Introducing a Teaching Technique for Reducing Students' Mistakes in Simplifying Algebraic Expressions

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Abstract: The present study investigates the effect of the separator lines on the learning of 8th grade students in simplifying algebraic expressions with parenthesis. An experimental study was designed to achieve this goal involving 60 girl students in 8th Grade (13 and 14 years old) randomly selected and assigned to two experimental and control groups. After taking the pre-test, both groups were taught by one teacher. The control group was led as usual and the experimental group was taught by using the separator lines. As a result of the covid-19 disease, students were taught online using WhatsApp. In the end, a post-test was carried out for both groups. Data was also collected using WhatsApp. For the analysis of the data, a Covariance test was conducted. The results showed positive effects of separator lines on the student's performance, as well as reducing their mistakes when working with parenthesis.

Keywords: Simplification, Algebraic Expressions, Students Mistakes, Separator Lines, WhatsApp.

INTRODUCTION

Algebra is an essential component of the mathematics curriculum because it is the language of generalization by which we describe patterns; algebra is a language for expressing the relationship between quantities; and without algebra, we cannot understand many concepts in basic sciences such as chemistry, physics, and geology (Kaput, 1995). Simplifying algebraic expression is one of the most important topics of algebra in middle school math textbooks (Hibi & Assadi, 2022). According to Bush (2011), simplifying algebraic expressions is a prerequisite for many mathematical subjects in higher education. Because of this, students who do not learn how to simplify algebraic expressions will have difficulty solving equations. Therefore, students' mistakes in this topic prevent them from learning other topics in math textbooks, such as equations, solving verbal problems, or calculating variable values. As a result, it is important to correct and reduce students' mistakes. Mamba (2012) and Seng (2010) found that students do poorly when simplifying algebraic expressions and make errors and misconceptions1, in using + and - symbols, parenthesis, power, addition, and subtraction of like sentences. Examples of these mistakes are given in Figure 1.

$$+3(6a-7)=-18a+21$$

$$4+5y^2=4+25y^2$$

$$4a^2+3a^2=7a^2=14a$$

$$5a+5b=10ab$$

$$2a\times 3a=6a$$

$$-6a+3a=-9a \ or \ +3a$$

$$5ab-6+4ba+7=9ab-13 \ or \ 9ab-1$$

$$2(3a+2)+3+4a=6a+4+6+8a$$

Figure1: Examples of mistakes in simplifying algebraic expressions

Many math teachers had encountered these types of mistakes in their classroom experiences. She has observed the examples above numerous times in her classroom. There are three reasons for choosing the topic of simplifying algebraic expressions in the current study. Firstly, simplifying algebraic expressions is a topic that students face at the beginning of the middle school program. Secondly simplifying algebraic expressions is one of the most important topics at all levels of the middle school mathematics curriculum in Iran. This is dealt with in the chapter on algebra in the 7th grade mathematics books. In the 8th grade, this topic is discussed in a separate section, and the 9th grade, simplifying expressions is a prerequisite for solving equations. Therefore, if the students do not learn to simplify algebraic expressions, they will find it difficult to solve equations or systems of equations later. Thirdly, the content analysis of Iranian middle school mathematics

[1] . mistakes that occur due to disruption in students' conceptual and procedural understanding

textbooks (Research and Educational Planning Organization, 2020), show that simplifying algebraic expressions has the central role.

In Table 1, the relationships displayed are according to the content of the books of this educational program. As shown in the above table, the study of all these topics requires learning the topic of simplifying algebraic expressions. For example, for solving a simple inequality, the student may need to be able to solve the linear equation and this solution usually requires simplifying algebraic expressions on both sides of the equation.

