that a high Hurst exponent results in a lower fractal dimension, indicating more short-term predictability, while a low Hurst exponent results in a higher fractal dimension, indicating more short-term volatility. To examine the time varying properties of the Hurst exponent in a time series, we apply a one year rolling window. This creates a series of Hurst exponents, which is applicable in time series analysis.

3.4 Regression analysis

The primary objective of this thesis is to provide an explanation for market efficiency through the utilization of industrial input factors. To achieve this goal, commodity future data will be incorporated as exogenous variables in regression models, while market efficiency metrics will serve as endogenous variables. The relationship between these two datasets will be assessed by analysing the direction and significance of the slope ratios. Given the distinct nature of the market efficiency models, it is most appropriate to employ separate regression models for their analysis. The AMIM data will be applied in a binomial logistic model, while the Hurst exponent data will be used in a multinomial logistic model. The two models produce results in different measurement units, namely odds ratios and relative risk ratios. Although these ratios are subject to slightly different interpretations, they share the same relationship with the number 1. Specifically, in both ratios, values close to 0 indicate a negative relationship, values close to 1 indicate no relationship, and values greater than 1 indicate a positive relationship.

3.5 Binomial Logistic Regression

The AMIM metric differentiates between efficient markets, indicated by a value smaller than or equal to 0, and inefficient markets, indicated by a value above 0. Consequently, the regression analysis will be structured as a binomial or binary logistic regression within a generalized linear model, where the response variable takes on the dichotomous values of 0 or 1. In the binomial logistic regression, the continuous independent variables are utilized to generate output ranging between 0 and 1, following a logistic distribution. The logarithm of the odds ratio (log odds ratio) is expressed as:

$$\ln\left(\frac{\mu}{1-\mu}\right) = \beta_0 + \beta_1 x_1 + \cdots + \beta_i x_i \tag{11}$$

Where the μ is the probability of the dependent variable being 1, β_0 is the intercept, and the slope coefficient is gathered from $\beta_i = (1, 2, 3 \ldots)$. Consider a sample size of 100. The observations are classified as 1 for inefficient, and 0 for efficient. If we identify 70 observations of value 1, and 30 of value 0, the μ would then be $\frac{70}{100} = 0.70$. Putting this result in equation (11), we get $\ln\left(\frac{0.70}{1-0.70}\right) \approx 0.85$ which is the log odds ratio of success. As log odds ratios can be challenging to interpret, both sides of equation (11) can be exponentiated to provide the odds ratio:

$$\frac{\mu}{1-\mu} = e^{\beta_0+\beta_1 x_1+\cdots+\beta_i x_i} \tag{12}$$

Where the same example as above would present the odds ratio of $\frac{0.70}{1-0.70} \approx 2.33$. As such, we can discern that the odds of the binary outcome variable being 1, are 2.33 times greater than of it being 0. Note that odds ratio and relative risk ratios, as described in chapter 3.6, are not the same, nor is the interpretation. As an example, the odds of a coin flip returning heads is 1 $\left(\frac{0.50}{1-0.50} = 1\right)$, while the risk of returning heads is 0.5 $\left(\frac{1}{2} = 0.5\right)$. This contrast occurs due to the odds ratio dividing the odds of the desired outcome on the odds of the undesired outcome, while the risk divides the probability of the desired outcome on all possible outcomes. Followingly, odds ratios may appear heavily inflated opposed to risk/probability.

In this study, the response variable will be classified as 0 or 1 dependent on the AMIM:

$$Y = \begin{cases} 0, & AMIM \leq 0 \\ 1, & AMIM > 0 \end{cases}$$

As the market is deemed efficient if AMIM is 0 or below, and inefficient when AMIM is above 0, the two outcomes are classified as Y =0 and Y = 1, respectively. The binomial logistic regression model will predict the probability of a specific market being efficient.

3.6 Multinomial logistic regression

For the Hurst exponent the properties of a time series are determined through analysis of the exponent's oscillation around a 0.5 benchmark, arriving at polytomous conclusions. Therefore, it is not appropriate to use an ordinary linear regression. Instead, a generalized linear model is constructed using multinomial logistic regression. In this case, the results are matched to a natural integer of three numbers, representing each possible conclusion and stored into a variable $y(\lambda)_i$.

$$U = \begin{cases} 1, & Hurst > 0.525 \\ 2, & 0.475 \leq Hurst \leq 0.525 \\ 3, & Hurst < 0.475 \end{cases}$$

Here the value of two represents the conclusion that the time series follows a Brownian motion. Correspondingly, the values one and three represent the conclusion that the market is inefficient, or specifically anti-persistent and persistent respectively. In the multinomial logistic model, these values, and the data of logistic returns in commodities will be processed through multiple binary regressions $\beta_k X_i$ to arrive at a marginal probability that observation i is equal to value in U. If the probabilities add up to one, for outcome k with K possible outcomes, the marginal probability is given as:

$$\pi_i{}^{(U)} = P[\mathrm{y}(\lambda)_i = k] = \frac{e^{\beta_k X_i}}{1 + \sum_{k=1}^{K-1} e^{\beta_k X_i}} \tag{13}$$

The probability of being in the baseline category is calculated by subtraction.

$$\pi_i{}^{(2)} = 1 - \sum_{k=1}^{K-1} \pi_i{}^{(k)} \tag{14}$$

The multinomial logistic regression model with one covariate $x(\theta)_i$ is expressed in the following equation:

$$\log\left(\frac{\pi_i{}^{(U)}}{\pi_i{}^{(2)}}\right) = \beta_0{}^{(U)} + \beta_1{}^{(U)} \mathrm{x}(\theta)_i + \varepsilon_i \tag{15}$$

Through statistical software, neural network iterative methods are applied to find the unknown slope coefficients $\beta_1^{(U)}$. Because the beta coefficients follow logarithmic scaling it can complicate the interpretation of the results, and it is therefore more appropriate to convert the coefficients to relative risk ratios (RRR), which range from zero to infinity. Deriving the RRR can be done simply by exponentiating the coefficient, as seen in equation 16.

$$RRR_{\beta_1{}^{(U)}} = e^{\beta_1{}^{(U)}} \tag{16}$$

The ratio is interpreted as if the explanatory variable has a one-unit increase, then a RRR = 1 indicates no relationship, RRR < 1 indicates negative relationship, and RRR > 1 indicates positive relationship. Consider the following results from a multinomial logistic regression (Crude oil: $RRR_{\beta_1{}^{(1)}}$ = 0.5, $RRR_{\beta_1{}^{(3)}}$= 2). The interpretation here is that following a 100% increase in crude oil prices, the number of Hurst exponents classified as anti-persistent are 50% of what they were under no price fluctuation, while the number of Hurst exponents classified as persistent are 2 times as many. The distance from 1 of the RRR will be used to conclude on the dependence of the categorical Hurst exponent data $y(\lambda)_i$ on logistic returns in commodity future contracts $x(\theta)_i$.

3.7 Summary statistics and correlations:

In Table 1.1, the descriptive statistics of the cumulative log returns of the five regional indices are displayed. The correlation matrix in Table 1.2 provides indication of some highly correlated market pairs, like World-North America, and World-Europe which are 92.4%, and 81.3% correlated, respectively. This is not considered an issue for this study, as the indices are treated and analysed separately.

Descriptive statistic of regional indices

	W	NA	FE	EM	EU
Mean	3.3%	4.4%	2.5%	4.7%	1.1%
Standard deviation	16.3%	19.3%	18.4%	18.6%	20.9%
Kurtosis	13.4	13.4	7.6	10.5	11.7
Skewness	-0.57	-0.45	-0.23	-0.51	-0.36
JB	28910	28575	5692	15009	19788
p-value	0.000	0.000	0.000	0.000	0.000
ADF	-18.3	-18.4	-18.0	-17.2	-18.6
p-value	0.010	0.010	0.010	0.010	0.010
Maximum drawdown	19.5%	23.2%	18.1%	19.6%	24.8%
Cumulative returns	80.3%	108.0%	61.0%	114.2%	26.3%

Table 1.1: Descriptive statistics of the MSCI equity indices World (W), North America (NA), Far East (FE), Emerging Markets (EM), and Europe (EU). The table includes annualized means and standard deviation, as well as kurtosis, skewness, two tests relating to normality, and the minimum and maximum value in each time series.

Correlation matrix for indices

	W	EU	NA	FE	EM
W	1	0.813	0.924	0.401	0.649
EU	0.813	1	0.573	0.364	0.658
NA	0.924	0.573	1	0.172	0.457
FE	0.401	0.364	0.172	1	0.721
EM	0.649	0.658	0.457	0.721	1

Table 1.2: Correlation matrix for the five indices.

Table 1.3 provides the descriptive statistics of the cumulative log returns of commodity futures. The standard deviation unveils the increased volatility present in the energy materials, compared to the other three futures contracts. The correlation matrix of the commodity futures contracts is displayed in Table 1.4.

Descriptive statistic of commodity futures contracts

	Live cattle	Corn	Copper	Crude oil	Natural gas
Mean	4.8%	5.9%	9.3%	12.3%	0.6%
Standard deviation	20.9%	27.6%	26.4%	51.5%	56.4%
Kurtosis	152.7	9.0	7.1	487.5	7.7
Skewness	0.24	-0.30	-0.15	-9.06	0.30
JB	5874289	9581	4346	61665365	5891
p-value	0.000	0.000	0.000	0.000	0.000
ADF	-20.3	-17.4	-16.8	-17.8	-17.0
p-value	0.010	0.010	0.010	0.010	0.010
Maximum drawdown	51.6%	31.5%	23.1%	204.7%	53.6%
Cumulative returns	116.6%	144.4%	222.5%	289.2%	14.0%

Table 1.3: Descriptive statistics of commodity futures contracts for five industrial input factors. The table includes annualized means and standard deviations, as well as kurtosis, skewness, two tests relating to normality, and the minimum and maximum value in each time series.

Correlation matrix for commodity futures

	Copper	Live cattle	Corn	Crude oil	Natural gas
Copper	1	0.096	0.199	0.260	0.083
Live cattle	0.096	1	0.057	0.059	0.018
Corn	0.199	0.057	1	0.161	0.093
Crude oil	0.260	0.059	0.161	1	0.183
Natural gas	0.083	0.018	0.093	0.183	1

Table 1.4: Correlation matrix for the five commodity futures contracts.

3.8 Participants and instruments

Since we will study the behaviour of large financial markets, where the data are mostly anonymous and public, there are very few ethical implications for this study. Even if the names of the managers and people in positions of power are public information, it will not be necessary to mention anyone by name. On the topic of software, for this study, we will primarily utilize R to process our data into graphs, statistics, and regressions. To efficiently process data, Microsoft Excel will be used to prepare data for import into R, as well as refining tables. The R packages used for this thesis include “zoo”, “ggplot2”, “pracma”, “tseries”, “nnet” and “moments”.