Algebraic topics	Prerequisite
Number patterns, the Nth sentence, the idea of a variable	Arithmetic in school elementary
Algebraic expressions, monomials, and like sentences	Number patterns
Simplification of algebraic expressions	Like sentences, monomials
Finding the numerical value of algebraic expressions	Simplification of algebraic expressions, variable
Linear Equations	Simplification of algebraic expressions
Identities	Simplification of algebraic expressions
Factorization	Identities, Simplification of algebraic expressions
Inequalities	Linear Equations, Simplification of algebraic expressions

Table 1: Algebraic topics in middle school and their prerequisites

According to Fleisch (2008), some algebraic errors made by students are the result of improper teaching methods. A teacher's main goal is to help students understand and use mathematical concepts and procedures correctly. Research in other countries has recently focused on the professional knowledge of teachers and found that one of the most important predictors of student success is the knowledge and teaching method of teachers (Hill, Rowan & Ball, 2005). To improve learning, teachers need to use new teaching methods. In most classrooms, teachers still use traditional methods and are unmotivated to teach differently. Norton and Irvin (2007) demonstrated that traditional methods are not useful, thus other teaching methods may be able to improve students' learning. Perhaps teachers' inappropriate teaching methods contribute to students' poor performance in simplifying algebraic expressions. The above content created the question in the minds of the authors how we can improve the performance of students on this topic and reduce their mistakes?

Having found that students' mistakes in simplifying algebraic expressions are an obstacle to learning mathematics in higher grades, and on the other hand, the research conducted in this field has focused on students' mistakes and their origin, without proposing a way to reduce them. Therefore, the authors of the current research suggest separating algebraic expressions by separator lines to reduce students' mistakes. In an algebraic expression, separating the sentences involves putting a slant line (/) before each positive and negative. Positive and negative are not considered inside the parenthesis (See formula 1).

$$3x - 4x(2 + 8x) - 7x^2 = 3x\ / {-4x}(2 + 8x) / {-7x^2} \qquad (1)$$

When we use slant lines, an algebraic expression is divided into several parts, and we know which parts to operate on. In the formula 1, the separated parts are 3x and there is no need to operate on this part since it has no parenthesis. In the next part, we have $-4x(2 + 8x)$ that there is a parenthesis in this part, so we have to remove it. To do this, we have to multiply the sentence before the parenthesis in it. As a result, the expression -4x is multiplied by 2 and +8x upon the associate property. Finally, for $-7x^2$, there is no need to operate on this part since it has no parenthesis. Therefore, separator lines divide algebraic expressions into several parts to recognize which parts should be operated on. Essentially, the separator lines indicate what should be multiplied in parenthesis and in which sentences it should be performed. So, below research question lead the current study.

- Do separator lines affect the learning of students to simplify expressions with parenthesis and reduce their mistakes?

The reason for choosing algebraic expressions with parenthesis is that most expressions in the algebra chapter of the math textbooks are in many countries. Due to this, parenthesis plays an important role in simplifying algebraic expressions, because students cannot add or subtract similar sentences until parenthesis are removed. In other words, removing parenthesis is the first step toward simplifying algebraic expressions with parenthesis. According to Seng (2010) when deleting parenthesis, students make mistakes, such as, they do not recognize what and how should multiply in parenthesis (2), in which sentences they should multiply (1), or to which sentence they should continue multiplying (3). Examples of these mistakes are given in Figure 2.

$$3x - 4x(2 + 8x) - 7x^2 = 3x - 8x + 8x - 7x^2 = 3x - 7x^2$$

$$3x - 4x(2 + 8x) - 7x^2 = 6x - 32x^2 - 7x^2 = 6x - 39x^2$$

$$3x - 4x(2 + 8x) - 7x^2 = 3x - 8x - 32x^2 - 28x^3 = -5x - 32x^2 - 28x^3$$

Figure 2: examples of mistakes in simplifying algebraic expressions with parenthesis

LITERATURE REVIEW

So far, no research has been done on teaching algebraic expressions differently from the textbook, and most research has focused on students' misconceptions of algebra. In this part, we discuss some challenges in learning algebra, particularly simplifying algebraic expressions, as well as the role of teaching methods in education. Mamba (2012) pointed out that understanding and working with algebraic expressions by accepted rules, procedures, and algorithms create challenges for students because of using letters. Students are faced with a lot of mistakes in the multiplication of algebraic expressions and equations, because of the lack of understanding of symbols and letters (Hall, 2002b). Some students are not even able to read algebraic expressions. Many students do not understand the idea of letters as a number, they tend to interpret the letters as a particular number and believe that different letters must necessarily represent different numbers (Seng, 2010). Using letters has caused students to find algebra challenging, according to Booker (1987). Students are unsuccessful in algebraic topics such as recognizing sentences, decomposing, simplifying, and calculating the numerical value of algebraic expressions, and for this reason, they have difficulty in solving verbal problems, forming, and solving equations.

A mistake is a simple lapse of care or concentration which almost everyone makes at least occasionally (Marpa, 2019). Misconceptions are one of the most important challenges in learning algebra .Russell, Odwyer, and Miranda (2009) believe the deep misconceptions that are created in algebraic concepts are not accidental and they lead to mistakes made by students. Sisman and Aksu (2016) while examining the mistakes and misconceptions of students presented them as evidence of a lack of basic concepts and lack of learning. Welder (2012) pointed out that student mistakes in algebra may be because the knowledge of students is not complete or is not understood well. Misconceptions of students have roots in the mental structures of individuals, for this reason, knowing how the effects of mental schemas in creating misconceptions can improve learning. Booth, Barbieri, Eyer, and Pare-Blagoev (2014) identified six mistakes categories when students solved algebraic Questions that include: Misconception of the variable concept; Use of the negative sign; False solving equation and inequality; Incorrect application of features such as displacement law; Fractions; and Do not observe the order of operation. Jupri, Drijvers, and Van den (2014) divided the mistakes in the initial algebra into seven categories which are applying an arithmetic operation, understanding the variable concept, understanding the concept of algebraic expressions, mathematization which goes back to the mistakes in describing the real situation mathematically, understanding the meaning of the sign of equality involves mistakes about the meaning of this sign in arithmetic and algebra, understanding the concept of algebraic expressions, and finally understanding the meaning of the sign of equality that in arithmetic this sign requires a numerical answer, but in algebra, it may require an algebraic expression.

Simplifying an algebraic expression means calculating a simpler and shorter algebraic equation than the original expression (Owusu, 2015). Students must understand that the question and answer of the algebraic expression can be variables and that the letter in the answer of an algebraic expression is a generalized number, which means that the letter can be any number (Hall, 2002b). Kieran (1992) observed that many people see a letter as a generalized number hardly, and this may be because the letter in the solution of the equation is only one specific number. In Malaysia, which calculated the frequency of student mistakes when simplifying algebraic expressions, was observed that the mistake in the order of the operation and the negative sign have the most frequent among the mistakes (Seng, 2010). Reyes (2012) finds that the common mistake of most students in simplifying is that they equalize algebraic expressions with zero. In other words, students know algebraic expressions are incomplete. For example, students cannot accept the expression of $6 + 8x$ as a solution to a question, (See formula 2).

$$6 + 5x + 3x = 6 + 8x \Rightarrow 6 + 8x = 0 \Rightarrow x = \frac{-6}{8} \tag{2}$$

This mistake may be due to the lack of understanding of the difference between the algebraic expression and the equation. Other mistakes are also made by students in simplifying algebraic expressions which we quote them below.

- Students tend to add and subtract similar sentences while simplifying algebraic expressions, for example; 5x+3=8x. The reason for this mistake may be that students collect non- similarity sentences based on the concept of the + sign, to write a reply.
- Sometimes a student writes 3x-2x = 1, whose reason may be that the student tends to deal with numbers and letters separately; hence 3-2=1 and x-x=0 (Matz, 1980, cited in Gunawardenna, 2011).
- In simplifying the sentence, a×a to the form 2a and a + a to the form a2, the student may be wondering about the rules related to collecting and multiplying and remind them (Owusu, 2015), But the other reason for this mistake is the application of correct rules in inappropriate situations (Matz, 1980, cited in Gunawardenna, 2011).

The reason to make mistakes in simplifying algebraic expressions is that mathematics teachers do not care about the origin of these mistakes (Guler & Celik, 2016). Mathematics teachers not only need to have arithmetical skills, but also must have the ability to think, select aimful training strategies, and curriculum algebraic ideas to help students move from arithmetic to algebra (Anne Hayata, 2012). In most classrooms, traditional methods dominate, and these traditional approaches have failed to teach algebra. (Norton & Irvin, 2007). Studies repeatedly highlight the role of teaching as an important variable, which affects learners' performance in mathematics as well as an effective factor in causing student mistakes (Shulman, 1987). It is reasonable to say that traditional methods do not provide meaningful educational options for addressing learners' mistakes in mathematics, and particular algebra (Owusu, 2015). Creating a new educational method that improves students learning (Doerr, 2004).