3.9 External and internal validity

The commodity future contracts will be tested against multiple global and regional equity indices, and it is therefore fair to assume that the results of the study will be applicable to most developed countries. Special consideration must be made to developed countries with relatively large publicly listed sectors, such as Norway, where the energy sector makes up 31,6% of the index as of March 31st (Euronext, 2023). However, it is possible to make some assumptions regarding such markets based on the results from the relevant regional market index and commodity future contract.

It is important for the reader to remember that market efficiency is an extensive and complex topic, and its relationship with commodity future contracts is minimally explored. Therefore, it is important to keep in mind the exploratory nature of this research. In extension, this means that omitted variable bias needs to be kept in mind both while writing and reading this study. In conclusion, this study cannot claim to fully explain variations in market efficiency. Instead, it will aim to expose the existence of a relationship between industrial input factors and market efficiency under benign and volatile regimes.

A drawback related to data in this study is the application of broad market equity indices. The implications following are first that we do not know for certain whether results for these indices are applicable to specific markets. Furthermore, the behaviour of the selected indices and commodities could have been the result of an unidentified anomaly which affects our study. Awareness also needs to be raised regarding the exclusivity of commodity futures contract data. The data used in this study includes five open license data sets, which are deemed representative of the general commodity market. Despite this, researchers with more funding could solidify the results of this study by inclusion of additional commodity futures contracts.

A strength of this study is that it includes a long enough period to include multiple events of varying characteristics. Events explored in this study include the Global Financial Crisis of 2007-08, European Debt Crisis of 2009-11, Global Oil crisis of 2014-16, Covid-19 pandemic of 2020-21, European energy crisis of 2022, and the Banking crisis of 2023, among several other periods of heightened volatility in the financial markets.

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Scientific article

Commodity Impacts on Time-Varying Market Efficiency

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Abstract

This study builds on the Efficient market Hypothesis (EMH) and the Adaptive Market Hypothesis (AMH) to explore the time-varying characteristics of market efficiency in global equity markets. The research applies Adjusted Market Inefficiency Magnitude (AMIM) and the Hurst exponent to measure efficiency in various markets. This study additionally investigates the impact of commodity futures prices on market efficiency, revealing a heightened sensitivity during periods of market turmoil. The relationship between commodity futures and market efficiency is explored through binomial and multinomial regression in generalized linear models. We find that market efficiency and its dependency on commodities varies substantially over time, with large and abrupt changes during times of global market turmoil. In North America, Far East, and Europe, market efficiency's dependence on corn during covid-19 is consistent across our models. Further discrepancies between models show that there is evidence on long memory under volatile regimes.

1 Introduction

The Efficient Market Hypothesis (EMH) was introduced in the seminal paper by Fama (1970), where he describes markets to be of varying degrees of efficiency, categorized as weak, strong, and semi-strong. The EMH states that participants in the market make decisions on account of information open and accessible for all. As such, the financial markets become rational, and subject to stochastic price movement. Market efficiency has proven to be a complex topic, and research indicates that market efficiency exists in all the forms and is varying across time, sector, and asset classes (Ito & Sugiyama, 2009; Ito et al., 2015; Kim et al., 2011; Lim & Brooks, 2011; Lim et al., 2013; Mensi et al., 2021; Naeem et al., 2022; Țiţan, 2015; Urquhart

& McGroarty, 2016; Yen & Lee, 2008). In this paper we compute several established measures of market efficiency and analyse how and why these change over time.

In alignment with EMH, numerous researchers demonstrate that market efficiency tends to increase as information availability rises (Broadstock & Zhang, 2019; Chen et al., 2014; Gu & Finnerty, 2002; L. Jiang et al., 2018). Building upon this, Drake et al. (2017), and Polyzos and Wang (2022) highlight the intricate relationship between market efficiency and information availability. As evidence of efficient markets, Malkiel (2003) asserts that market anomalies self-destruct upon discovery. This idea is also supported by earlier work from Schwert (2003), Banz (1981), and Chan and Chen (1991), who argue that persistent market anomalies could merely result from model misspecification. Fama (1991) contends that stock prices are random and independent of past values at each increment, resulting in no discernible pattern or predictability. In contrast, various researchers suggest that predictability exists in stock markets (Jegadeesh & Titman, 1993; Reinganum, 1983; Urquhart & McGroarty, 2014; 2016). If stock prices are predictable, then past prices and returns are significantly correlated with current and future prices and returns. As such, tests for efficiency examine whether past and future returns are related. In this study, we apply two such measures and investigate the degree of efficiency across a wide range of markets.

Extending on EMH, Lo (2004; 2005) proposes the Adaptive Market Hypothesis (AMH). It offers an alternative to the EMH by incorporating insights from behavioural finance and evolutionary biology. The AMH suggests that financial markets are adaptive systems in which participants, driven by competition and natural selection, evolve their behaviour over time. According to the AMH, investors use heuristic techniques to adapt to changing market conditions. As a result, their behaviour is influenced by cognitive biases and psychological factors, leading to market inefficiencies and anomalies. The AMH argues that market efficiency is not a fixed characteristic but rather varies over time due to changing market conditions, participants' behaviour, and the evolution of investment strategies. Examples provided suggest that cognitive biases persist in asset pricing (Ariel, 1987; 1990; De Bondt & Thaler, 1985; 1990; Gu & Finnerty, 2002; Kahneman et al., 1982; Mensi et al., 2022). This research investigate how market efficiency varies over time and find that there is substantial variation in how efficient our selected financial markets are in the specified period.

Application of autoregressive models to examine the market efficiency for differing markets exist in multiple examples (Fama, 1970; Ito et al., 2015; Khuntia & Pattanayak, 2018; Noda, 2016). However, there are several drawbacks of the autoregressive approach, including insignificance of lagged variable (Ito et al., 2015; Nyblom, 1989), linear dependence on sample size (Parfenov, 2022), time and market-varying volatility clustering (Tseng & Li, 2012), and stationarity assumptions in the selection of optimal lag length (Cont, 2007; Granger & Ding, 1995, 1996). Tran and Leirvik (2019) proposed a solution to these drawbacks by deriving the Adjusted Market Inefficiency Magnitude (AMIM), with multiple examples existing for differing markets (Lauter & Prokopczuk, 2022; Okoroafor & Leirvik, 2022; Puertas et al., 2023; Tran & Leirvik, 2019; 2020). In this paper, one of the measures for market efficiency we use, is the AMIM. We find that the AMIM indicates that markets are often efficient, but with periods of inefficiency.

Mandelbrot and Van Ness (1968) challenged the sufficiency of an ordinary Brownian motion model in describing financial markets, suggesting that a fractional Brownian motion model is better suited for capturing heterogeneity and volatility clustering. They illustrated the advantage of fractional Brownian motion in quantifying market efficiency using the Hurst exponent. This exponent serves as a measure of the persistence or memory exhibited by a time series. Ilalan (2016) provided evidence through Elliot waves to justify the use of fractional Brownian motion in equity markets, with extensions made to model equity markets (Angstmann et al., 2019), commodity markets (Ibrahim et al., 2021), and option markets (Azmoodeh et al., 2009). However, Rogers (1997) argued that the utilization of fractional Brownian motion enables the potential for statistical arbitrage, and therefore, cannot effectively describe an efficient market. Despite this, empirical evidence suggests that practical implementation of statistical arbitrage using fractional Brownian motion has demonstrated inferiority to existing trading models (Bui & Ślepaczuk, 2022). Applications of the Hurst exponent in market efficiency analysis exist in both static (Aloui et al., 2018; Gaio et al., 2022; Galluccio et al., 1997; Mensi et al., 2021; Scalas, 1998) and dynamic approaches (Engelen et al., 2011; Y. Jiang et al., 2018; Morales et al., 2012; Tzouras et al., 2015). However, drawbacks of utilizing fractional Brownian motion to gauge market efficiency relate to sensitivity of sample size (Couillard & Davison, 2005; Ghazani & Ebrahimi, 2019; Vogl, 2023). In this paper we compute the Hurst exponent for several markets over a relatively long period of time. We find that the Hurst exponent varies over time, indicating time-varying market efficiency.

Mandelbrot (1985), and Voss (1989) defined the fractal dimension of the Hurst exponent. The authors showed that as the Hurst exponent approaches one, the time series converges towards a one-dimensional flat structure indicating positive autocorrelations, while as it approaches zero, the structure becomes a two-dimensional jagged one indicating volatility and negative autocorrelation. Practical examples in Gaussian processes are shown by Bassingthwaighte and Bever (1991), and Chen et al. (2011). On the other hand, research indicates that a positive linear relationship is not consistent for all models and processes (Gneiting & Schlather, 2004; Li & Lim, 2008).

Various research indicates that spot prices and future contract prices are positively correlated and that future contract prices lead spot price for multiple asset classes (Asche & Guttormsen, 2002; Bannigidadmath & Narayan, 2022; Choi & Hammoudeh, 2010; Huth & Abergel, 2014; Zhang & Liu, 2018). Expected supply tightening roughly explains divergence of future contract price from spot price (Fama & French, 1988; Gorton et al., 2013; Ng & Pirrong, 1994), implicative of commodity future contract prices reflection of uncertainty regarding its underlying commodity's availability. Further studies on co-movement in the commodity future contract markets are listed regarding intersectoral correlation (Dai et al., 2022; Fan & Qiao, 2023; Wadud et al., 2023), correlation to related equity (Billah et al., 2023; Diaz & de Gracia, 2017), for varying volatility (Berger & Uddin, 2016; López, 2014; Nguyen et al., 2020; Wadud et al., 2023), and transnational interconnectedness (Fan & Qiao, 2023; Li & Lu, 2012). This study explores the impact of commodity futures prices on market efficiency, considering them a possible indicator of market sentiment. Although there is limited evidence to support any significant relationship under benign markets, the market appears to be more sensitive towards futures prices during market turmoil.

2 Data

To account for regional differences, events with dissimilar cross-country effects on financial markets, and the exploratory element of this paper, the markets selected for investigation are broad and diversified indices. Utilizing a range of statistical testing, this study aims to examine and elucidate the variation in market efficiency present within the selected dataset. The data contains ten timeseries of 6310 daily observations over the period March 1999 through March 2023, formatted in logarithmic returns $\ln(P_t/P_{t-1})$. Utilizing daily observations is considered the optimal approach for examining high liquidity equity markets through the implementation of responsive autoregressive and fractal models. The five market indices selected for this study are MSCI-World, MSCI-North America, MSCI-Europe, MSCI-Far East, and MSCI-Emerging Markets, while the five futures contracts comprise of copper, corn, crude oil, live cattle, and natural gas.

2.1 Market indices

The MSCI World index represents 23 developed markets, with United States the largest member by some margin. For European equity, The MSCI Europe index is spread across 15 developed markets across Europe. The MSCI Far East index involves 3 developed Asian markets, heavily weighted on China. The MSCI Emerging Markets Index consist of 24 emerging markets countries. The MSCI North America index inhabits mostly United States and some Canadian publicly traded firms. All indices capture large and mid-cap firms within their specific markets. The data for the indices and the futures contracts is gathered from the MSCI website.