METHOD

The quantitative method was found to be appropriate for this study. To evaluate the effectiveness of the designed method in simplifying algebraic expressions compared to traditional methods, an experimental method was employed with two groups of control and experiment, as well as pre-test and post-test. The innovation of this work is in providing an educational method for simplifying algebraic expressions that reduces students' mistakes. Since other research has focused on students' mistakes and their causes. Furthermore, WhatsApp was used to collect data and teach students during this research. Due to its features such as the possibility of two-way communication, the ability to form groups, and the ability to share files, the WhatsApp environment is convenient for teaching (Barhoumi, 2015). Users can send text, audio, and video messages with this messenger. According to the study by Rafiepour, Abdolahpour, and Farsani (2021), the WhatsApp environment can be used to teach mathematical concepts.

As the statistical population, we selected eighth-grade students in the academic year 2021-2022 who were 13 and 14 years old. The sample consists of students from a girls' school in Kerman. This school had 150 eighth-graders, 60 of whom were randomly selected and placed in control and experimental groups. The Covid-19 pandemic caused education in villages and city schools to be virtual, making it difficult to reach students from other schools.

To determine the effects of separator lines on learning to simplify algebraic expressions, an experimental study was designed and conducted in which 8th grade students were divided into two experimental and one control group .60 students were randomly divided into two groups of 30 each. The teacher who teaches control and experimental groups of students has 14 years of experience teaching mathematics at middle school. In 6 sessions of 45minutes, she taught both groups how to simplify algebraic expressions. According to the experimental design, the experimental group was taught using the new method, while the control group was taught using the traditional method. Teacher in both groups recalled concepts that students had previously read and needed to simplify algebraic expressions, such as how to multiply and empower parenthesis. The teaching method was flipped classroom. In this way, the teacher sent clips of the content of each group to the students in every session of class. Students were given time to watch the clips. Then, the questions of the students were answered through a conversation. An example of a flip classroom assignment and communication about that come in appendix (figure 3). In the pre-test and post-test Students sent photos of their answers on WhatsApp.

The data were collected through tests designed by the researchers. First, In the WhatsApp environment, all students took a pre-test related to simplifying algebraic expressions taught in the 7th grade. In the pre-test, 5 problems about simplifying algebraic expressions from the 7th math book were raised. In the classroom, students worked individually. If a student asked a question, everyone could answer it. After teaching simplifying expressions, In the WhatsApp environment, the students again completed a post-test. Based on the training content, five questions were raised about simplifying algebraic expressions in the post-test. Five experienced math teachers evaluated and approved the validity of the tests. Cronbach's alpha was used to determine the reliability of the test, and the result was α=0.79. To analyze the quantitative data for investigating the effect of the method on students we carried out an analysis of covariance (ANCOVA) using SPSS 20.

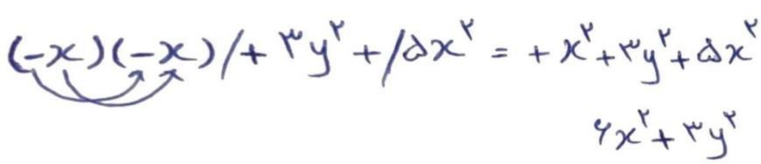
RESULTS

The mean and standard deviation of students' scores in the control and experimental groups for the pre-test and post-test are displayed in Table 2. As shown in the above table, the difference in the mean post-test and pre-test scores is much greater for the experimental group than for the control group.

Group	Test stage	Number	Mean	Std. Deviation
Experimental	pre-test	30	16.46	3.60
	post-test	30	18.06	2.90
Control	pre-test	30	16.93	3.74
	post-test	30	17.31	2.83

Table 2: Descriptive Statistics

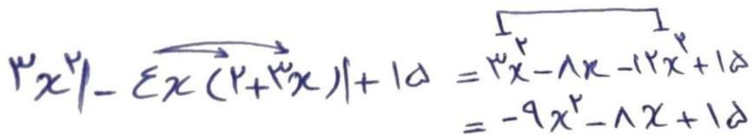
For checking the research hypothesis, we used to analyze covariance (ANCOVA) with the post-test score as the independent variable, pre-test score as covariate, and group as a fixed factor. We first need to check the main assumptions needed. Figures 3, 4, and 5 show examples of the answers given by the experimental group's students.