Table 2.1 illustrates the descriptive statistics of the regional indices, the Jarque-Bera (JB) and Augmented Dickey-Fuller (ADF) test stats, and calculations of the maximum drawdown and cumulative returns of each respective market index. The distribution of logarithmic returns is presented in histograms in Figure 2.1.

Descriptive statistic of regional indices

	W	NA	FE	EM	EU
Mean	3.3%	4.4%	2.5%	4.7%	1.1%
Standard deviation	16.3%	19.3%	18.4%	18.6%	20.9%
Kurtosis	13.4	13.4	7.6	10.5	11.7
Skewness	-0.57	-0.45	-0.23	-0.51	-0.36
JB	28910	28575	5692	15009	19788
p-value	0.000	0.000	0.000	0.000	0.000
ADF	-18.3	-18.4	-18.0	-17.2	-18.6
p-value	0.010	0.010	0.010	0.010	0.010
Maximum drawdown	19.5%	23.2%	18.1%	19.6%	24.8%
Cumulative returns	80.3%	108.0%	61.0%	114.2%	26.3%

Table 2.1: Descriptive statistics of MSCI equity indices World (W), North America (NA), Far East (FE), Emerging Markets (EM), and Europe (EU). The table includes annualized means and standard deviations, as well as kurtosis, skewness, two tests relating to normality, and the minimum and maximum value in each time series. Cumulative returns from December 1999 through March 2023.

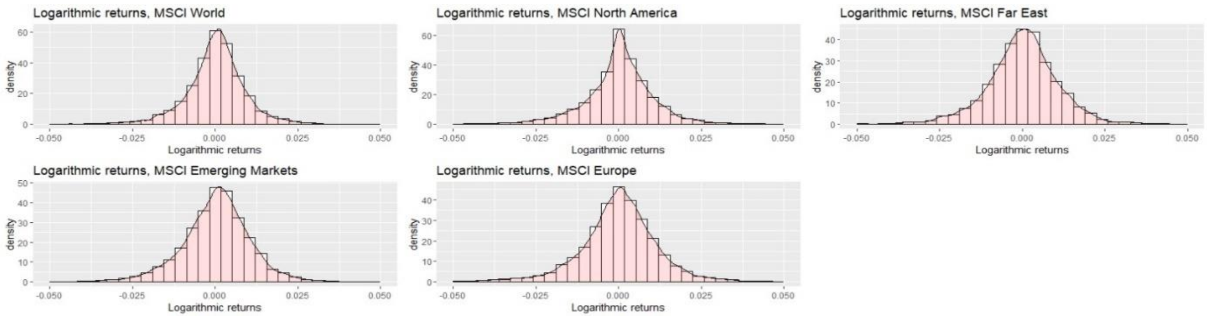

Figure 2.1: Distribution of log returns of five MSCI equity indices (World, North America, Far East, Emerging markets, and Europe). Outliers greater than 5% or less than -5% not included.

2.2 Commodity futures contracts

Commodities play a pivotal role in equity markets, exerting wide-ranging influences. They have a direct impact on corporate earnings, as changes in commodity prices can significantly affect the profitability of businesses operating in commodity-dependent industries. Additionally, commodities serve as important economic indicators, reflecting shifts in global supply and demand, inflationary pressures, and overall economic health. Certain sectors, such as energy, mining, and agriculture, have a more direct relationship with commodities and are highly sensitive to price movements. Moreover, commodities offer investment opportunities for hedging and diversification. Due to this, we investigate how commodities impact the variation in market efficiency.

To capture both prices and expectations about future prices, we apply commodity futures contracts, which incorporate common industrial input factors. These input factors comprise metals, agricultural and food resources, as well as fossil energy sources. The following five futures have been selected to achieve the research objectives: copper, corn, crude oil, live cattle, and natural gas. The data for the commodity futures contracts is gathered from investing.com. Table 2.2 presents the descriptive statistics of the selected commodity futures, while the distribution of logarithmic returns is displayed in histograms in Figure 2.2. The intended period for this study will extend from 1999 up until March 2023, the time of the analysis.

Descriptive statistic of commodity futures contracts

	Live cattle	**Corn**	**Copper**	**Crude oil**	**Natural gas**
Mean	4.8%	5.9%	9.3%	12.3%	0.6%
Standard deviation	20.9%	27.6%	26.4%	51.5%	56.4%
Kurtosis	152.7	9.0	7.1	487.5	7.7
Skewness	0.24	-0.30	-0.15	-9.06	0.30
JB	5874289	9581	4346	61665365	5891

p-value	0.000	0.000	0.000	0.000	0.000
ADF	-20.3	-17.4	-16.8	-17.8	-17.0
p-value	0.010	0.010	0.010	0.010	0.010
Maximum drawdown	51.6%	31.5%	23.1%	204.7%	53.6%
Cumulative returns	116.6%	144.4%	222.5%	289.2%	14.0%

Table 2.2: Descriptive statistics of commodity futures contracts for five industrial input factors. The table includes annualized means and standard deviations, as well as kurtosis, skewness, two tests relating to normality, and the minimum and maximum value in each time series. Cumulative returns from December 1999 through March 2023.

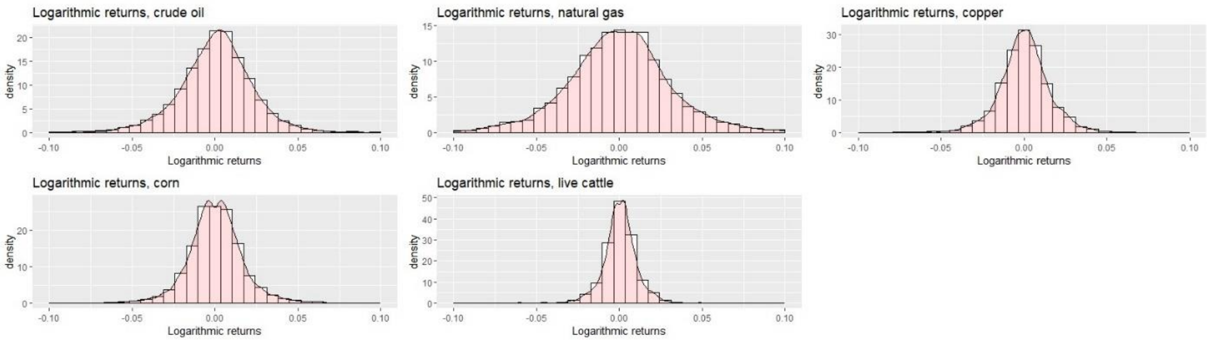

Figure 2.2: Distribution of log returns of five commodity future contracts (Crude oil, natural gas, copper, corn, and live cattle). Outliers greater than 10% or less than -10% not included.

3 Methodology

3.1 Market efficiency

Market efficiency is examined by employing the Adjusted Market Inefficiency Magnitude (AMIM) and the R/S Hurst exponent.

3.1.1 Adjusted market inefficiency magnitude

The adjusted market inefficiency magnitude (AMIM) as described in the paper by Tran and Leirvik (2019), will be applied to examine the properties of the time-varying market efficiency in the data. The AMIM model assumes that the lagged values of the AR(q) model seen in Equation (1) in an efficient market will not have any predictive value. Backwards selection through the Akaike information criterion will be applied to determine the number of lags.

$$r_t = \alpha + \beta_1 r_{t-1} + \cdots + \beta_q r_{q-1} + \varepsilon_t \qquad (1)$$

The quantification of market inefficiency, denoted as the Market Inefficiency Magnitude (MIM), is calculated based on the standardized beta coefficients obtained from Equation (1). See appendix A for derivation of MIM. Nonetheless, it is necessary to adjust the MIM as it tends to yield higher values due to its positive correlation with the number of lags in the underlying autoregressive AR(q) equation. This adjustment is performed using the adjusted MIM (AMIM) at time t, which is expressed in Equation (2).

$$AMIM = \frac{MIM_t - R_{CI}}{1 + R_{CI}} \tag{2}$$

Here, MIM corresponds to the Market Inefficiency Magnitude at time t, and CI denotes the range of confidence intervals for the MIM. The denominator provides a standardized basis for comparison across different time periods, assets, and regions. This process is applied in a one year rolling window analysis. The derivation of the equation and the specific range of confidence intervals employed are elaborated upon in the work of Tran and Leirvik (2019). When interpreting the model, a negative value of AMIM signifies market efficiency, while positive values indicate the presence of inefficiency within the asset or market.

3.1.2 Dynamic Hurst exponent

The Hurst exponent is a non-parametric measure calculated based on the rescaled range, which captures the normalized fluctuation amplitude in a time series. Through its relationship with power-law behaviour, the Hurst exponent provides a quantification of the long-term memory and fractal structure of the data. It is a valuable tool for analysing the persistence or anti-persistence of time series by assessing autocorrelations and their decay rates.

Consider a time series x(i), divided into s subsets of length n. For each subset, α = 1, 2, …, s, $x_{k,\alpha}$ signifies the element in subset α where k = 1, 2, …, n. The rescaled range is calculated as follows:

1. Calculate the cumulative deviate series using a mean adjusted series:

$$y_{k,\alpha} = \sum_{i=1}^{k} (x_{i,\alpha} - \frac{1}{n}\sum_{k=1}^{n} x_{k,\alpha}) \tag{3}$$

2. Calculate the range and standard deviation of the subsets:

$$R_\alpha = \max(y_{k,\alpha}) - \min\,(y_{k,\alpha}) \tag{4}$$

$$S_\alpha = (\frac{1}{n}\sum_{k=1}^{n} x_{k,\alpha} - E_\alpha)^2)^{0.5}, \alpha = 1,2,\dots,s \tag{5}$$

3. Calculate the mean of the rescaled range for every subset of length n:

$$\left(\frac{R}{S}\right)_n = \frac{1}{s}\sum_{\alpha=1}^{s} (R_\alpha / S_\alpha) \tag{6}$$

4. Fitting the rescaled range asymptotically to the power-law relation $(R/S)_n \propto cn^H$, extract the Hurst exponent H through log linear regression:

$$\log\left(\left(\frac{R}{S}\right)_n\right) = \log c + H \log n \tag{7}$$

According to the model of fractional Brownian motion H=0.5 indicates a Brownian motion, H>0.5 indicates persistence, and H<0.5 indicates anti-persistence (Mandelbrot & Van Ness, 1968). If the time series is persistent, it is evidence of long-range dependencies, and if it is anti-persistent it is evidence of short-term dependencies. According to the process of fractional Brownian motion, the scaling properties shift inversely between fractal dimensions as H moves between zero and one (Mandelbrot, 1985). This means that a high Hurst exponent results in a lower fractal dimension, indicating more short-term predictability, while a low Hurst exponent results in a higher fractal dimension, indicating more short-term volatility. The Hurst exponent process is applied in a one year rolling window analysis.

3.2 Regression analysis

3.2.1 Binomial logistic regression

The AMIM distinguishes between efficient markets with a value smaller or equal to 0, and inefficient markets with values above 0. Hence, the regression analysis will be structured as a binomial or binary logistic regression in a generalized linear model, where the response variable takes the dichotomous value of 0 or 1. The binomial logistic regression applies the continuous independent variables to produce output between 0 and 1 in a logistic distribution. The log odds are given as:

$$\ln\left(\frac{\mu}{1-\mu}\right) = \beta_0 + \beta_1 x_1 + \cdots + \beta_i x_i \tag{8}$$

Where the μ is the mean of the dependent variable being 1, β_0 is the intercept, and the slope coefficient is gathered from $\beta_i = (1, 2, 3 \ldots)$. Consider a sample size of 100. The observations are classified as 1 for inefficient, and 0 for efficient. If we identify 70 observations of value 1, and 30 of value 0, the μ would then be $70/100 = 0.70$. Putting this result in Eq. (8), we get $\ln\left(\frac{0.70}{1-0.70}\right) \approx 0.85$ which is the log odds of success. As log odds can be challenging to interpret, both sides of Eq. (8) can be exponentiated to provide the odds ratio:

$$\frac{\mu}{1-\mu} = e^{\beta_0 + \beta_1 x_1 + \cdots + \beta_i x_i} \tag{9}$$

Where the same example as above would present the odds ratio of $\frac{0.70}{1-0.70} \approx 2.33$. As such, we can discern that the odds of the binary outcome variable being 1, are 2.33 times greater than of it being 0. Note that odds ratio and relative risk ratios, as described in chapter 3.2.2, are not the same, nor is the interpretation. As an example, the odds of a coin flip returning heads is 1 $\left(\frac{0.50}{1-0.50} = 1\right)$, while the risk of returning heads is 0.5 $\left(\frac{1}{2} = 0.5\right)$. This contrast occurs due to the odds ratio dividing the odds of the desired outcome on the odds of the undesired outcome, while the risk divides the probability of the desired outcome on all possible outcomes. Followingly, odds ratios may appear heavily inflated opposed to risk/probability.

In this study, the response variable will be classified as 0 or 1 dependent on the AMIM:

$$Y = \begin{cases} 0, & AMIM \leq 0 \\ 1, & AMIM > 0 \end{cases}$$

As the market is deemed efficient if AMIM is 0 or below, and inefficient when AMIM is above 0, the two outcomes are classified as Y =0 and Y = 1, respectively. The binomial logistic regression model will predict the odds of a specific market being efficient.

3.2.2 Multinomial logistic regression

The Hurst exponent characterizes time series properties by analysing its deviation from the 0.5 benchmark, leading to polytomous categorical conclusions. Multinomial logistic regression is used to build a generalized linear model. The resulting outcomes are assigned to one of three integer values representing possible conclusions of arbitrary limits and saved as a variable $y(\lambda)_i$.

$$U = \begin{cases} 1, & Hurst > 0.525 \\ 2, & 0.475 \leq Hurst \leq 0.525 \\ 3, & Hurst < 0.475 \end{cases}$$

In this study, a value of two is indicative of a Brownian motion for the time series analysed. On the other hand, a value of one or three implies market inefficiency with anti-persistent and persistent behaviour, respectively. To process these values and logarithmic returns data for commodities, a multinomial logistic model is employed, where multiple binary regressions $\beta_k X_i$ are utilized to obtain a marginal probability of the observation i equating to a value in U. Given K possible outcomes, the marginal probability for outcome k is expressed under the assumption that the probabilities add up to one, as follows:

$$\pi_i^{(U)} = P[y(\lambda)_i = k] = \frac{e^{\beta_k X_i}}{1 + \sum_{k=1}^{K-1} e^{\beta_k X_i}} \tag{10}$$

The probability of being in the baseline category is calculated by subtraction.

$$\pi_i^{(2)} = 1 - \sum_{k=1}^{K-1} \pi_i^{(k)} \tag{11}$$

The multinomial logistic regression model with one covariate $x(\theta)_i$ is expressed in the following equation:

$$\log\left(\frac{\pi_i^{(U)}}{\pi_i^{(2)}}\right) = \beta_0^{(U)} + \beta_1^{(U)} x(\theta)_i + \varepsilon_i \tag{12}$$

Through statistical software, neural network iterative methods are applied to find the unknown slope coefficients $\beta_1^{(U)}$. Because the beta coefficients follow logarithmic scaling it can complicate the interpretation of the results, and it is therefore more appropriate to convert the coefficients to relative risk ratios (RRR), which range from zero to infinity. Deriving the RRR can be done simply by exponentiating the coefficient. The ratio is interpreted as if the explanatory variable has a one-unit increase, then a RRR = 1 indicates no relationship, RRR < 1 indicates negative relationship, and RRR > 1 indicates positive relationship. The distance from 1 of the RRR will be used to conclude on the dependence of the categorical Hurst exponent data $y(\lambda)_i$ on logistic returns in commodity future contracts $x(\theta)_i$.

4 Empirical results

Whitin the timeline investigated in this paper, there are several major events affecting global financial markets. The Dot-com bubble of 2000, the 2001, September 11th terrorist attacks, the financial crisis of 2008, the Covid-19 pandemic, and the Russian invasion of Ukraine in 2022 are some of the major events that caused severe and rapid downturns in global equity markets. Although the entire dataset is explored, the periods of market turmoil will be particularly scrutinized as they, assumably, challenge the assumption of unpredictable, random price movement under the EMH.

4.1 AMIM Rolling window analysis

Figure 2.3 illustrates the AMIM measure of all indices in a one year rolling window, 250 observations per year. The oscillating nature of the timeseries, illustrating the time-varying propensity of market efficiency. Considering periods of market turmoil, the global financial crisis and Covid-19 are easily identifiable in the time periods of 2008-2010 and 2020 to mid-

2021, respectively. The impact of these crises led to inefficiency in most markets. The descriptive statistics of AMIM are displayed in Table 2.3, where there are some dissimilarities. Noticeably, the mean score of AMIM differs between -0,55 in the European market to 0,27 in Emerging Markets. Considering that a negative AMIM score expresses efficient markets and positive AMIM expresses inefficient markets. This is signalling that overall, European, North American, and Far East markets have in the selected period been the more efficient markets. Simultaneously, the Emerging Markets and World indices have been less efficient, while also being less normally distributed. Figure 2.4 exhibit histograms of the observed AMIM scores in specific markets.

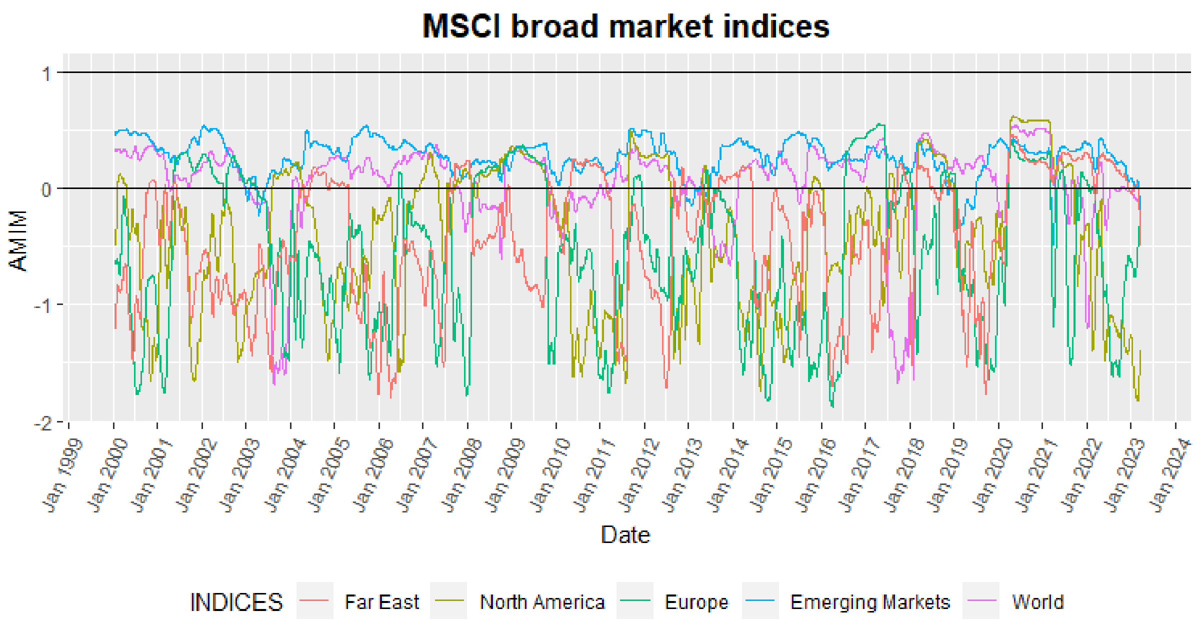

Figure 2.3: Graph of time varying AMIM statistics in five markets (World (W), North America (NA), Far East (FE), Emerging Markets (EM), and Europe (EU)). Black horizontal lines passing through one and zero indicates maximum AMIM and limit between efficiency and inefficiency respectively.

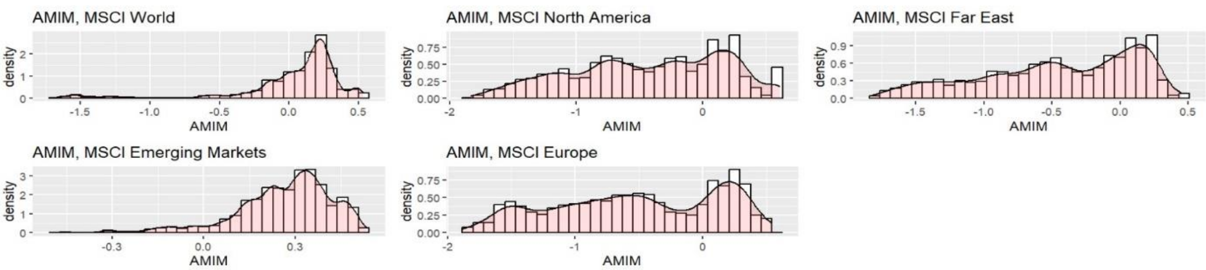

Figure 2.4: Distribution of AMIM in five markets (World (W), North America (NA), Far East (FE), Emerging Markets (EM), and Europe (EU)).