$$(-x)(-x)/+3y^2/+5x^2$$
$$= x^2 + 3y^2$$
$$+ 5x^2$$
$$= 6x^2 + 3y^2$$

Figure 4: The answer of student number 10 in the experimental group at the post-test

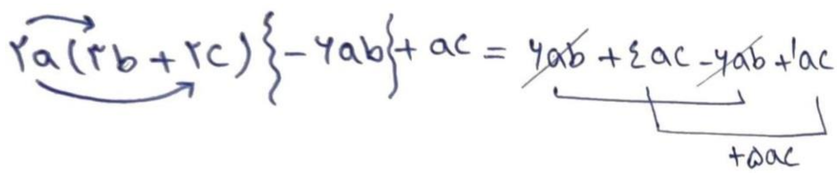
As shown in Figure 4, the student has drawn a slant line before the positive sign to separate the sentences of the algebraic expression. His operation has been performed correctly on the first part, which has parenthesis, of the algebraic expression that is divided into three parts.



$$3x^2/-4x(2+3x)/+15 = 3x^2 - 8x - 12x^2 + 15 = -9x^2 - 8x + 15$$

Figure 5: The answer of student number 14 in the experimental group at the post-test

This algebraic expression is divided into three parts because of the slant line used by the student in Figure 5. The student correctly recognized that the second part requires an operation and correctly multiplied -4x in parenthesis.



$$2a(3b+2c)/-6ab/+ac = 6ab + 4ac - 6ab + ac = 5ac$$

Figure 6: The answer of student number 26 in the experimental group at the post-test

6 shows the student divided the algebraic expression into three parts by drawing a line and correctly recognized that the first part required multiplying 2a by two sentences inside the parenthesis. The research hypothesis was then tested.

Research Hypothesis: Compared to the traditional method, the explained method has a greater impact on students' learning of simplifying algebraic expressions with parenthesis in 8th grade. In this paper Kolmogorov-Smirnov test was used for checking the normality of data and obtained $F = 1.39$ and $p = 0.083 > 0.05$; also the Levene test was used for checking Homogeneity (equality) of variances and obtained $F\ (1, 58) = 0.63$ and$p = 0.281 > 0.05$. Furthermore, the sampling method in this study guarantees Random sampling. The random sampling, normality, and homogeneity of variances allowed the necessary assumptions for an analysis of covariance to be made, thus this test was conducted. Table 3 shows the results of the analysis of covariance (ANCOVA).

According to Table 3, $F = 4.34$ and $p = 0.02 < 0.05$ for groups, i.e. after removing the effect of the covariate, there is a significant difference between the mean post-test scores of the two groups (experiment and control). So, the hypothesis of the research i.e. “effect on learning in middle school students when our method is used for teaching simplification of algebraic expressions is greater than when using traditional methods” is confirmed.

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Pre-test	292.341	1	292.341	89.72	.000
group	14.820	1	14.820	4.34	.02
Error	185.711	57	3.258		
Total	491.828	60			

Table 3: Results of analysis of covariance (ANCOVA)

DISCUSSION AND CONCLUSIONS

According to the findings of the research in Table 3, separating sentences in algebraic expressions by separator lines facilitates the learning of how to simplify algebraic expressions and reduces girl students' mistakes in performing algebraic expression calculations. The innovation of separator lines explains this positive effect. By separating algebraic expressions with slant lines, students were able to perform algebraic calculations correctly. See, for example, figures 3 and 2. You can see the role of separator lines in these figures. It seems that these lines caused the students to pause, or reflect, to focus on the calculation of the expression that has priority over other parts, leading to more accurate calculations and fewer errors. In general, Separator lines for separating algebraic expressions have a positive effect on solving algebraic expressions and also on reducing students' errors. The findings of this study are consistent with the results of Seng (2010) and Sisman and Aksu (2016). In their research, these people stated that it could be effective in improving algebra learning by students. Although these researchers did not examine the effects of a new method in their research, they claimed that new teaching methods can improve algebra learning in students. New methods of teaching can enhance students' problem-solving skills and improve their academic performance (Seng, 2010). Sisman and Aksu (2016) point out that despite recent advances in learning theories and the emergence of constructivist approaches, most mathematical teaching methods are traditional and new ones are seldom utilized.