Descriptive statistics, AMIM

	W	NA	EU	EM	FE
Mean	0.04	-0.43	-0.55	0.27	-0.43
Standard deviation	0.41	0.63	0.68	0.17	0.58
Kurtosis	10.4	2.2	1.9	14.0	2.5
Skewness	-2.56	-0.40	-0.31	-1.85	-0.63
Jarque Bera	20337	343	404	33996	469
p-value	0.000	0.000	0.000	0.000	0.000
Augmented Dickey-Fuller	-5.7	-5.6	-5.9	-5.4	-6.2
p-value	0.010	0.010	0.010	0.010	0.010
Min	-1.96	-1.96	-1.96	-1.79	-1.96
Max	0.62	0.63	0.55	0.57	0.54

Table 2.3: Descriptive statistics of AMIM statistics in five markets (World (W), North America (NA), Far East (FE), Emerging Markets (EM), and Europe (EU)). The table includes annualized means and standard deviations, as well as kurtosis, skewness, two tests relating to normality, and the minimum and maximum value in each time series.

4.2 R/S Hurst exponent rolling window analysis

Figure 2.5 presents the Hurst exponent in a one year rolling window with a window size of 250 observations. From the figure it is assessable that equity markets have highly varying market efficiency over the sample period, with a tendency towards persistence. Multiple periods of synchronous movement are observed, such as in 2001-2003, 2008-2010, 2018, 2019-2020, 2020-2021, and 2022. Judging by the descriptive statistics in Table 2.4, North America inhabits the overall most efficient market. Emerging Markets is also deemed the least efficient market by the Hurst exponent. The histograms in Figure 2.6 explain the distribution of Hurst exponents in specific markets. The shape of the bell curve for Emerging Markets reinforces the index' deviation in normality from the other market indices.

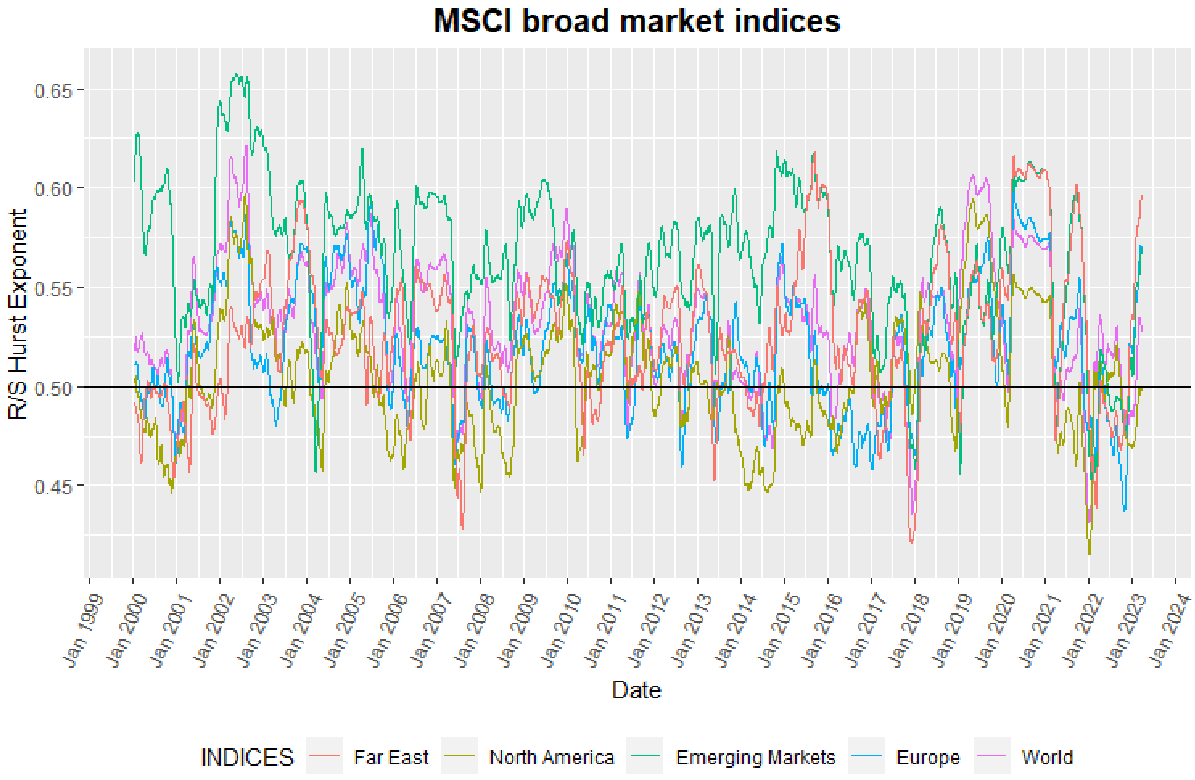

Figure 2.5: Graph of time varying of R/S Hurst exponents in five markets (World, North America, Far East, Emerging Markets, and Europe). Black horizontal line passing through 0.5 indicates fractional Brownian.

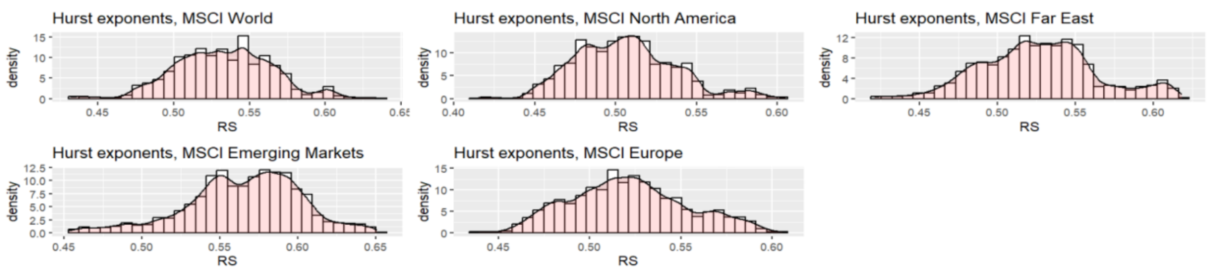

Figure 2.6: Distribution of R/S Hurst exponents in five markets (World, North America, Far East, Emerging Markets, and Europe).

Descriptive statistic, Hurst exponent

	World	North America	Europe	Emerging Markets	Far East
Mean	0.53	0.51	0.52	0.57	0.53
Standard deviation	0.03	0.03	0.03	0.04	0.04
Kurtosis	3.1	3.1	2.5	3.5	3.2
Skewness	-0.07	0.30	0.10	-0.45	0.08
Jarque Bera-test	10	97	70	271	12
p-value	0.010	0.000	0.000	0.000	0.000
Augmented Dickey-Fuller	-6.2	-5.2	-5.8	-6.3	-6.0
p-value	0.010	0.010	0.010	0.010	0.010

Min	0.42	0.41	0.42	0.44	0.41
Max	0.65	0.62	0.62	0.67	0.65

Table 2.4: Descriptive statistics of Hurst exponents in five markets (World, North America, Far East, Emerging Markets, and Europe). The table includes annualized means and standard deviations, as well as kurtosis, skewness, two tests relating to normality, and the minimum and maximum value in each time series.

4.3 Frequency analysis

As this study enables two market efficiency-models, we analyse if there is any difference between them. The 6046 observations in the dataset are classified with categorical values for the Hurst and AMIM. For the Hurst exponent, the values 1, 2 and 3 express persistent, efficient, and anti-persistent market characteristics. For AMIM, 0 and 1 represent efficient and inefficient markets, respectively. Table 2.5 presents a comparison of the frequency of the AMIM and Hurst. Generally, the two measurements largely coincide in amount of efficiency detected in the respective markets. However, for Far East and Europe, AMIM considers markets efficient more often than the Hurst Exponent. This indicates the presence of more long memory for these two respective markets, in comparison to the remaining three.

Frequency table of categorical variables

Region	Hurst	Freq.	Percent	Cum.	AMIM	Freq.	Percent	Cum.
World	1	238	3.94	3.94				
	2	2,171	35.91	39.84	0	1,734	28.68	28.68
	3	3,637	60.16	100	1	4,312	71.32	100
North America	1	1,041	17.22	17.22				
	2	3,462	57.26	74.48	0	3,998	66.13	66.13
	3	1,543	25.52	100	1	2,048	33.87	100
Far East	1	532	8.8	8.8				
	2	2,421	40.04	48.84	0	4,123	68.19	68.19
	3	3,093	51.16	100	1	1,923	31.81	100
Emerging markets	1	149	2.46	2.46				
	2	666	11.02	13.48	0	400	6.62	6.62
	3	5,231	86.52	100	1	5,646	93.38	100
Europe	1	511	8.45	8.45				
	2	2,891	47.82	56.27	0	4,028	66.62	66.62
	3	2,644	43.73	100	1	2,018	33.38	100

Table 2.5: Table displaying the frequency Hurst exponents in categories 1 (anti-persistence), 2 (fractional Brownian motion) and 3 (persistence) and AMIM in binary categories 0 (efficiency) and 1 (inefficiency) for five markets (World, North America, Far East, Emerging Markets, and Europe).

4.4 Logistic regression covariate analysis

4.4.1 Binomial logistic regression

Results of the logistic regression in the binomial generalized linear model are displayed in Table 2.6. The regression investigates the relationship between AMIM market efficiency and commodity futures contract log returns in the selected indices. The strongest indication of a link is uncovered in the pairing of live cattle futures and the efficiency level of the European market. The constant of 0.5 signifies that the odds of the binary response variable being 0 is twice as high as the odds of the same variable being 1, implying a greater possibility of an efficient European market, when not considering the impact of any predictor variables. The odds ratio of 16.35 for live cattle can be interpreted as for every 100% increase in the log returns of live cattle futures, the odds of the market being inefficient escalates by 16.35, which when considering scale of the input data is close to 1. In addition to odds ratios varying close to 1, low statistical significance suggests there is an overall absence of compelling evidence that futures prices impact market efficiency.

Table of odds ratios (OR)

		World	North America	Far East	Emerging Markets	Europe
Live cattle	**cons**	2.49	0.51	0.47	14.11	0.50
	OR	0.16'	1.12	1.17	1.71	16.35'''
Corn	**cons**	2.49	0.51	0.47	14.13	0.50
	OR	0.20'	2.14	1.36	0.12	0.27'
Copper	**cons**	2.49	0.51	0.47	14.11	0.50
	OR	2.37	0.87	0.52	2.10	0.98
Crude oil	**cons**	2.49	0.51	0.47	14.11	0.50
	OR	0.83	1.33	1.16	1.19	1.85
Natural gas	**cons**	2.49	0.51	0.47	14.13	0.50
	OR	0.99	1.01	2.09'	3.85'	1.47

Table 2.6: Odds ratios (OR) of the binomial generalized linear model for the efficiency of five markets (World, North America, Far East, Emerging Markets, and Europe), and commodity futures contracts. The efficiency gathered from AMIM is classified as 0 or 1 for efficient and inefficient markets. Cons represent the constant, or baseline odds. OR are noted with their respective sign of significance. Range of p-values: *** $p<0.01$, ** $p<0.05$, * $p<0.1$, ''' $p<0.2$, '' $p<0.3$, ' $p<0.4$

The results of model optimization through Akaike Information Criterion (AIC) of the binomial logistic regression including every continuous variable is presented in Table 2.7. The optimization returns models with no exogenous variable. From this it can be ascertained that no explanatory relationship exists between commodity future contract prices and market efficiency using an autoregressive approach.