The findings of the study are related to the subject of algebra. About mathematical content, algebra occupies a very special place, both because it is a powerful tool for solving problems or modeling situations (Watson, 2016), and also because it is an essential building block for learning other types of mathematics and the various sciences (Chea & Baba, 2021). Capraro and Joffrion (2006) observed in their research that one of the obstacles to learning mathematics among middle school students was that they could not make a connection between their knowledge of arithmetic and the knowledge needed to understand algebraic concepts. When algebraic topics are discussed in middle school, students encounter many challenges and mistakes. Mistakes seem a common phenomenon among students with algebra. Mistakes have been a concern for researchers and mathematics teachers (Kshetree et al., 2021). Therefore, this study can provide a solution to these challenges and reduce mistakes.

The next topic addressed in this study was simplifying algebraic expressions with parenthesis. When simplifying algebraic expressions with parenthesis, students make more mistakes. The reason is that they don't know what to multiply in parenthesis or from which sentence to start and which to continue this multiplication. Therefore, the present study examined the effect of separator lines on the learning of 8th grade students in simplifying algebraic expressions with parenthesis. This way, clips were sent on What Sapp to two control and experimental groups, and the data was collected by pre-and post-tests that were taken on WhatsApp. The findings of the teaching of this topic by separator lines have shown its positive effect on improving 8th grade students' learning. Using this method to simplify algebraic expressions helped students make fewer mistakes while removing parenthesis. They learned how to divide algebraic expressions into parts and which to operate on. Students learned what should be multiplied in parenthesis or in which sentences the sentence before parenthesis should be multiplied. As a result of this success in algebra, students will be able to recognize the number of sentences in algebraic expressions, solve verbal problems, and solve equations and systems of equations. Hence teachers should help students to learn topics of algebra. Therefore, teachers should determine the most appropriate way to facilitate students' learning based on purposes, content, students' needs, and available resources (NCTM, 2000).

Mathematical books in all countries cover simplifying algebraic expressions (NCTM, 2000). We hope that this research has helped the international mathematics education society and the mathematics performance of students in all countries. It is suggested that those interested in mathematics education research develop new teaching methods to reduce students' misconceptions in other algebra topics such as addition and subtraction of like sentences.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

[1] Anne Hayata, C. (2012). *The development of Algebraic Reasoning in undergraduate Elementary Preservice Teachers* [Unpublished doctoral dissertation]. North Texas University.

[2] Barhoumi, C. (2015). The Effectiveness of WhatsApp Mobile Learning Activities Guided by Activity Theory on Students Knowledge Management. *Contemporary Educational Technology*, 6(3), 221-238. https://files.eric.ed.gov/fulltext/EJ1105764.pdf.

[3] Booker, G. (1987). Conceptual obstacles to the development of algebraic thinking. In J. Bergeron, N. Herscovics & C. Kieran (Eds.), *Proceedings of the eleventh international conference*

Psychology of Mathematics Education (Vol. 1, pp. 275-281). Montreal: PME. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.897.7817&rep=rep1&type=pdf#page=303

[4] Booth, J., Barbieri, C., Eyer, F. & Pare-Blagoev, J. (2014). Persistent and Pernicious Errors in Algebraic Problem Solving. *Journal of Problem Solving*, 7 (1), 10-23. doi: 10.7771/1932-6246.1161. https://docs.lib.purdue.edu/jps/vol7/iss1/3/

[5] Bush, S. B. (2011). *Analyzing common algebra-related misconceptions and errors of middle school students* [(Doctoral dissertation]. Retrieved from https://doi.org/10.18297/etd/187.

[6] Capraro, M. M. & Joffrion, H. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? *Journal of Reading Psychology*, 27 (1), 147-164. doi:10.1080/02702710600642467.

[7] Chea, S. & Baba, T. (2021). Analyzing Students Conceptions Underlying the Errors in Algebra Expression: A case Study on Cambodian Ninth Grade Students. *Journal of Science and Mathematics Education in Southeast Asia*, 44, 22-40.