Table of optimized binomial logistic models

OR	World	North America	Far East	Emerging Markets	Europe
Constant	2.5***	0.5***	0.5***	14.1***	0.5***
Residual deviance	7246	7741	7562	2946	7700
AIC	7248	7743	7564	2948	7702

Table 2.7: Results of the AIC optimized binomial generalized linear model for the efficiency of five markets (World, North America, Far East, Emerging Markets, and Europe), and commodity futures contracts. The efficiency gathered from AMIM is classified as 0 or 1 for efficient and inefficient markets. Constants are noted with their respective sign of significance. Range of p-values: *** p<0.01, ** p<0.05, * p<0.1, ''' p<0.2, '' p<0.3, ' p<0.4

4.4.2 Multinomial logistic regression

Results of the logistic regression in the multinomial generalized linear model are displayed in Table 2.8. The regression investigates the relationship between R/S Hurst market efficiency and commodity futures contract prices in the selected indices. Here, a Hurst category of one indicates anti-persistence and a Hurst in category three indicates persistence, while the baseline category two is a fractional Brownian motion. Constants greater than one indicate higher probability of inefficient markets and are consistent with the results from the frequency analysis. Relative risk ratios indicate that there is a generally weak relationship between commodity future contract prices and R/S Hurst exponents in the selected equity indices. Simultaneously, it is found that positive price movements in live cattle and copper is accommodated by anti-persistence in Emerging Markets. There is also an identifiable weak positive relationship between price movements in crude oil, copper and corn in Emerging Markets and anti-persistence, as well as for persistence and copper in Emerging Markets and North America. However, none of the relative risk ratios from the regression are supported by an alpha lower than 0.12.

Tables of Relative Risk Ratios (RRR)

	Region	**W**	**W**	**NA**	**NA**	**FE**	**FE**	**EM**	**EM**	**EU**	**EU**
	Hurst	**1**	**3**	**1**	**3**	**1**	**3**	**1**	**3**	**1**	**3**
Crude oil	**cons**	0.11	1.67	0.30	0.45	0.22	1.28	0.22	7.85	0.18	0.91
	RRR	0.86	1.98	1.01	4.90'''	0.77	0.90	24.02''	1.66	0.59	1.64
Natural gas	**cons**	0.11	1.68	0.30	0.45	0.22	1.28	0.22	7.86	0.18	0.91
	RRR	0.70	2.53''	0.78	3.21'''	0.66	0.59	2.18	5.21'''	0.63	1.70
Copper	**cons**	0.11	1.67	0.30	0.45	0.22	1.28	0.22	7.86	0.18	0.91
	RRR	2.48	2.81	0.24	10.06''	0.05'	3.47	120.46	40.19'''	1.98	10.59'''
Live cattle	**cons**	0.11	0.64	0.30	0.45	0.22	1.28	0.22	7.85	0.18	0.91
	RRR	7.97	1.67	7.73	3.80	1.14	0.27	2522.14'''	3.72	3.67	1.99
Corn	**cons**	0.11	1.67	0.30	0.45	0.22	1.28	0.22	7.86	0.18	0.92
	RRR	0.65	4.55'	0.10''	1.06	8.16	6.19''	16.72	0.25	0.14	0.16''

Table 2.8: Relative risk ratios (RRR) of the multinomial generalized linear model for the efficiency of five markets (World (W), North America (NA), Far East (FE), Emerging Markets (EM), and Europe (EU)), and commodity futures contract prices. The market inefficiency gathered from the R/S Hurst exponent is classified as 1 or 3 for anti-persistence and persistence, respectively, while the baseline category represents a fractional Brownian motion. Cons represent the constant. Range of p-values: *** p<0.01, ** p<0.05, * p<0.1, ''' p<0.2, '' p<0.3, ' p<0.4

The findings of the model optimization, based on the Akaike Information Criterion (AIC) for multinomial logistic regression, are displayed in Table 2.9. The optimized models, which include only continuous variables, do not incorporate any exogenous variables. Consequently, it can be inferred that there is no discernible explanatory relationship between commodity future contract prices and market efficiency using a fractal approach.

Table of optimized multinomial logistic models

Hurst	RRR	World	North America	Far East	Emerging Markets	Europe
1	**Constant**	-32.4**	-34.0***	-31.6***	-16.5***	-36.1***
3	**Constant**	19.0***	-26.4***	9.0***	50.1***	-3.3***
Residual deviation:		9684	11738	11164	5557	11165
AIC:		9688	11742	11168	5561	11169

Table 2.9: Results of the AIC optimized multinomial generalized linear model for the efficiency of five markets (World, North America, Far East, Emerging Markets, and Europe), and commodity futures contracts. The efficiency gathered from the R/S Hurst exponent is classified as 1 or 3 for anti-persistence and persistence, respectively, while the baseline category represents a fractional Brownian motion. Constants are noted with their respective sign of significance. Range of p-values: *** p<0.01, ** p<0.05, * p<0.1, ''' p<0.2, '' p<0.3, ' p<0.4

4.5 Logistic regression indicator variable analysis

To extend on the finding from chapter 4.4, indicator variables of nine volatile regimes are included. By involving periods of market turmoil in the regression, we can investigate whether the relationship of commodity futures prices and market efficiency differs under the duration of the crisis. The volatile regimes included are the Dotcom-bubble (DC), Twin Tower terrorist attack (TT), Global Financial Crisis (GO), Eurozone sovereign debt crisis (SD), Global oil crisis (GO), Chinese stock market turbulence (CS), Covid-19 (C19), Russo-Ukrainian war (RU), Silicon Valley banking crisis (SV). See Appendix B for extended information regarding the time interval of the indicator variables.

4.5.1 Volatile regime indicator variables in binomial regression

Table 2.10 exhibits the results of introducing dichotomous indicator variables in the binomial logistic regression analysis. Although not revealing considerable association between futures prices and efficiency in market indices outside of crisis periods, this same relationship does, in fact, reveal some interesting statistics during market turbulence. The interaction of the C19 and

SD volatile periods uncovered a certain degree of connectivity among efficiency and the rising prices of corn and copper. As the odds ratios for corn and copper in North America, Far East, and Europe suggest, a surge in these futures prices would greatly increase the odds of inefficient markets during C19. The same scenario is witnessed in the corn-Far East pairing during SD. Conversely, amidst SD, higher futures prices of corn would increase the odds of efficient markets in North America and Europe. Figure 2.7 shows how corn and copper trends upwards, while the AMIM statistic displays inefficiency for during C19, and similarly for corn during SD.

Table of odds ratios (OR)

Region	**D=k**	**Crisis**	**Live cattle**	**Corn**	**Copper**	**Crude Oil**	**Natural Gas**
NA	**1**	**C19**	2088.72'	6.89E+07**	8.08E+10***	1.24	47.47'''
NA	**1**	**SD**	2.77	2.83E-04**	0.06	0.58	0.05''
NA	**0**	**C19**	0.62	1.26	0.38	1.41	0.78
NA	**0**	**SD**	1.06	12.81'''	1.35	1.38	1.44
FE	**1**	**C19**	14.98	6.97E+08**	6.22E+11***	0.96	31.41''
FE	**1**	**SD**	2.59E-03	5005.64**	14.85	2.07	3.75
FE	**0**	**C19**	0.96	0.73	0.21'	1.39	1.73
FE	**0**	**SD**	1.73	0.26	0.29	1.13	1.95
EU	**1**	**C19**	4529.72''	2.20E+08**	3.93E+11***	1.42	101.24'''
EU	**1**	**SD**	2.78E-04'	2.45E-04**	0.11	0.62	2.19
EU	**0**	**C19**	10.35''	0.14''	0.41	2.31	1.09
EU	**0**	**SD**	33.85'''	1.06	1.42	1.95	1.40

Table 2.10: Odds ratios (OR) of the binomial generalized linear model for the efficiency of five markets (World (W), North America (NA), Far East (FE), Emerging Markets (EM), and Europe (EU)), and commodity futures contracts, with crisis periods as categorical indicator variables. D=1 and D=0 is indicative of the presence or absence of a recognized crisis period. The table only includes results where, at least one combination of market, commodity, and crisis, achieved statistical significance at the 95% confidence level. Crises represented in the table are the Eurozone sovereign debt crisis (SD), Covid-19 (C19), Range of p-values: *** p<0.01, ** p<0.05, * p<0.1, ''' p<0.2, '' p<0.3, ' p<0.4

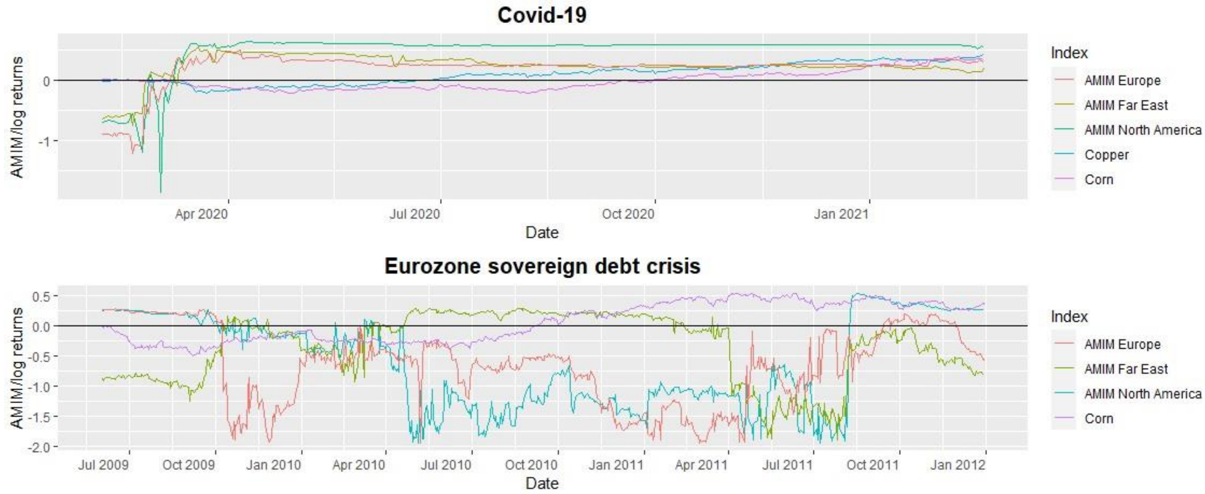

Figure 2.7: Timeseries plots displaying the evolution of AMIM for three markets (North America, Far East, and Europe) and log returns for two commodities (corn and copper) during Covid-19 and the Eurozone sovereign debt crisis. Black horizontal line indicates the separation between efficient and inefficient test results.