[8] Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education*, 13 (1), 16-30. dio: 10.2307/748434. https://pubs.nctm.org/view/journals/jrme/13/1/article-p16.xml

[9] Doerr, H. (2004). Teacher's knowledge and the teaching of algebra. In K. Stacey, H. Chick & Kendal (Eds.), The Future of Teaching and Learning of algebra. *The 12th ICMI Study* (pp. 267-290). Boston: Kluwer. https://link.springer.com/chapter/10.1007/1-4020-8131-6_10

[10] Fleisch, B. (2008). *Primary education in crisis: Why South African Schoolchildren underachieve in reading and mathematics*. Juta Academic: Cape Town, South Africa. http://www.amazon.co.uk.

[11] Guler, M. & Celic, D. (2016). A research on future mathematics teacher's instructional explanations: the case of algebra. *Journal of Educational Research and Reviews*. 11(16), 1500-1508. 10.5897/ERR2016.2823.

[12] Gunawardenna, E. (2011). *Secondary school students' misconceptions in algebra* [doctoral dissertation, Toronto University], Canada. Retrieved from Google Scholar.

[13] Hall, R. D. G. (2002b). An analysis of thought processes during simplification of an algebraic expression. https://. People.ex.ac.uk/PErnest/pome 15/r_hall_expressions.pdf.

[14] Hibi, W. & Assadi, N. (2022). Mistakes in Simplifying Algebraic Expressions, Their Repetition Patterns and Thinking Strategies Associated with These Mistakes in Nine Grades. *International Journal of Mechanical Engineering*, 7 (1), 1585-1596.

[15] Hill, H. C., Rowan, B. & Ball, D. L. (2005). Effects of teacher's mathematical knowledge for teaching on student achievement. *Journal American Education Research*, 42(2), 371-406.doi:10.3102/00028312042002371. https://journals.sagepub.com/doi/abs/10.3102/00028312042002371

[16] Jupri, A., Drijvers, P. & Van den Heuvel-Panhuizen, M. (2014). Difficulties in initial algebra learning in Indonesia. *Journal of Mathematics Education Research*, 26(4), 683-710. dio: 10.1007/s13394-013-0097-0. https://link.springer.com/article/10.1007/s13394-013-0097-0

[17] Kaput, J. J. (1995). A research base supporting long-sentence algebra reform. In D. T. Owens, M. K. Reed & G. M. Millsaps (Eds.), Proceedings of the Seventeenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (Vol. 1, pp. 71-94). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education. https://files.eric.ed.gov/fulltext/ED389539.pdf

[18] Kieran, C. (1992). The learning and teaching of school algebra. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 390-419). New York: Macmillan. https://psycnet.apa.org/record/1992-97586-017

[19] Kshetree, M. P., Acharya, B. R., Khanal, B., Panthi, R. K., & Belbase, S. (2021). Eighth grade student's misconceptions and errors in mathematics learning in Nepal. *European Journal of Educational Research*, 10 (3), 1101-1121. https://files.eric.ed.gov/fulltext/EJ1307676.pdf

[20] Mamba, A. (2012). *Learner's errors when solving algebraic tasks: A case study of grade 12 mathematics examination papers in South Africa* [Unpublished doctoral dissertation]. Johannesburg University, South Africa.

[21] Marpa, E. p. (2019). Common Errors in Algebraic Expressions: A Quantitative-Qualitative Analysis. *International Journal on Social and Education Sciences*, 1(2), 63-72. https://files.eric.ed.gov/fulltext/EJ1264037.pdf

[22] National council of teachers of mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

[23] Norton, S. & Irvin, J. (2007). A concrete approach to teaching symbolic algebra. In J. M. Watson& K. Beswick (Eds.), *Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia.* (pp. 561-570). Hobart, Sydney: MERGA. https://files.eric.ed.gov/fulltext/ED503746.pdf#page=554

[24] Owusu, J. (2015). *The Impact of Constructivist- Based Teaching Method on Secondary School Learners Errors in Algebra (Master dissertation)*. http://hdl.handle.net/10500/19207.