The results of model optimization through Akaike Information Criterion (AIC) of the binomial logistic regression including every continuous and indicator variable is found in Table 2.11. It reveals that for World, North America, Far East, and Europe, the inclusion of C19 as an indicator variable reinforces AMIM's dependence on corn prices. Especially, it is observed that the dependence of market efficiency on corn in World and Europe switches from being weakly negatively in benign periods, to strong positive dependence during C19. It is also found that the inclusion of the CS improves AMIM market efficiency's dependence on Crude Oil in the Far East market. For Emerging Market, market efficiency is not better explained by inclusion of the volatile regimes included in this study. Figure 2.7 visualizes how the model fit for crude oil and corn is improved by the inclusion of indicator variables in four markets.

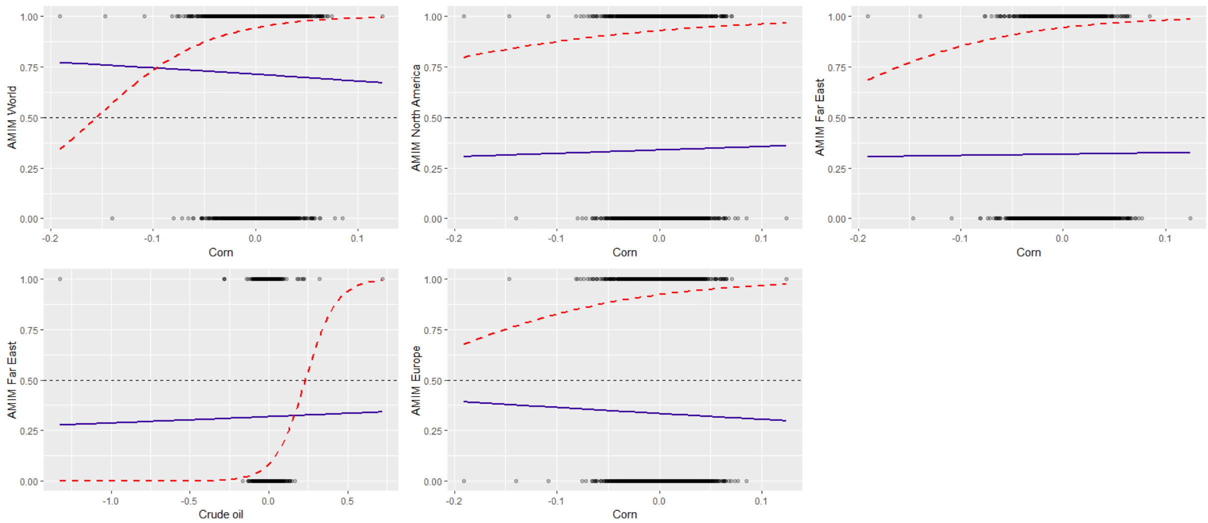

Figure 2.8: Graphs displaying the fit of the logistic models including indicator variables (red dotted line) as specified in table 2.10 and excluding an indicator variable (purple solid line) for four markets (World, North America, Far East and Europe), for two industrial input factors (crude oil and corn).

Table of odds ratios (OR)

	World	North America	Far east	Emerging markets	Europe
Constant	2.49***	0.51***	0.47***	14.11***	0.50***
Corn	0.13***	1.26	0.75		0.14***
Crude oil			0.76		
Corn (C19)	9.11E+05***	5.48E+07***	1.31E+09***		1.56E+09***
Crude oil (CS)			7329***		
Residual deviation	7243	7737	7555	2945	7694
AIC	7249	7743	7565	2947	7700

Table 2.11: AIC optimized binomial log models for five markets (World, North America, Far East, Emerging Markets, and Europe) and five commodity future contracts (Crude oil, natural gas, copper, live cattle, and corn), including indicator variables for nine volatile regimes (Dotcom-bubble (DC), Global Financial Crisis (GO), Silicon Valley banking crisis (SV), Covid-19 (C19), Russo-Ukrainian war (RU), Twin Tower terrorist attack (TT), Global oil crisis (2014-16) (GO), Chinese stock market recession (CS), Eurozone sovereign debt crisis (SD)). Range of p-values: *** p<0.01, ** p<0.05, * p<0.1, ''' p<0.2, '' p<0.3, ' p<0.4

4.5.2 Volatile regime indicator variables in multinomial regression

Table 2.12 displays relative risk ratios for anti-persistence in binary indicator variables in the multinomial logistic regression analysis. The table reveals increased polarization of ratios, as well as a considerable increase in statistical significance, for variables indicating volatile regimes. DC specifically increased the dependence of anti-persistence's in North America for all five commodities, although only with strong statistical significance for natural gas and live cattle. Similar relationships are also observed to exist in this period for live cattle in World and

Far East, although surges in crude oil future contacts prices are observed to follow a reduction in anti-persistence in Europe. Further it is also observed that during GO. anti-persistence in World, North America, and Europe moved inversely of corn future contract prices. Finally, during RU, increases in natural gas prices has been observed to follow a reduction in anti-persistence in Far East, while copper increases anti-persistence in Emerging Markets.

Table of relative risk ratios (RRR)
(Anti-persistence)

Region	D=k	Event	Live cattle	Corn	Copper	Crude Oil	Natural Gas
			RRR	RRR	RRR	RRR	RRR
W	1	DC	1.09E+31**	855.2	2.5	5.0E-6'''	0.7
W	1	RU	0.0	0.2	6.48E+10'''	6.6	25.1
W	1	GO	676.9	2.0E-12**	0.1	2.22E-03	0.2
W	1	CS	7747.7	17.8	5.21E+07	12249.0	26248.3
W	0	DC	1.3	0.5	2.4	1.4	0.3
W	0	RU	10.7	0.7	0.7	0.8	0.5
W	0	GO	5.0	12.8	3.1	1.4	0.8
W	0	CS	6.5	0.6	1.7	0.7	0.6
NA	1	DC	4.78E+16**	1.12E+8*	8854585''	4564.6'''	25842.3***
NA	1	C19	6.8	1002730	38.9	2.1	2.2
NA	1	RU	4510.7	0	2.06E+8**	0.1	0.6
NA	1	GO	928373.5'''	1.0E-6**	0.01	0.1	0.3
NA	0	DC	3.1	0.04'''	0.1'	0.7	0.3''
NA	0	C19	7.9	0.1''	0.2	0.9	0.8
NA	0	RU	6.9	0.1''	0.1'	1.1	0.8
NA	0	GO	2.1	0.3	0.3	1.3	0.9
FE	1	DC	1.53E+26***	3.92E+8'''	0.003	0.02	60.6'
FE	1	RU	5.19E-11'	2.49E+8'''	6.0E-8'''	0.1	2.0E-4**
FE	0	DC	0.2	3.7	0.1'	0.9	0.4
FE	0	RU	1.8	3.6	0.1	0.8	1.7
EM	1	RU	10359.2	5.09E+11''	1.69E+22**	0.3	73.4
EM	1	SD	2.43E+07	0.0	2027.2	852.2	0.8
EM	0	RU	2775.8'''	5.0	9.4	28.2''	1.5
EM	0	SD	939.1''	57.5	73.4	20.4''	2.5
EU	1	DC	1.28E+13'''	2.5	421.7	7.0E-7**	0.1
EU	1	GO	0.6	1.0E-8**	0.2	4.7	1.5
EU	0	DC	1.7	0.1	1.7	1.0	0.7
EU	0	GO	4.5	0.8	2.3	0.5	0.6

Table 2.12: Relative risk ratios (RRR) for anti-persistence from the multinomial generalized linear model for five markets (World (W), North America (NA), Far East (FE), Emerging Markets (EM), and Europe (EU)), and commodity futures contracts with crisis periods as categorical indicator variables. D=1 and D=0 is indicative of the presence or absence of a recognized crisis period. Crises represented in the table are the Eurozone sovereign debt crisis (SD), the Russo-Ukrainian war (RU), the Global Financial Crisis (GFC), Covid-19 (C19), the Chinese stock market recession (CS), and the Global oil crisis (GO). Range of p-values: *** p<0.01, ** p<0.05, * p<0.1, ''' p<0.2, '' p<0.3, ' p<0.4

Table 2.13 displays relative risk ratios for persistence in dichotomous indicator variables in the multinomial logistic regression analysis. The table reveals increased polarization of ratios, as well as a considerable increase in statistical significance, for variables indicating volatile regimes. Increases in natural gas future contract prices followed a reduction in persistence in Far East during RU, but increased persistence in World during both RU and the CS. Increases in the price of live cattle future contracts follows an increase in the prevalence of persistence in Emerging Markets during SD. However, corn, another agricultural commodity, has an inverse relationship to persistence in North America during C19. It can also be noted that an increase in copper prices often increased the prevalence of persistence in every market for various volatile regimes, although none are statistically significant.

Table of relative risk ratios (RRR)
(Persistence)

Region	D=k	Event	Live cattle	Corn	Copper	Crude Oil	Natural Gas
			RRR	RRR	RRR	RRR	RRR
W	**1**	**DC**	4.6	4.6'	0.53	2.0	2.5''
W	**1**	**RU**	121.2	21.3	3857''	2.5	331.1**
W	**1**	**GO**	0.9	1.4	387.5'	0.1	2.9
W	**1**	**CS**	826.1	4302	995492''	2321'''	297630**
W	**0**	**DC**	0.6	3.7	3.6	1.9	3.3'''
W	**0**	**RU**	0.6	4.2'	2.0	2.0	1.4
W	**0**	**GO**	0.6	5.1'	1.9	2.5''	2.5''
W	**0**	**CS**	0.5	3.9'	2.1	1.7	2.1'
NA	**1**	**DC**	0.9	13.8	3.3	74.6'	6.2
NA	**1**	**C19**	1.0E-5'	2.3E-20**	7.82E-06	2.3	1.1
NA	**1**	**RU**	0.02	0.03	887.7	2.5	5.9
NA	**1**	**GO**	46.8	0.2	140009*	7.9	1.5
NA	**0**	**DC**	3.9	0.9	10.4''	4.4'''	3.0''
NA	**0**	**C19**	6.4	1.4	11.7'''	6.1'''	3.3'''
NA	**0**	**RU**	4.2	1.3	8.1''	5.0'''	3.0''
NA	**0**	**GO**	2.8	1.3	4.8	4.6'''	3.5'''
FE	**1**	**DC**	193445'	47.7	960.4	2.3	2.6
FE	**1**	**RU**	0.02	17.0	53.1	0.9	3.0E-3**
FE	**0**	**DC**	0.2	5.7''	2.9	0.9	0.5
FE	**0**	**RU**	0.3	5.9''	3.0	0.9	1.1
EM	**1**	**RU**	3.71E-07	101.8	3.08E+11'	29.5	3.2
EM	**1**	**SD**	2.95E+11**	0.1	45563'	7526'''	0.6
EM	**0**	**RU**	5.0	0.2	12.2**	1.5	5.5'''
EM	**0**	**SD**	0.7	0.3	12.2*	1.2	6.7'''
EU	**1**	**DC**	129.0	4.0	24678''	4.5	0.3
EU	**1**	**GO**	0.3	37.8	0.8	0.2	0.04''
EU	**0**	**DC**	1.7	0.1''	8.5'''	1.6	2.0'
EU	**0**	**GO**	2.5	0.1'''	13.1'''	2.0	2.5''

Table 2.13: Relative risk ratios (RRR) for persistence from the multinomial generalized linear model for five markets (World (W), North America (NA), Far East (FE), Emerging Markets (EM), and Europe (EU)), and commodity futures contracts with crisis periods as categorical indicator variables. D=1 and D=0 is indicative of the presence or absence of a recognized crisis period. Crises represented in the table are the Eurozone sovereign debt crisis (SD), the Russo-Ukrainian war (RU), the Global Financial Crisis (GFC), Covid-19 (C19), the Chinese stock market recession (CS), and the Global oil crisis (GO). Range of p-values: *** p<0.01, ** p<0.05, * p<0.1, ''' p<0.2, '' p<0.3, ' p<0.4

The results of model optimization through Akaike Information Criterion (AIC) of the multinomial logistic regression including every continuous and indicator variable is found in Table 2.14. The table finds that inclusion of indicator variables improves the overall quality of the optimized models. For the World index, increasing crude oil future contract prices were accompanied by a reduction of inefficiencies during GO, and increased the prevalence of inefficiencies during CS. Price increases in corn future contracts reduces inefficiencies with some significance during GO, and specifically for anti-persistence during the GO. Natural gas future price increases the prevalence of inefficiencies during DC and for RU, although only persistence in the latter is statistically significant. For live cattle during DC, the model indicates that price increases follow anti-persistence.

For the North America index, anti-persistence is observed to accompany increases in live cattle and natural gas future contract prices for DC. Market inefficiencies are observed to increase with increasing prices in future contracts for copper during RU and corn during C19, although only statistically significant for anti-persistence and persistence respectively.
For the Far East index, market efficiencies are reduced following price increases in corn during C19 and natural gas during RU, while it increases with price increases in live cattle during DC. For the Emerging Markets index, market efficiencies increase when prices in live cattle and natural gas increases, for SD and RU, respectively.

For the Europe index, price increases in crude oil future contracts followed a reduction in anti-persistence and an increase in persistence during CS. Similar behaviour is spotted for DC, however here there is no effect on persistence, indicating that crude oil uncertainty improved market efficiency in Europe. During GO, crude oil price surges followed an increase in anti-persistence and a reduction in persistence. Corn future contracts display positive relative risk ratios for anti-persistence during CS, and for persistence during C19, but during GO were high for persistence and low for anti-persistence. It is also seen that live cattle future contracts prices had high relative risk ratios for persistence and low for anti-persistence, but with low statistical significance.

Table of relative risk ratios (RRR)

	World		North America		Far east		Emerging markets		Europe	
	1	**3**	**1**	**3**	**1**	**3**	**1**	**3**	**1**	**3**
Constant	0.11***	1.68***	0.30***	0.44***	0.22***	1.22***	0.22***	7.90***	0.17***	0.91***
Crude Oil	1.19	1.96							0.75'''	2.49'
Natural gas	0.15'	1.73	0.34'	2.65''	1.08	0.91				
Live cattle	1.63	0.57	4.50	2.68	0.24	0.18	635.7''	0.46	19.7	0.96
Corn	50.9'	18.3'''	0.10''	0.20'	43.5''	26.5*			1.24	0.03**
Copper			0.15'	8.81''			4.29	8.22		
Live cattle (SD)							2055	4.65E+11**	1.57E-07	699426'''
Crude oil (CS)									6.60E-10	19759
Corn (CS)									2.19E+20**	0.03
Crude oil (GO)	3.81E-05''	9.36E-04*							5840'''	1.92E-03*
Corn (GO)	6.45E-14**	0.10							7.47E-13***	6162'''
Corn (C19)			408.3	2.12E+14***	2.89E-04''	6.30E-04*			18.8	7.76E+08**
Crude Oil (DC)									1.29E-06**	1.85
Crude oil (CS)	7.39E+08'''	9.71E+05**								
Crude Oil (GO)										
Natural gas (RU)	129.5	156.9**			3.92E-04*	2.29E-03**				
Corn (GFC)	5.11E-05	1.49E-05**								
Natural Gas (DC)	21243'''	0.05''	57746***	2.48						
Live cattle (DC)	8.49E+28**	20.3	1.51E+15*	0.23	1.78E+25**	2.44E+05'				
Copper (RU)			2.99E+09**	100.0'			8.33E+20**	3.16E+10**		
Residual deviation	9649		11702		11142		5542		11126	
AIC	9697		11738		11170		5562		11170	

Table 2.14: AIC optimized multinomial logistic models for five markets (World, North America, Far East, Emerging Markets, and Europe), five commodity future contracts (Crude oil, natural gas, copper, live cattle, and corn), and indicator variables for nine volatile regimes (Dotcom-bubble (DC), Global Financial Crisis (GFC), Silicon Valley banking crisis (SV), Covid-19 (C19), Russo-Ukrainian war (RU), Twin Tower terrorist attack (TT), Global oil crisis (2014-16) (GO), Chinese stock market recession (CS), Eurozone sovereign debt crisis (SD)). Range of p-values: *** p<0.01, ** p<0.05, * p<0.1, ''' p<0.2, '' p<0.3, ' p<0.4

5 Conclusion

Over the past decades market efficiency has proven to be a complex and dynamic concept. For many investors, understanding the impact of various market signals and anticipating the reaction of other market participants is crucial in their portfolio management. The purpose of this paper has been to investigate how market efficiency is impacted by changes in the commodity market. By deduction, it is inferable that since a commodity's price is inflated by scarcity; and scarcity in industrial commodities causes information asymmetry about how to obtain materials; that therefore price inflation in commodities will impact market efficiency. Additionally, it was examined whether scarcity in resources is tied to external shocks and events which could disrupt financial markets and cause inefficiencies.

The analysis reveals the presence of regional disparities in market efficiency within global equity markets. We observe frequent synchronized movements across all five regional indices, implying potential spill-over effects and interdependence among markets in terms of market efficiency. In terms of normality, the deviation of the Emerging Markets index from the four alternate indices may be contributed to its flexible composition. Moreover, our findings indicate that World and Emerging Markets are more prone to inefficiency, while North America exhibits a higher frequency of efficiency. Discrepancies between Hurst exponents and AMIM statistics in Far East and Europe, suggests persistence of long memory in these markets. Overall, equity markets demonstrate a higher frequency of persistence compared to anti-persistence.

We find that the relationship between market efficiency and uncertainty regarding industrial input factors is complex. While statistical evidence does not strongly support the hypothesis that uncertainty in availability of industrial input factors affect market efficiency in the selected markets and commodities, there are indications that specific commodity future contracts and market efficiency are influenced by volatile regimes.

During Covid-19, the relationship between corn and copper had a strong, reinforcing impact on the AMIM in North America, Far East, and Europe indices. Similarly, during the Eurozone sovereign debt crisis market efficiency in North America, Far East, and Europe had an increased dependency on copper. Model optimizations employing Akaike information criterion solidify the effect of Covid-19 on the relationship between corn and AMIM-estimated market efficiency in North America, Europe, and Far East.

Multinomial logistic regression results reveal that Hurst exponents depend on various volatile regimes and underlying commodities without an easily predicted structure. It provides evidence that certain commodity futures contracts can have efficiency-strengthening effects, wherein increases in commodity uncertainty seemingly enhances market efficiency. Noteworthy examples include corn in the World and Far East markets during the Global Financial Crisis and Covid-19, respectively, as well as natural gas in the Far East market during the Russo-

Ukrainian war. Conversely, we find the opposite effect for copper in the Far East market during the Dotcom bubble period, copper in Emerging Markets during the Russo-Ukrainian war, and crude oil in the World market during the Chinese stock market turbulence of 2015-16. In these instances, uncertainty in commodities followed an increase in volatility and trend predictability in equity markets. The contrasting findings of the multinomial logistic regression underscore that uncertainty in commodities alone does not fully explain variations in equity markets under volatile regimes.

The added complexity in the AIC optimized models for the Hurst exponents, compared to the AMIM-estimates, suggests the existence of long memory in global equity markets that are statistically dependent on changes in commodity futures contract prices. Furthermore, the results indicate a connection between the Hurst exponents of the North American, Far East, and European equity markets and corn futures during Covid-19. This is consistent with the findings of AMIM, hence reinforcing the conclusion that these variables are connected.

In conclusion, we find that market efficiency is not impacted by commodities during benign periods. Presumably, this could be because benign periods are characterized by little variation and low information asymmetry. Therefore, slowly increasing scarcity impacts valuations appropriately. However, improved fit and polarized ratios strengthen the hypothesis that there exists a relationship between commodity futures contracts and market efficiency under volatile regimes. This is as expected because volatile regimes are characterized by disruption and uncertainty for both commodity and equity markets. In addition, we are successful in ascertaining the existence of long memory in several global equity markets for various commodity types and volatile regimes. This indicates that under volatile regimes where similar events occur that affect the market, investors will remember and look to the last time the event occurred, and potentially sell before the market crashes, causing self-fulfilling prophecies.

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Appendix

Appendix A

Equation (13) includes the Market Inefficiency Magnitude (MIM). MIM is defined as:

$$MIM_t = \frac{\sum_{j=1}^{q} \left| \widetilde{\beta_{J,t}}^{standard} \right|}{1 + \sum_{j=1}^{q} \left| \widetilde{\beta_{J,t}}^{standard} \right|} \tag{13}$$

Where MIM represents the Market Inefficiency Magnitude at time t, and $\widetilde{\beta_{J,t}}^{standard}$ is the standardized beta coefficient obtained from an autoregressive model with q number of lags AR(q), as described in Equation (1).

Appendix B

Chapter 4.5 introduces indicator variables for various volatile regimes. Time intervals and number of observations included for each indicator variable is included in Table 2.15.

Table of indicator variables

Description	From	To	Observations
DotCom bubble	2000 March 13th	2001 September 10th	388
Twin Tower terrorist attack	2001 September 11th	2001 October 11th	21
Global Finacial Crisis	2007 December 3rd	2009 June 1st	390
Eurozone sovereign debt crisis	2009 June 2nd	2011 December 30th	673
Global oil market turbulence	2014 January 1st	2016 December 30th	782
Chinese stock market turbulence	2015 June 12th	2016 February 1st	166
Covid-19	2020 February 7th	2021 February 19th	270
Russo-Ukrainian war	2022 February 24th	2023 March 23rd	281
Silicon Valley banking crisis	2023 March 10th	2023 March 23rd	10

Table 2.15: The crisis periods investigated in the study, with specification of time intervals and number of observations.