[25] Research and Educational Planning Organization. (2020). *Eighth-grade math for middle school.* Iranian Textbook Publishing Company: Tehran. http://chap.sch.ir › books

[26] Research and Educational Planning Organization. (2020). *ninth-grade math for middle school*. Iranian Textbook Publishing Company: Tehran. http://chap.sch.ir › books

[27] Research and Educational Planning Organization. (2020). *Seventh-grade math for middle school*. Iranian Textbook Publishing Company: Tehran. http://chap.sch.ir › books

[28] Reyes, J. (2012). *Equal or not? An exploration of eighth-grade students' experience of Algebra*. http://digitalcommons.georgiasouthern.edu/etd/789.

[29] Rafiepour, A., Abdolahpour, K. & Farsani, D. (2022). Developing Conceptual Understanding of Irrational Numbers Based on Teaching through Activity System. *Mathematic Teaching Research Journal,* 14(1), 114-137. https://www.researchgate.net/profile/Abolfazl-Rafiepour/publication/360182024_Developing_Conceptual_Understanding_of_Irrational_Numbers_Based_on_Technology_through_Activity_System/links/626702731b747d19c2a73d88/Developing-Conceptual-Understanding-of-Irrational-Numbers-Based-on-Technology-through-Activity-System.pdf

[30] Russell, M., Odwyer, F. & Miranda, H. (2009). Diagnosing student's misconceptions in algebra: Results from an experimental pilot study. *Journal of Behavior Research Methods*. 41(2): 414-424. doi: 10.3758/BRM. 41.2. 414.

[31] Seng, L. K. (2010). An error analysis of Form 2 (Grade 7) students in simplifying algebraic expressions: A descriptive study. *Electronic Journal of Research in Education Psychology*, 8(1): 139-162. https://ojs.ual.es/ojs/index.php/EJREP/article/view/1398

[32] Shulman, L.S. (1986). Those who understand: knowledge growth in teaching. *Journal of Educational researcher*, 15 (2), 4-14. doi: 10. 3102/0013189X015002004. https://journals.sagepub.com/doi/abs/10.3102/0013189x015002004?journalCode=edra

[33] Sisman, G. T. & Aksu, M. (2016). A Study on Sixth Grade Students Misconceptions and Errors in Spatial Measurement: Length, Area, and Volume. *International Journal of Science and Mathematics Education*, 14 (7): 1293- 1319. doi:10.1007/S10763-015-9642-5. https://link.springer.com/article/10.1007/s10763-015-9642-5

[34] Watson, S. (2016). Algebra teaching around the world. *Research in mathematics education*, 18 (2), 211-218.

[35] Welder, R. M. (2012). Improve Algebra Preparation: Implications from Research on Student Misconceptions and Difficulties. *School Science and Mathematics*, 112 (4), 255-264. https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1949-8594.2012.00136.

Appendix

In one of the sessions of the experimental group, after sending the educational clip and watching it for 15 minutes by the students, the algebraic expression $2x - 3(x - 2y) + 2y$ was sent on WhatsApp and discussed with the students about it; a part of this conversation is below. Of course, in the educational clip, simplifying 5 algebraic expressions was taught using separator lines and the method of using these lines was fully explained to the students.

Teacher: Maryam, how many separator lines did you use to simplify this algebraic expression?

Maryam: 2

Teacher: Anahita, where did you draw the separator lines?

Anahita: before -3 and before $+2y$

Teacher: Nasim, dividing lines divide the algebraic expression into how many parts?

Nasim: in 3 parts

Teacher: Saba, says the three parts

Saba: 2x, $-3(x - 2y), +2y$

Teacher: Are all three parts single?

monomial expression?

Hasti: No, we have to remove the parenthesis, that is, multiply -3 by x and by $-2y$ to convert them into a monomial expression.

Teacher: Nazanin, what will be the result of this multiplication?

Nazanin: $-3x - 2y$

Teacher: Fatima, is Nazanin's answer correct?

Fatima: No, $-3x + 6y$

Teacher: Yes, Fatima's answer is right. Now identify similar terms.

Yasna: Now we have $2x - 3x + 6y + 2y$, so $2x$ and $-3x$ are similar terms and $6y$ and $2y$ are similar terms to each other.

Teacher: What is the answer of $-3x + 2x$?

Maryam: $-1x$

Teacher: That's right. What is the result of $2y + 6y$?

Bahar: $8y$

Teacher: what is the final answer?

Fatima: $-1x + 8y$

Figure 3: An example of a flip classroom assignment and communication about that